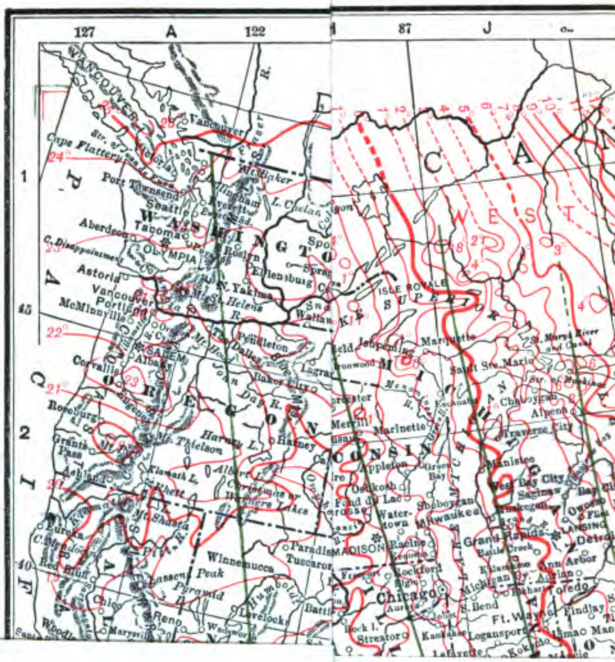

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Consulting

Author

RACTICAL SURVEYING

FOR

SURVEYORS' ASSISTANTS,
VOCATIONAL, AND HIGH SCHOOLS

BY

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of Contracting"; "Engineering as a Vocation"*

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To

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IN RECOGNITION OF THE WORK
IT IS DOING

PREFACE

Modern texts on surveying assume the student to have completed the mathematical work in high school and college in algebra, plane and solid geometry, trigonometry and analytical geometry. This book is a serious attempt on the part of the author to meet the needs of students whose mathematical preparation does not extend beyond the arithmetic given in the grade schools. It is intended therefore to be a text book in high schools, vocational schools and evening classes. It is intended also as a text for self-tutored men in the employ of surveyors who wish to become surveyors.

The author knows the need for such a book is real. He is entering upon his twenty-eighth year of practice as a civil engineer, sixteen years of which were spent in the far western states in the Rocky Mountains and along the Pacific Coast. He has served as city engineer in medium-sized cities, as a deputy county surveyor and as a United States Deputy Mineral Surveyor. He taught surveying in evening classes in schools operated *not for profit* and the lessons prepared for his students have been revised as the result of the teaching and are now placed in book form. It is hoped the book will satisfy the large number of inquirers for a simple work on the subject.

A text book should be accurate and clear. It should be well balanced, especially so when intended for men who will obtain little help from teachers. Above all it should be readable. The author in attempting to comply with these specifications trusts he has prepared a book in which the treatment is not lacking in rigor because of the assump-

tion of little previous mathematical preparation on the part of the student. It contains more than is commonly given in schools and colleges where some instruction is given in surveying as a part of the course in mathematics, usually under instructors who have had little or no practical surveying experience. The author hopes it will supplant a number of books too evidently written by teachers having little or no knowledge of difficulties arising in practical field work.

An attempt has been made to give a logical presentation of the subject suitable for men of limited mathematical preparation. Necessary instruction in mathematics is given step by step as the need is felt for it, and not before. The appendix on the essentials of algebra should serve as a useful introduction to the study of algebra. The chapter on trigonometry contains what should be properly considered the *minimum* a surveyor should possess although many surveyors do earn a living who know only a few of the formulas there given.

The author trusts he has succeeded in showing there is more to surveying than the ability to solve triangles and read a vernier, important as these items are. The insistence from the first chapter on the large part played by unescapable errors and the action of courts and juries in passing upon the work of surveyors, gives a view of the subject that is seldom obtained until after the classroom is a memory and actual work begins. Land surveying is emphasized, for land survey methods underlie all work done by instrumentmen. Enough is given of engineering surveying to help local surveyors over many hard places. The work is complete in everything with which the average surveyor will have to deal, and names and prices are given of books dealing fully with special branches of survey work. The first-class land surveyor the author has

found is a respected person in his community, and rightly so. Every reader of this book is counselled to buy and read "Boundaries and Landmarks," by A. C. Mulford, to obtain the point of view of the successful surveyor proud of his calling, the most ancient of the learned arts and sciences. To read that book gives something all young men need and should appreciate, the nearest possible equivalent to association with an elderly, well-read, kindly gentleman of broad experience in his calling.

To make the book readable and hold the interest of the student the author has dropped at times into the vernacular. Idiomatic expressions may be encountered which will trouble fastidious readers. It must be remembered that many who use this book will not have the benefit of personal contact with professors, and much that is said by professors in class would not be regarded as the best of English without considerable editing. The author has a mental twist which makes him unafraid of the split infinitive. It is such a relief from regularity.

Thousands of men have contributed to the advancement of the science and art of surveying. Many good writers in the past have handled the subject well. It is impossible to write anything not already written. The surveyor's knowledge of his work is common knowledge and it is therefore not possible to give credit to past writers except to say that in preparing this book the literature of the subject has been well searched, some of the books being yellowed with age, some so old that the *ſ* was used for *s* and some so recent that the title page bears the date of 1914. Credit has been given when only one writer is involved, matters dealt with by several writers being assumed as common knowledge. The author believes he has contributed nothing original but hopes his method of presentation may serve to give the book some popularity

and make it of real service. If the men for whom it is intended make use of the opportunity now offered them the result cannot help but be beneficial to surveying as a calling. The reproach under which surveyors suffer is that many have picked up the business in a haphazard way for lack of comprehendible texts, the majority of graduates in engineering preferring to engage in engineering work and not liking the work of the surveyor. This leaves the field then to self-tutored men working without adequate instruction, or at least instruction not as complete as it should be.

Opportunity we are told is half of life. An old engineer once gave the author a recipe for success which he said consisted of:

Opportunity......................	$\frac{1}{2}$ part
Common sense....................	$\frac{1}{3}$ part
Special training..................	$\frac{1}{6}$ part

which implies that given opportunity, success is due to a mixture of two-thirds natural ability and one-third special training. This book is intended to give the necessary technical knowledge to self-tutored young men of natural ability who obtain positions with surveyors and like the work. Hints and suggestions for the improvement of future additions and corrections of any errors which may be found will be appreciated.

THE AUTHOR.

CHICAGO, ILL., *June*, 1915.

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PRACTICAL SURVEYING

CHAPTER I

INTRODUCTORY

Gravitation is a natural force acting on all material bodies with the effect of attracting them to each other.

Terrestrial gravitation (gravity) is the operation of the law of gravitation so that all bodies within the range of its influence tend to be drawn to the center of the earth. That is, a straight line drawn from any point on the surface of the earth in the direction of its center is said to lie in the direction of gravity. In many cases gravity means simply weight, which is a measure of the force of gravity acting on a body.

A cord having a plumb-bob attached to the lower end points in the direction of gravity and is said to define a vertical line.

Differences in elevation are measured on vertical lines.

A line forming a right angle with a vertical line is defined in plane surveying as being level, or horizontal.

A point has position without length, breadth or thickness. A line has position and length without breadth or thickness. A line is terminated by points at the ends; it may be said to be composed of a number of points touching each other; it may be said to be generated by a point changing position in space, the first position being the beginning of the line, the final position of the point being the other end, the line between the two points marking the path traveled. This is illustrated every time a line is drawn with a pencil or pen.

A curved line changes direction at each point and a straight line is the shortest distance between two points.

A broken line consists of a number of short straight lines joined end to end and changing direction at the junction points. The changes in direction are angular, an angle being the amount of divergence between two lines that join or cross.

A surface has position, length and breadth without thickness. A plane surface, or plane, is perfectly flat so

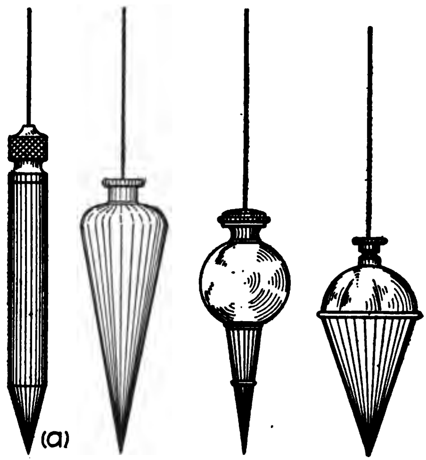


FIG. 1. Types of plumb-bobs used by surveyors.

that a straight line may be drawn to connect any two points in the plane and each point in the line will touch the plane.

In plane surveying the portion of the earth's surface measured is small in comparison with the circumference of the earth, so the curvature of the earth is safely neglected, an assumption which simplifies operations. Consequently a horizontal plane is dealt with.

In geodetic surveying the curvature of the earth is taken into account, for the operations are so extensive that boundaries between nations are determined. The use of geodetic methods is confined almost entirely to the highest grade of Government work. Large cities are surveyed by a combination of geodetic and plane surveying methods.

The methods of plane surveying are used on work of the following character:

Land surveys to determine boundaries of fields or lots and areas of same.

Canal, road and railway surveys to determine routes to be followed and the quantities of materials to be moved in forming the excavations and embankments.

Construction surveys for the purpose of setting stakes for the location of buildings, bridges and other structures; computations of quantities, etc.

Mining surveys to guide miners in driving tunnels, shafts and other workings. These are a combination of land and construction survey work.

Topographical surveys made to obtain data for maps on which are shown all natural physical characteristics, such as the shape and heights of hills, extent of lowlands, routes of rivers, streams, roads, etc., so that improvements may be planned. On extensive topographical surveys a framework may be used of lines fixed by geodetic methods, the "filling in" being done by plane surveying methods.

LAND SURVEYS

The river Nile in Egypt annually overflows its banks and deposits upon the adjacent bottom lands many tons of silt, which, being washed down from rich land on the mountains of the interior of Africa, makes the valley wonderfully fertile. This silt covers the stones and posts set for landmarks and swirling water often removes them, so that from the most ancient times the boundaries of land in Egypt have been surveyed annually. The science of geometry arose in consequence according to Diodorus and others, although some authorities claim the science of geometry was of indigenous growth, appearing in each country as the people reached a proper development. That surveying preceded geometry is indicated by the Latin name, geometria, from the Greek γεωμετρια (the measurement of land). The first geometers were land surveyors.

When a portion of a tract of land is sold the seller and buyer proceed to "establish" the corners where the angular boundary lines join and a surveyor is employed to determine

the lengths and directions of the lines between corners. A description of the corners, together with the directions and lengths of all lines, is written in the deed conveying title to the land and sometimes a map is made. Such a survey is termed "original," for by it the boundaries and marks are first described.

In an original survey the corners are established. This can never again be done. If by any chance the marks are lost surveyors are employed to "re-locate" if possible the position of the corners, but no man can "re-establish" anything once established. A surveyor, making a re-survey, can only say that he has gone over the lines to the best of his ability from information given to him and believes he has set marks as nearly as possible where the original marks were placed.

A surveyor, not being an interested party, cannot presume to put in a new corner having the force and effect of an original corner. Some time after he is gone good evidence may be found showing that the true corner lies at a considerable distance from the corner set by him. The owners being the only parties having any vital interest in the matter may destroy, if they wish, all evidences of the re-survey and abide by the boundaries originally fixed. The surveyor however should be notified so he may alter his notes. If this is not done trouble may arise in the following generation.

Sometimes, after several unsuccessful attempts to re-locate a missing corner, the owners affected agree to accept a certain point in lieu of the corner originally set. This, however, can be done only by mutual agreement duly recorded and it does not constitute a "re-establishing" of a corner, but is the "establishing" of a new corner. If the agreement is not a mutual one between all the parties at interest and no proper record is made of it, a discovery at a future time of the original corner may cause trouble. In some states corners mutually held to become fixed after the lapse of a certain number of years, but this does not act always as a final settlement. In the absence of records it is easy for some person to deny that the agreement was mutual. There may be unearthed records to show that the agreement was based on fraud, so it is better to have a

carefully drawn up agreement placed on record than to depend on the statute of limitations. The fact a student must never forget is that no surveyor, by virtue of any official position he may hold, can "fix," "re-establish," or "establish," any old corner.

A surveyor on an original survey is employed to describe the intentions of a grantor in selling and of a grantee in buying. A surveyor on a re-survey tries to retrace the lines run by the surveyor on the original survey. If the first man did his work well no troubles should develop even though nearly all the original corner marks have been destroyed. If the first surveyor made mistakes trouble cannot be avoided unless the second surveyor possesses a good knowledge of surveying, a good knowledge of court decisions affecting surveys, and plenty of common sense.

It is a physical impossibility to make surveys positively free from error, but by proceeding carefully and taking all the time that is necessary errors may be reduced to a small amount which will be satisfactory to all concerned. Good work takes time and surveyors are generally paid by the day. It follows that the accuracy of surveyors' work must be governed by the value of the land surveyed. Various authors give limits of accuracy based on experience, which the author a number of years ago reduced to the following rule:

The limit of error in land surveys may be expressed by a fraction having a numerator of 1 *and a denominator equal to one-tenth the value of the land per acre expressed in cents, the maximum limit being* $\frac{1}{500}$, *corresponding to a value of* $50 *per acre.*

When a compass is used to obtain directions and a chain in the hands of unskilled men is used to measure distances the error may be as large as 1 in 500. A very little experience will enable men to do better work, so this limit of error should never be considered satisfactory. A limit of 1 in 750 should be attempted.

The limit of error for land worth $100 per acre will be 1 in 1000 by the above rule and for land worth $1000 per acre the limit of error will be 1 in 10,000. An error of 1 in 20,000 is permissible in the business district of any large city, except possibly in the hearts of cities having a popula-

tion of more than 1,000,000 where a limit of 1 in 50,000 may be worth attempting.

An old legal maxim reads, "Monuments govern courses and courses govern distances." This arose from the fact that monuments are marks set by the grantor as visible evidences of intention. The original survey was an operation performed to obtain data for a record of intention. It was not considered that chaining was a particularly skilled avocation and untrained men usually did such work. It was not uncommon to have the owner and one of his sons act as chainmen in order to impress upon the memory the location of the corners. In Great Britain "whipping the bounds" has not entirely gone out of style, and this was as common in the early history of America as it was in older countries. On a certain day in each year, usually just after the spring planting, boys were taken around every piece of property and whipped soundly at each corner after the names of the owners whose lands the fences and corner determined, were read to them. The evidence painfully so secured was often invaluable in lawsuits years later, sometimes when the whipped boys were almost in their second childhood. In the early days chaining, therefore, was considered to be on a par with the reading of bearings for surveyors were not always well educated and many knew little, or nothing, about the variation (declination) of the needle. The effect of local attraction was generally ignored; compasses were not always well made and readings were seldom taken closer than the nearest half degree of bearing.

Nevertheless, with all the shortcomings of the angle work the bearings were given first consideration for they at least gave a close idea of direction, without which the most careful measurement is worthless.

The compass is not used today for surveys where land is valuable for readings cannot be taken closer than one-quarter of a degree with the best made compass. When skilled chainmen are employed the errors in chaining are apt to be much smaller than the errors in angle. Modern instruments are well made and angles can be read to half minutes on even the lower-priced transits, while many men have instruments graduated to read to ten seconds,

twenty seconds being the usual degree of accuracy for engineering surveys. Modern surveyors are better trained than the surveyors of preceding generations, so the instrument work leaves little to be desired in point of accuracy. Errors today are most apt to occur in the chaining, for only by experience can men be trained to measure with a chain or tape. Young surveyors should impress this fact upon employers who want to save expenses, and object to paying a good price to an experienced chainman. The picking up of some unemployed laborer to do the measuring is a fruitful cause of lawsuits.

In reading angles the plate of the instrument must be level, and the chain or tape must be level and be drawn

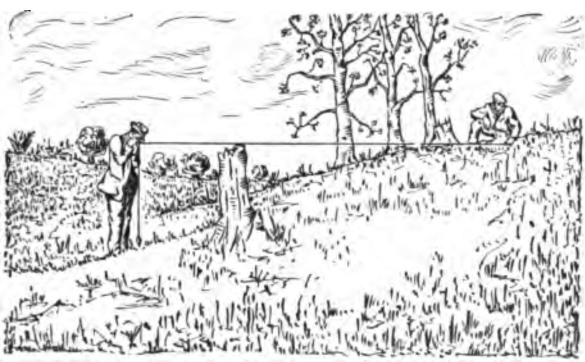


FIG. 2. Measuring on sloping ground.

taut. The chainmen are carefully "lined in" by the instrument man in order that the measured line may be a straight, not a broken, line. Each man has a plumb-bob on a cord by means of which he fixes each tape length, for the tape must be held high enough to clear all bushes, stones, etc. On level ground the tape is usually held about the height of the waist. On sloping ground one man holds his end as close as possible to the ground, the other man raising his end until the tape is level, letting the plumb-bob line slide between his fingers slowly until the point of the plumb-bob touches the ground. When the

slope is steep short measurements are taken. In measuring up or down hill in short sections like steps the additive process is used to avoid errors. For example assume it is necessary to measure in lengths of approximately 20 ft. The head chainman first pulls the tape (or chain) ahead the full length, then returns to the 20 ft. mark and sets a pin while the rear chainman holds his end on the starting point. Then the rear chainman drops his end and goes forward to the head chainman, gives him a pin in place of the one just set, holds at the 20 ft. mark while the head chainman goes to the 40 ft. mark, when the process is repeated. In this manner the full length is measured without any adding being done mentally. When the full tape length is set off the number of pins held by the rear chainman represents the number of full lengths, for the pins used temporarily for the short lengths have been exchanged each time.

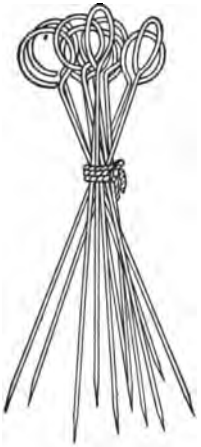


FIG. 3. Survey pins.

In commencing to measure a line a steel or iron pin is stuck in the ground, or, if the mark is plain, the pin is held by the rear chainman. When a tape length is measured the head chainman sticks a pin in the ground at the point marked by the plumb-bob. Another length is then measured, the rear chainman pulling up the pin at his end after the head chainman has set his pin. The number of pins held by the rear chainman always indicates the number of tape, or chain, lengths measured, the pin in the ground not being counted for it merely fixes the point from which to measure.

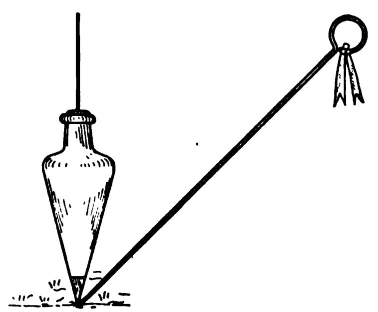


FIG. 4.

The head chainman starts with ten pins, the total number being eleven. When ten lengths have been measured the rear chainman goes forward and hands ten pins to the head chainman and they check by counting the pins, making a record of the "tally," so that the correct number of tape lengths can be told when the line is fully measured. The head chainman holds the zero end of the tape and thus the number of feet, or links, past the last full tape length may be read directly. Pins usually have a length of about one foot and are stuck in the ground slanting to one side so the plumb-bob may be held over the point where the pin enters the ground. In order to find the pins readily it is customary to tie white, or brightly colored, strips of cloth in the ring at the upper end.

Correct measuring cannot be done when weighted pins are used instead of plumb-bobs, although weighted pins were formerly used. A pin cannot be accurately dropped, no matter how well weighted.

ROUTE SURVEYS

Route surveys for roads, ditches, railways, etc., are marked by stakes driven in the ground at each tape length, the distances between stakes 100 ft. apart being termed "stations," the numbering of the stations proceeding from zero. Each stake being numbered, the number on any stake multiplied by 100 gives the distance in feet from Station 0, the starting point.

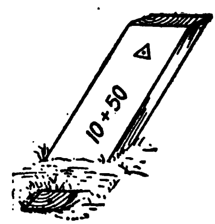


FIG. 5. Hub and witness stake.

In land surveys only the distance between corners is wanted so pins are used merely as counters of tape lengths. In route surveys the elevation of the ground at each station point is required for the purpose of fixing grade lines and computing quantities of earthwork, therefore the stakes are numbered and left in the ground for the use of the leveler and slopeman. These stakes are set slanting for the same reason that pins are thus set. At stations where the instrument is placed for the purpose of reading angles a square stake, called a hub, is driven flush

with the ground and a tack driven in the top to mark the point exactly. A stake is driven to one side as a witness and on the witness stake the necessary identification marks are placed.

After the stations have been set, by the transit (or compass) party, elevations of the ground are taken at each stake and recorded in a book. These elevations when plotted on ruled paper give a "profile" of the route and on the profile is fixed the grade line. The slope of the ground to the right and left of each station is also measured and from the slope-notes are plotted cross-sections from which earthwork quantities are computed after a grade is adopted.

MEASURING ON SLOPES

If all ground sloped the same degree all measurements might be made on the surface and no errors would arise. Since some ground is level and some is very steep, with many degrees of slope between, it is necessary to adopt a standard, so 100 ft. will be 100 ft., no matter what the nature of the ground. This is accomplished by holding the tape level and taut. A level line is parallel to the surface of still water and as the earth is practically a sphere a level line is really the circumference of a great circle. The diameter of the circle is so great and the distances, com-

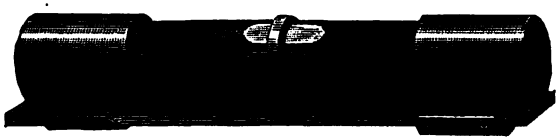


Fig. 6. McCullough tape level.

paratively, so short that horizontal lines are assumed for all practical purposes to be level, a horizontal line being perpendicular to a vertical line.

Unskilled men experience considerable difficulty in holding a tape level. The error is cumulative and the sloping tape measures "short" each time, so the total length is recorded as greater than it is actually marked on the ground. A number of devices are used to assist inexperienced chainmen, the author, in 1892, patenting a small level for the

purpose. The chief merit of this device is that it may be carried in the vest pocket and be quickly put on or taken off any tape. The patent having expired a number of instrument dealers now advertise similar levels, a few crediting the inventor by name. The level is placed on the tape about one foot from the end and the tape is pulled tight. By first experimenting with the tape and level on a level surface, as a floor, the positions of the bubble for different tape lengths and different amounts of sag are noted. Without some such device a tape may be held horizontally as follows: While the rear chainman holds his end stationary the head chainman raises and lowers his end until he ascertains the longest distance which can be obtained, each man using a plumb-bob. On route surveys it is usual to give each man a hand level, the proper use of which will be explained later.

The numbering of stations has been described. Stakes set between are marked with the number of feet from the

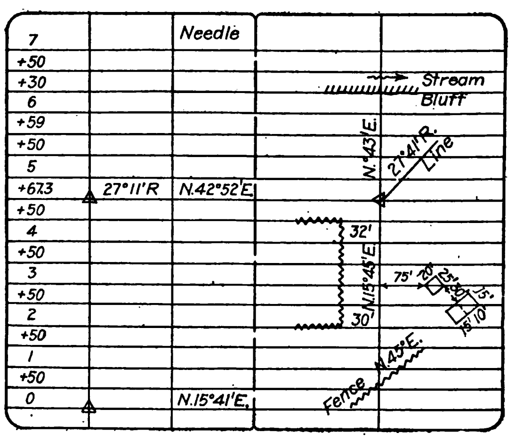


FIG. 7. Proper methods for keeping notes of line survey.

preceding station, preceded by a + sign. Thus 4 + 50 indicates a point on the line 450 ft. from the starting point. It is read "Station 4 plus 50." The transitman enters his

notes in a book starting on the bottom line on the left-hand page, the notes reading from the bottom up. On the right-hand page the center line represents the line of survey and sketches may be made with objects shown in the proper relative positions. A leveler makes no sketches so stations are numbered from the top down in his field book in order to add and subtract readily.

Fig. 7 shows some notes of a transit survey. At Sta. 0 a hub is set and the △ mark placed on line indicates an instrument point. The correct bearing is set down in the third column and the reading given by the needle is placed on the sketch line of the survey as a check on the "right" (R) and "left" (L) readings. The needle is always read to check angle readings taken on the horizontal plate. At Sta. 4 + 67.3 a change in direction occurred and this was shown on the sketch line in order to give the draftsman a check, but the sketch line is always kept straight. Bearings are obtained from preceding angles by addition or subtraction.

When very accurate work is required all measurements are made parallel to the surface of the ground, between marks in the tops of hubs driven at each tape length. With a level and rod the elevations of the tops of the hubs are taken. Then

$$H = \sqrt{L^2 - D^2},$$

in which expression

H = horizontal length,
L = length on slope,
D = difference in elevation between ends of tape on slope.

(*Note.* — In all problems involving squares or square roots of numbers use the table of squares in Chapter II. Information on the solving of formulas is given in Appendix A.)

Problem. — A line was measured on the surface of the ground with a tape 50 ft. long, hubs being set at each tape length. Elevations were taken on each hub and the differences in elevation thus obtained. What is the horizontal length of the line?

Stations.	Elevations above base.	Difference in elevations.	Stations.	Elevations above base.	Difference in elevations.
0	93.2	0.0	+50	109.2	0.2
+50	95.6	2.4	4	114.5	5.3
1	98.2	2.6	+50	115.2	0.7
+50	101.4	3.2	5	115.8	0.6
2	105.1	3.7	+50	117.0	1.2
+50	108.1	3.0	6	119.4	2.4
3	109.0	0.9	+50	124.3	4.9

ANGULAR MEASURING ON SLOPES

In rough country where extreme accuracy is not required and speed is essential it is a common practice to measure on the slope. Sometimes the tapes used for this work are several hundred feet long. They are held at a height to go over small brush, etc., and plumb bobs at each end transfer the length to the ground. With a clinometer, or transit, the vertical angle is measured from the horizontal. The secant of an angle is the length of the hypothenuse of a triangle having a horizontal base = 1. Take from a table of secants (pp. 194–200) the secant of the angle; then

$$H = \frac{L}{\text{Sec } A}$$

in which H = horizontal distance.
L = length on the slope.
A = angle of slope.

The following table shows deductions to make in feet for each one hundred feet measured on a slope provided the surveyor has an instrument for obtaining the angle of slope. Column A shows the angle and column B the number of feet to be deducted.

A	*B*	*A*	*B*	*A*	*B*	*A*	*B*
1°	0.015	6°	0.548	11°	1.837	16°	3.874
2	0.061	7	0.745	12	2.185	17	4.37
3	0.137	8	0.973	13	2.563	18	4.894
4	0.244	9	1.231	14	2.970	19	5.448
5	0.381	10	1.519	15	3.407	20	6.031

CHAPTER II

CHAIN SURVEYING

Steel tapes have practically supplanted chains for measuring, but the word chain will be used for years to come and the men who make linear measurements will no doubt always be called chainmen. The term "chain surveying" applies only to surveying done without the use of instruments to measure angles or read bearings.

The surveyors' chain was invented by Edward Gunter in 1620 and is often referred to as Gunter's chain. It is 66 ft. long and is divided into 100 links. One acre contains 160 square rods and the invention of the chain by Gunter simplified area calculations. Ten square chains equal one square acre, so the division of the chain into 100 equal parts was a practical demonstration of the value and simplicity of the decimal system of notation. In common units one link is equal to 0.66 ft. or 7.92 inches.

FIG. 8. Surveyors' chain.

The surveyors' chain is four rods long, the rod being an old unit for land measurement in Great Britain. An old legend describes the origin of the rod for this purpose as due to the fact that in olden times farmers carried ox-goads, or rods, which were conveniently used for measuring land and a royal decree fixed a standard length. In common units of today the length of the rod is sixteen and one-half feet, this being the rod used by Gunter. One mile contains eight furlongs and a furlong has a length of forty rods. Gunter divided the furlong into ten chains, which thus made the chain four rods long.

As long as the acre and mile remain in our system of measurements the Gunter divisions will be retained, but as time passes more men will use the 100-ft. tape with 100 links each one foot long. Every piece of land in the United States was originally surveyed with a Gunter chain, so to reduce the descriptions to modern terms involves considerable labor and already has given rise to mistakes. In this work the word "chain" will refer to the surveyors', or Gunter, standard with a link of 7.92 ins. as the unit. The word "tape" will refer to the engineers' standard, a tape 100 ft. long with the decimally divided foot as a unit.

The surveyors' standard for land measurement made in the form of a chain is heavy and the sag in consequence is so great that each length is less than it should be, considering a horizontal line to be a universal standard. When a standard is short it is applied oftener than necessary when measuring a line, thus the recorded length is too great. Some manufacturers tried to overcome the effect of sag by making each chain a trifle long, or by having one link at each end attached to a threaded bar by means of which the chain could be lengthened to overcome the effect of sag: or to shorten the chain when the rings and ends of links became worn. Some chains of very light steel have links connected end to end but this arrangement lacks flexibility. It is usual to make the links less than 7 ins. long and connect them by using two rings between, which provides 600 wearing points. When a chain has three rings between the links there are 800 wearing points. A slight pull has a tendency to open the joints and in the better grade of chains all joints are closed by brazing.

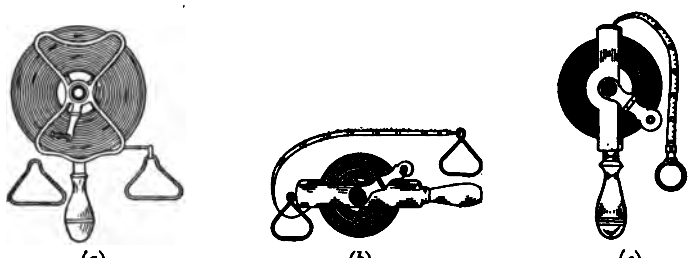
When a standard is long it is not applied as often as necessary when measuring a line, so the recorded length is short. The weight of a chain tending to make it short and the wearing of the joints tending to make it long there never was a definite standard of length in the days when the surveyors' unit of measure was made in the form of a chain. Adjusting screws at the ends only affected the whole length and fractional chain lengths were never correct. Links frequently became bent and the small rings were often snarled, thus shortening the chain. The numerous joints made it difficult to pull a chain through brush

and places filled with small obstructions. Fractional parts of links had to be estimated, so for close measurements surveyors carried rods ten links long with the links divided decimally, to measure the ends of lines, the poles being convenient for sighting purposes. In cities where a chain

FIG. 9. Contact rod.

would be out of place it was customary up to quite recent times to measure with contact rods. These were made from properly seasoned wood and were shod with brass at each end. The length was a matter of individual preference, usually ten feet. In the top was inserted a spirit level, in some rods a level being placed at each end. Two or more rods were used, being placed carefully end to end. A later type had marks near the ends and plumb-bobs were used with one rod, instead of using several rods placed end to end.

Tapes were used for many years in Europe before they were adopted in the United States. Since 1880 tapes have



(a) (b) (c)

FIG. 10. Steel tapes: (a) Graduated in links. (b) Graduated in feet and tenths. (c) Graduated in feet and hundredths. Open reels protect against rusting of tapes.

so grown in favor that the old-style chain is regarded as a curiosity. Tapes are much lighter than chains so the sag is less. The length is standard at a temperature of practically 60° F. with the tape lying on a level surface with a pull of 10 lbs. applied at the ends. The coefficient of expansion for steel is 0.0000067 the length for one degree F. so

that a tape 100 ft. long at a temperature of 60° F. will be 99.981 ft. long at a temperature of 32° F. and will be 100.0134 ft. long at 80° F., the formula being

$$L_t = L\,(1 \pm CT),$$

in which

L_t = length of tape at assumed temperature,
L = length of tape at standard temperature,
C = coefficient = 0.0000067,
T = difference in degrees between standard and assumed temperature.

When a tape is sold the maker should state the temperature and pull at which the length is standard. The United States Bureau of Standards, Washington, D. C., will test and certify tapes for a small fee. When work of a very important character is to be performed by an engineer or surveyor the temperature should be read and the proper correction applied for each measurement. The effect of temperature is to shorten a tape when cold and lengthen it when warm. Errors continually alter in amount and direction during the day but they practically balance in ordinary work. The most accurate measurements are made during cool foggy weather or at night, the latter being the time selected by Government surveyors for the highest-grade work. The "Invar" tape is made of an alloy of nickel and steel possessing a very small coefficient of expansion. It is used on work of the highest grade.

The effect of sag is to shorten measurements and being constant is important. When a new tape is purchased it should be laid on a level floor or walk at a temperature as near the standard as possible. A spring balance should be fastened to one end and the tape pulled until the standard pull is indicated, the ends being carefully marked on the level surface. Two men should now hold the tape at a definite height above the level surface and, holding a plumb-bob at each end, pull the tape until the points of the bobs strike the marks, thus eliminating sag. The pull then registered on the spring balance will be from 50 to 100 per cent greater than the standard pull and should be the pull used with this tape thereafter. Only on exceedingly important work is it necessary to use a spring balance, a chain-

man after a few tests becoming expert in applying a pull close enough for ordinary work. Having obtained the proper pull as described, this becomes the standard pull for this particular tape in order to eliminate the effect of sag. There will be some sag with any less pull and the tape will measure short of the proper length. By using ordinary care no correction is required to overcome the effect of sag due to lack of tension but a wind blowing a tape to one side makes it curve, or deflect, and this is a case calling for a correction on important work. The deflection must be obtained and the correction added to the measured length.

Let C = correction for deflection or sag in feet,
d = amount of deflection or sag in feet,
L = length of tape in feet,

then $$C = \frac{8\,d^2}{3\,L}.$$

When a pull greater than the standard pull is used the tape will stretch. The correction must be subtracted from the measured length, on important work. The standard pull here referred to is the new standard obtained by measuring with the suspended tape above a level surface.

Let P = difference in pounds between the standard and actual pull,
A = area of cross-section of tape in square inches,
E = modulus of elasticity of steel = 30,000,000,
L = length of tape in feet,
S = stretch of tape in feet,

then $$S = \frac{PL}{EA}.$$

ERRORS IN MEASUREMENT

Errors in measurement are as follows:

1. The tape may not be held in a horizontal position. This shortens the length and the error is constant and "cumulative." A cumulative error is one that grows by repetition so errors of this sort cannot be too carefully guarded against. It requires long practice before men learn to hold a tape with the ends at the same elevation,

so work done with inexperienced chainmen is always faulty. A level on the tape, or a hand level, is desirable until experience is gained.

2. The tape is not held with a constant, or with a proper, tension. This causes an undue amount of sag, the error being cumulative.

3. Chainmen are not carefully kept to line ("lined in"). This gives a measured broken line instead of a straight line, the error being cumulative.

4. The plumb-bobs may be too light. The proper weight for a plumb-bob is one of the points on which it is easy to raise discussion between practical surveyors. Wind delays work when light plumb-bobs are used and the error is more likely to be cumulative than compensating (that is, balancing). A light plumb-bob weighs less than one pound. The writer prefers plumb-bobs weighing not less than one and one-half pounds with a decided preference for a two-pound bob.

5. Pins may be placed ahead, or back, of the point. If ordinary care is used this error will be compensating, for with a series of repetitions the number of pins placed ahead will equal the number placed back of a point, the law of averages applying in this case. The error is only important on short lines.

6. A mistake may be made in counting pins, thus omitting a chain or tape length. This often happens and can only be avoided by careful attention to the work, each man checking the count of the other at the ends of "tallies."

7. The chain or tape may not be of standard length. The length should be tested on a level surface side by side with a standard in order to determine a proper correction. The measure may be short or long.

Let M = length of line as measured with faulty tape or chain,
T = true length provided the line were measured with the standard,
l = assumed length of tape or chain (standard length),
a = actual length,

then
$$T = \frac{Ma}{l}.$$

Example. — A line was measured and the length recorded as 1122.5 ft. The tape was later discovered to be 99.8 ft. long instead of 100 ft. What is the true length of the line?

$$T = \frac{Ma}{l} = \frac{1122.5 \times 99.8}{100} \doteq 1120.26 \text{ ft.}$$

CORRECTING ERRONEOUS AREAS

A tract of land is sometimes surveyed with a chain or tape not of standard length and the area is computed before the error is discovered. It is not necessary to re-compute the area by first correcting each line. The correct area may be obtained by making use of the geometrical proposition that similar polygons are to each other as the squares of the like sides.

Let A = true area of field,
C = computed area of field,
l = standard length of chain or tape,
a = actual length of chain or tape,

then
$$A = \frac{Ca^2}{l^2} = C\left(\frac{a}{l}\right)^2.$$

PROBLEMS

1. A line was recorded as having a length of 43.80 chains but the chain was discovered to be 97 links long. What was the true length of the line?

2. A tape having 0.15 ft. broken off one end was used to measure a line which was reported as a result of the measurement to be 1345.2 ft. long. Assuming that the broken tape was used as if it were 50 ft. long what was the actual length of the line?

3. A tape standard at 70° F. with a pull of 18 lbs. when freely suspended was used to measure a line 2910 ft. long. What length was returned by the surveyor who used the same tape with a pull of 26 lbs. at a temperature of 96° F. and made no corrections?

4. A tape 200 ft. long was used to measure a line 7800 ft. long on a windy day with a deflection of 0.73 ft. for each tape length. What was the length of the line as returned

by the surveyor who made no allowance for sag caused by wind?

5. A line was measured with a 100-ft. tape and the length returned as 6700 ft., a stake being set at each tape length. The lining-in was done by the chainmen. Afterwards an instrument was used to project a perfectly straight line, when half the stakes were found to be 6 ins. on one side and half were found to be 7 ins. on the opposite side of the line. What was the actual length?

6. The area of a field was returned as 48.9 acres. The tape was later found to be half a link too long. What was the true area?

7. A chain 98.5 links long was used to measure a field, the area of which was known to be 93.2 acres. What was the area as found with the defective chain?

8. A field known to contain 83.4 acres was measured with a defective chain and the area given as being 80.7 acres. What was the actual length of the chain?

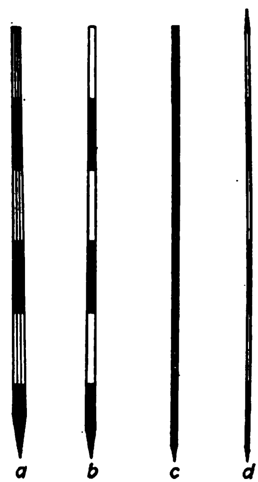


FIG. 11. Sighting poles and line rods.

RANGING LINES

Surveyors use poles of wood several feet long for ranging lines and for sights. These poles are shod with a metal point on one end and are painted with one foot spaces alternately red and white. The top diameter is usually one inch and the diameter at the bottom is one and one-half inches. For use with instruments having telescopes the sighting poles are of steel five-eighths of an inch in diameter. Steel poles are called line rods. For very accurate work line rods have a relatively short point at one end for setting hubs, the other end having a long point used in setting tack points on hubs.

To range a line with poles. — Three or more points are required. One man can do the work but time is saved

when two or more men are used. The general direction of the line being chosen a pole is set on line at any distance from the starting point and a second pole is set ahead in line with the first pole and the starting point. The first pole is then carried ahead and set in line with the second pole and the starting point, this operation being repeated until the starting point cannot be plainly seen. A third pole is then used and the line carried forward by moving one pole ahead and setting it in line with the two standing poles. A small stake is driven in each hole as the poles are lifted. It is better to measure the line and set the stakes at regular intervals. In setting the poles a plumb line should be used to insure verticality.

Assume that a line has been ranged with poles with the intention that it shall strike a certain point and it fails to do so. The offset distance is measured from the point to the line and each stake must be moved proportionately. In Fig. 12 the random line from A to C was ranged with

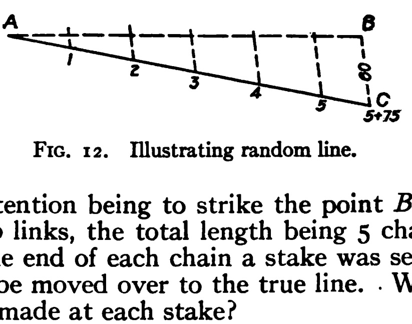


FIG. 12. Illustrating random line.

poles, the intention being to strike the point B which was missed by 60 links, the total length being 5 chains and 75 links. At the end of each chain a stake was set and these stakes must be moved over to the true line. What correction is to be made at each stake?

Solution. — $\frac{0.60}{5.75} = 10.435$ links per chain. The stakes will be moved 10.43 links at 1; 20.87 links at 2; 31.31 links at 3; 41.74 links at 4 and 52.175 links at 5, stake C being discarded.

To pass obstacles. — Measure to one side of the first pole a distance sufficient to enable a line to pass the obstacle. Measure an equal distance from the third pole and set a middle pole between these two by sighting. There will be three poles set in a straight line parallel to the line first

started. This second line is ranged forward until the obstacle is passed when the original line is resumed by measuring back to the line from two poles set in line on the offset line beyond the obstacle.

To range a line over a hill. — Let A and B, Fig. 13, be on the line. One man stands at C, where he can see both A and B. A second man is then lined in to d, between c

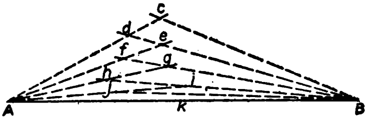


FIG. 13. Ranging a line over a hill.

and A. Standing at d and facing B, the second man lines the first man in to e, on line between d and B. From e the first man then lines the second man to f, on line between e and A. In this manner the poles are successively brought closer to line at each operation until finally one is set at j and the other at k on the line from A to B.

Further use of poles and tapes calls for the exercise of considerable ingenuity and the problems should be first worked out on a drawing board. When the principles are mastered the work may be repeated on a larger scale in the field.

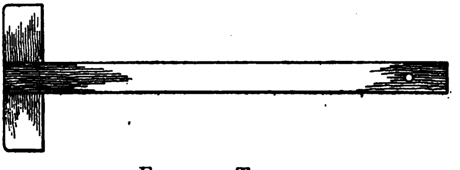


FIG. 14. T square.

PRACTICAL GEOMETRY

The student should have the following drafting tools:

1. A drawing board of soft pine, $\frac{3}{4}$ in. thick, preferably 18 ins. by 24 ins. in size. All edges should be straight and true, the lower left-hand corner being a perfect right angle.

2. A T square with a blade 24 ins. long. The best have transparent celluloid edges and the heads are fixed. A T

square has a head to slide along the edge of a drafting board so lines ruled along the edge of the blade as the head is moved will be parallel. The head should form a true right angle with the blade and if the edges of the board are true and two of the edges form a perfect right angle, lines drawn along the edge of the T square will form right angles when the head is placed first against one side and then against a side perpendicular to the first.

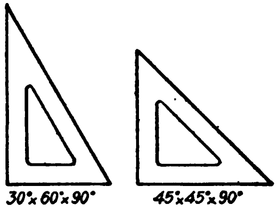


FIG. 15. Draftsmens' triangles.

For architectural and mechanical drafting and for map drafting where all angles are right angles, or are readily drawn from a horizontal or vertical line as a base, a T square is useful and well-nigh indispensable. For the surveyor a straight-edge is better than a T square. All lines do not so run that right angles are formed and a heavy steel straight-edge may be laid anywhere on a drafting board in any direction and it will not be disturbed when triangles are slid along the edge, provided proper care is exercised.

3. A 30-deg. triangle, 8 ins. long.

4. A 45-deg. triangle, 8 ins. long.

The three interior angles of a triangle equal two right angles. A 30-deg. triangle contains one right angle, one

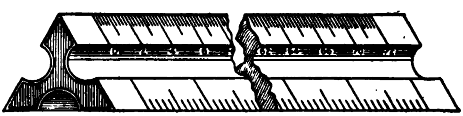


FIG. 16. Engineers' triangular scale.

angle of 30 deg. and one angle of 60 deg. A 45-deg. triangle contains one right angle (90 deg.) and two 45-deg. angles. Transparent celluloid triangles are better than triangles of rubber or of wood.

5. An engineers' triangular scale. The six edges are divided into one-inch spaces with each edge divided decimally for convenience in plotting lines. The usual gradua-

tions are $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, $\frac{1}{40}$, $\frac{1}{50}$, $\frac{1}{60}$ of an inch, enabling the draftsman to make maps with scales of 10, 100, 1000, 20, 200, 2000, 30, 300, 3000, etc., feet per inch, inches, feet or yards per mile, etc.

6. A 3-H lead pencil, although it is well to have a 4-H pencil also. The 4-H pencil will be used to mark points and draw lines lightly, the 3-H pencil being used to draw more prominent lines.

7. Pencil eraser.

8. Drawing pen, 5 ins. long.

9. Compass with pen and pencil points, for drawing circles. The length should be 6 ins. or 8 ins.

Other tools will be required when compass surveying is reached but for present use the foregoing will be sufficient. Use a good quality of drawing paper, the most restful color to the eyes being a light brown or buff. For holding the paper to the drawing board use thumb tacks of punched steel $\frac{3}{8}$-in. diameter.

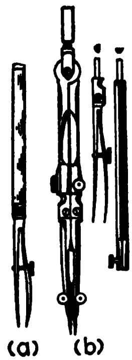


FIG. 17. (*a*) Ruling pen. (*b*) Compass with pen point and lengthening bar.

To draw parallel lines. — Place a triangle on the paper and hold it by pressing with the tips of the fingers of the

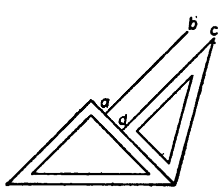


FIG. 18.

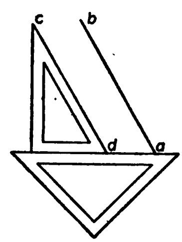


FIG. 19.

left hand. With the right hand slide another triangle along one side of the first.

In Figs. 18 and 19 the line *a . . . b* having been drawn, the triangles are held so the edge *c . . . d* touches the line *a . . . b*. Holding the larger triangle firmly the

smaller one is held closely in contact with it and moved to the point c, when a pencil drawn along the edge will describe the line $c \ldots d$ parallel with $a \ldots b$. Instead of two triangles a straight-edge or T square may be used with one triangle. By using two triangles in combination with a straight-edge or T square a number of angles may be formed as shown in Fig. 20.

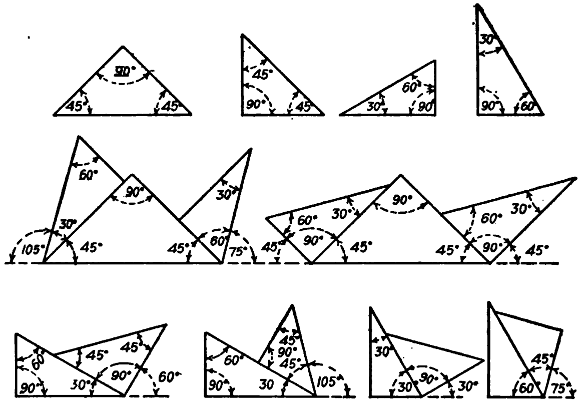


FIG. 20.

The surveyor seldom uses triangles for setting off angles in the manner shown in Fig. 20. The principal use to which he puts these useful tools is to transfer lines from one part of a map to another.

PROBLEMS

1. *To erect a perpendicular to a given line passing through a given point outside the line.* — Let $B–C$ be the line and A be the point.

1st method. — Draw the line $B–C$. Set the needle point of the compasses at A and describe the arc intercepting $B–C$ at D and E. Bisect $D–E$ and from the midway point F draw the line $F–A$, which will be the required perpendicular. Fig. 21.

2nd method. — Describe arc $D–E$ as before. With needle point at E set pencil point at A and describe arc $A–E$.

With needle point at D and pencil point at A describe arc $A-F$ intersecting at F the first arc drawn from E as the center. The line $A-F$ will be the required perpendicular. Fig. 22.

3rd method. — Let F be the point. From a point A describe an arc $D-E-F$ and a line connecting E and F will

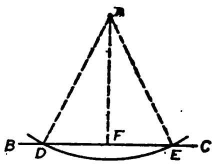


FIG. 21.

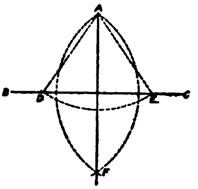


FIG. 22.

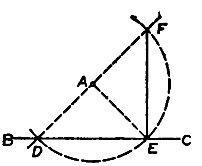


FIG. 23.

be the required perpendicular ***provided*** the three points D, A and F are in a straight line. To accomplish this draw a line from F to the line $B-C$ and mark D. Set off the middle point A, so that $DA = AF$. Fig. 23.

From the foregoing problems a perpendicular is seen to be a line intersecting another line in such a manner that the angle formed on one side of the line is equal to the angle on the other side. By the second method four equal angles are formed, each being a right angle. A perpendicular line is also called a normal line.

With the needle point set at the intersection of two lines normal to each other describe with the pencil point a complete circle. Fig. 24. Then

$$< AOB = < BOC = < COD = < DOA,$$

the sign $\angle$ standing for the word "angle."

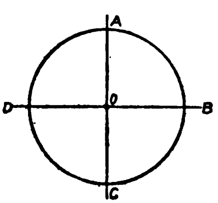


FIG. 24.

Circles are divided into 360 equal divisions called degrees and one-fourth of 360 = 90, so each of the equal angles formed by the intersection of perpendicular lines, or lines normal to each other, contains 90 deg. Such an angle is called a right angle, and contains a "quadrant," or quarter of a circle.

An angle is the amount of divergence of two intersecting

lines. The amount of divergence, that is, the difference in direction between the line *A–O* and the line *B–O*, is 90 deg. (90°). Between *A–O* and *C–O*, proceeding to the right in a clockwise direction, the amount of divergence is 180 deg., the angle being known as a straight angle. Between *A–O* and *D–O* the amount of divergence in a clockwise direction = 270 deg. and in an anticlockwise direction = 90 deg. The three figures show the line *A–D* to be equal to the line *A–E*. This is proof that the angles formed by the meeting of the perpendicular lines are equal, for lines opposite angles are proportional to the angles.

When the problem has been worked on paper the operations should be repeated on the ground. The lengths of the lines are unimportant except as they govern the accuracy of the work; longer lines producing more accurate results. The first method is generally used when the line *B–C* is along a wall and all the work must be performed on one side. Drive a stake or pin at *A* and hold the end of the tape there. Take any distance *A–D* to the wall and make a mark at *D*. On the other side measure *A–E* = *A–D* and mark the point *E*. Halfway between *D* and *E* make the mark *F*, which will be the point where a perpendicular through *A* will meet the wall.

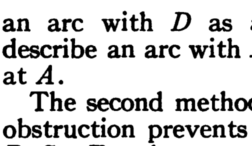


FIG. 25.

Sometimes it may be necessary to erect a perpendicular from a point. Let the point be *F* (Fig. 25). From *F* measure *F–D* and *F–E*, equal to each other. With any radius describe an arc with *D* as a center and with the same radius describe an arc with *E* as a center. The arcs will intersect at *A*.

The second method may be used when no wall or other obstruction prevents a measurement on both sides of line *B–C*. For the reason that long radii may be used and the required point is midway between two accurately determined points, this method is apt to be more accurate than the first method.

The third method is not so good as either of the two others, but is often used when a perpendicular is to be set off from a point ***on a line*** and not through a point ***off the line***.

Let E (Fig. 26) be a point on the line B–C, usually the end of the line, in which case the line is B–E. Select some point A as a center and with a radius = A–E describe the arc D–E–F intersecting the line at D and having F, as nearly as the eye can judge, on the line D–A prolonged. With E as a center with radius = E–D describe an arc intersecting the first arc at F. A line through E–F will be the required perpendicular through E.

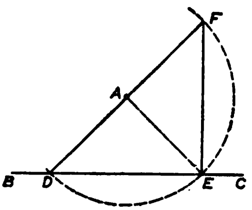


FIG. 26.

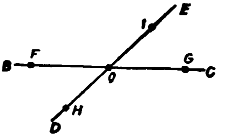


FIG. 27. Marking intersection of two lines.

2. *To mark the intersection of two lines in the field.* — In Fig. 27 let B–C be one line. It is assumed that the second line D–E will cross B–C near the point O. A pole may be set at C and the observer standing on line at B sights towards C over pegs driven on line at F and G, with small nails or tacks in the top to mark the exact line. Standing at D with a pole at E, similar pegs are set at H and I. Tie strings to the projecting nails, connecting F to G and H to I, drawing the strings taut. Under the intersection of the strings at O drive a peg and drive a small nail or tack in the top of the peg to mark the exact intersection.

If the line D–E is an arc, after setting the points F and G, connect them with a string and describe the arc, setting peg with tack at the intersection. In this case pegs H and I are not necessary. This latter method may also be used when line D–E is straight, the peg and tack at O being set by sighting an intersecting line after the string is tied to F and G. The best method to use depends upon circumstances, governed by the judgment of the surveyor.

3. *To erect from a point on a given straight line a perpendicular to the line.*

The square on the hypothenuse of a right-angled triangle is equal to the sum of the squares upon the other two sides; that is

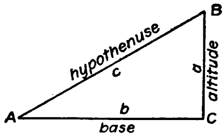


FIG. 28.

$$Hyp.^2 = base^2 + altitude^2,$$

or $$Hyp. = \sqrt{base^2 + altitude^2}.$$

Designating the angles by capital letters and the sides opposite by small letters

$$c^2 = a^2 + b^2. \quad \therefore \quad c = \sqrt{a^2 + b^2},$$
$$a = \sqrt{c^2 - b^2} \text{ and } b = \sqrt{c^2 - a^2}.$$

Assume values for some triangle as follows:

$$a = 3; \quad b = 4; \quad c = 5;$$

then $$a = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3,$$

which proves that when the sides of a triangle are to each other as 3, 4 and 5, the angle at C will be a right angle.

Using a multiple of 4, set off the base A–C. Using the same multiple of 5, describe an arc from A as a center, passing through B. Using the same multiple of 3, describe an arc from C as a center, intersecting the first arc at B. A line connecting B and C will be normal (perpendicular) to A–C.

4. *To set out on the ground a line parallel to another line.* — In Fig. 29 the line C–D is to be laid off parallel to the line A–B. Three methods are in common use.

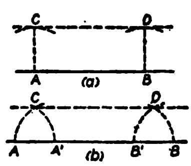


FIG. 29.

1st method. — With radius = A–C describe an arc from A and also from B. Sight across the two arcs on the line C–D and set pegs. The second method is preferable for it is difficult to obtain a good intersection on flat curves.

2nd method. — From A with a radius A–A' describe an arc intended to pass through C. With the same radius from A' describe an arc intersecting the first arc at C and drive a peg. Repeat these operations at B and B', setting a peg at D.

Whether to use the first or second method depends upon circumstances and the judgment of the surveyor, the accuracy desired being a governing factor. The distance A–B should be not less than ten times the distance A–C, the degree of accuracy depending upon the ratio adopted and the care with which the work is done.

3rd method. — This method is based on the geometrical truth that when a straight line crosses two parallel lines the alternate angles ABC and DCB are equal. At B and C, Fig. 30, set pegs and stretch a string to define the line B–C. From B as a center describe the arc e–f and measure the

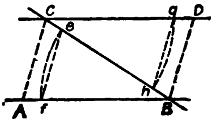


FIG. 30.

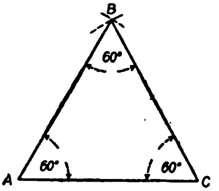


FIG. 31. Equilateral triangle.

chord (the straight line e–f). From C with the same radius describe the arc g–h and set off the chord g–h = chord e–f. From C a straight line through g will fix the location of C–D. This problem is most conveniently worked when an equilateral triangle is formed by the lines A–B, A–C and B–C.

5. *To describe an equilateral triangle.* — In Fig. 31 from C with a radius = A–C describe an arc through B. From A with the same radius describe an arc intersecting the first arc at B. Draw the lines A–B and A–C, forming the triangle, which has three equal sides and three equal angles.

(*Note.* — An isosceles triangle has two equal sides and two equal angles, Fig. 32.)

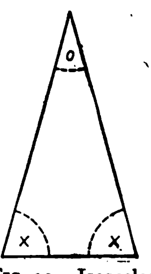


FIG. 32. Isosceles triangle.

6. *To pass an obstacle on line.* — When an obstacle is encountered in ranging a line it may be passed by offsetting as already described. A common method is shown

in Fig. 33. From A and B perpendiculars of equal length are measured and points A' and B' set. Sighting from A' past B' stakes are set at C' and D'. Perpendiculars from the offset line are measured from C' to C and from D' to D, equal in length to AA' and BB', then A, B, C, D are on the same line. Another method not often used is shown in Fig. 34. On the line A–B set the points 1 and 2. With a base equal to the distance between these points erect the equilateral triangle 1–2–3. Produce the side 1–3 to a point 5 opposite the obstacle and construct the equilateral triangle 4–5–6 with sides equal to 1–2–3. Produce the line 5–8 with 5–6 = 4–5 and 6–7 = 3–4. Make 7–8 = 5–6 and construct the equilateral triangle 7–8–9. The base 8–9 should be on the line A–B.

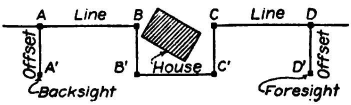


FIG. 33.

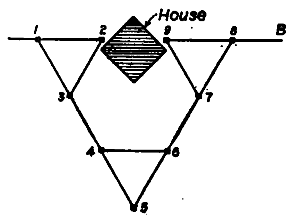


FIG. 34.

The method just described is clumsy and the base 8, 9 is very short so a slight error will throw the line off considerably. A parallel line set off by means of carefully measured perpendiculars is recommended as the best means for passing obstacles. The triangle method is used only when the survey is small and it is impossible to obtain long sights because of numerous obstacles.

7. *To divide a line into any number of equal parts.* —

1*st method.* — Measure the line carefully and divide the length by the number of spaces into which it will be divided. This is the field method.

2*nd method.* — This is an office method and is shown in Fig. 35. It is useful in making scales for drafting. Assume that on a map a line A–B is known to be a certain length but the paper has shrunk through age and the scale is not known. From one end lay off a line A–C of any length and divide it

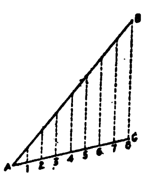


FIG. 35.

into the number of parts, by scaling, into which it is desired to divide A–B. Draw the line C–B and by using a straight-edge and triangle draw parallel lines through A–B from the points marked on A–C.

PROBLEM

A map is to be made to a scale of one-half mile to an inch and no scale is available for the purpose. One-half mile contains 2640 ft. so on a piece of paper draw a line A–B one inch long and the line A–C 2.64 ins. long, using the 100 per inch graduation of the triangular scale. On the line A–C a division of $\frac{1}{10}$ in. = 100 ft., so through each $\frac{1}{10}$-in. graduation on A–C draw a line through A–B parallel with C–B. Each space on A–B equals 100 ft., except the last space which equals 40 ft.

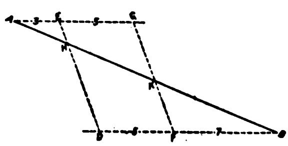


FIG. 36.

8. *To divide a straight line in any proportion, as* 3, 5, 7, *etc.* — Let the line be A–B in Fig. 36. From A draw the line A–C, making A–E = 3 and E–C = 5. From B draw the line B–D parallel to A–C, making B–F = 7 and F–D = 5. Join C to F and E to D, thus making A–H = 3, H–K = 5 and K–B = 7.

A number of problems might be given similar to those already treated but it is assumed the student has a scale with which to work in the office and a tape with which to work in the field, so some problems may be more readily solved by arithmetical methods than by strictly geometrical methods. By measurement the last problem would be solved as follows: Add $3 + 5 + 7 = 15$. Measure the line and divide by 15. Then A–H = $\frac{3}{15}$ of A–B; H–K = $\frac{5}{15}$ of A–B; K–B = $\frac{7}{15}$ of A–B. Annoying differences may creep into arithmetical and measuring work whereas the geometrical method is exact if proper intersections can be obtained. The geometrical method may be used in either the field or the office when no tapes, scales or other measuring instruments are available.

9. *To bisect a given angle ABC.* — From B in Fig. 37 with any radius cut the sides in A and C. From A with any

radius A–D, and from C with the same radius, describe arcs intersecting at D. Join B–D and then $\angle ABD = \angle CBD$.

10. *To divide a right angle into three equal parts.* — From A in Fig. 38 with radius A–B describe the arc B–E–D–C.

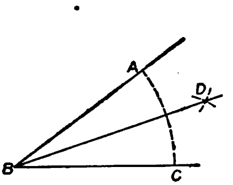


FIG. 37.

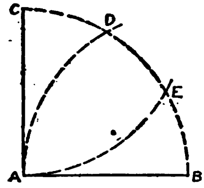


FIG. 38.

From B with the same radius cut this arc in D and from C with the same radius cut the arc in E. Then $\angle CAD = \angle DAE = \angle EAB = 30$ deg.

(*Note.* — No direct method is known for closer division except continual bisection, or dividing by trial. With a protractor, an instrument to be later explained, any required angle may be set off.)

11. *To construct a triangle when the three sides are known.* — In Fig. 39 lay off the line A–C to scale. From C with a radius = C–B, and from A with a radius = A–B, describe

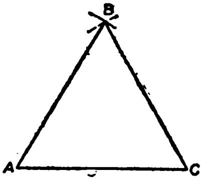


FIG. 39.

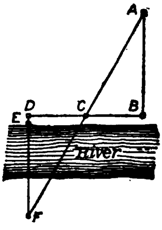


FIG. 40.

arcs intersecting at B. Then A–B–C is the required triangle.

12. *To measure an inaccessible distance, for example a line crossing a stream or swamp.* —

1*st method.* — The principle of equal triangles illustrated in Fig. 40 is used. Set out a triangle A–B–C

with right angle at B. Make $C\text{–}D = B\text{–}C$ and set a pole vertically at C. Opposite D set pole at E on a line normal to the line $B\text{–}D$. One observer stands at A and sighting past C directs an assistant with a pole into position at F. Another observer stands at D and sighting past E also directs the assistant at F. The lines $A\text{–}C$ and $D\text{–}E$ intersect at F and $D\text{–}F = A\text{–}B$. By measuring the length of $A\text{–}B$ the distance between D and F is found, the two triangles being equal.

2nd method. — By similar triangles. The line $B\text{–}C$ may be made equal to $\frac{\text{line } D\text{–}C}{2}$ when $D\text{–}F$ will be twice the length of $A\text{–}B$. If $B\text{–}C = \frac{\text{line } D\text{–}C}{3}$ then line $D\text{–}F$ will be three times the length of $A\text{–}B$. The divisor may be any number instead of 2 or 3. Let $B\text{–}C$ divided by $D\text{–}C$ be any ratio, then

$$D\text{–}F = \frac{A\text{–}B \times D\text{–}C}{B\text{–}C}.$$

In Fig. 41 is illustrated another form in which triangles may appear. The line $B\text{–}D$ is produced across the river and at B a perpendicular line, $B\text{–}C$, is set off. At C set off a line perpendicular to $C\text{–}D$, intersecting $B\text{–}D$ at A. Measure $A\text{–}B$, then

$$B\text{–}D = \frac{(B\text{–}C)^2}{A\text{–}B}.$$

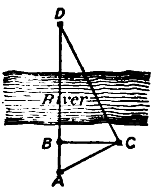


FIG. 41.

The length $A\text{–}B$ is proportional to $B\text{–}D$ and any error in measurement will be in the same proportion. Line $B\text{–}C$ should therefore be not less than one-half or two-thirds the length of $B\text{–}D$.

13. *To erect a perpendicular to a line from an inaccessible point.* — In Fig. 42 point C is a fence post from which a line is to cross the river to an intersection with the fence $A\text{–}B$, forming an angle of 90 deg. with the fence. At any convenient point, H, on the line $A\text{–}B$ erect perpendicular lines on both sides. By intersection fix F on the line $A\text{–}C$ and

make $H-F = G-H$. Sight from G to C and by intersection fix I on the line $A-B$. Sighting from F past I and from A past G, fix E by intersection. From E sight to C

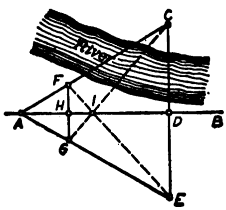


FIG. 42.

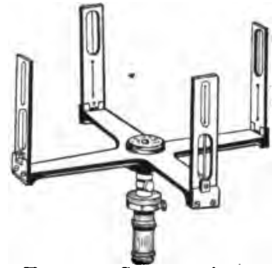


FIG. 43. Surveyors' cross.

and fix D on line $A-B$ by intersection. Then by similar triangles $C-D = D-E$ and the angle ADC is a right angle. By sighting from E to C the line connecting C and D may be staked out on both sides of the river.

14. *To erect a perpendicular with a surveyors' cross.* — The surveyors' cross was once a common instrument, made and sold by all instrument manufacturers. Several patterns were used. Today few dealers sell this instrument but it is very useful in chain surveys when no compass or transit is available. In Fig. 44 a homemade cross is illustrated. A piece of wood 2 ins. thick and 6 ins. square has two cuts in it normal to each other to a depth of about 1 in. It is fastened to a Jacob staff (a pole 5 ft. 3 ins. long, about $1\frac{1}{2}$ ins. diameter, pointed and shod at the lower end with metal) so the top surface makes a right angle with the staff. A small pocket level is used with the cross, generally circular in form. If a circular level cannot be obtained then two small levels are used, one parallel with each cut.

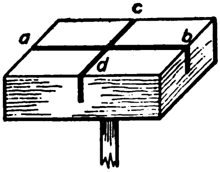


FIG. 44. Surveyors' cross.

The staff is set in the ground at the point on the line from which the perpendicular is to be set off. By means of the level the staff is brought to a truly vertical position and by

sighting forward and back through one slit this slit is placed on the line. Sighting through the other slit stakes may be set on a line normal to that on which the cross is set. It is practically impossible, except with costly tools, to make the two cuts form a perfect right angle, so if the perpendicular line is to be set off with more than ordinary care the principle known as "double-centering" must be used.

After sighting through the slit across the line and setting a stake turn the staff 180 deg. and get the first slit again on line, the ends being reversed. When the slit is on the line and the staff vertical sight again at the stake set on the perpendicular line. The line of sight will fall to one side if the two slits do not make a perfect right angle. The correct position for the center of the stake is between the first and second points fixed by the two sightings.

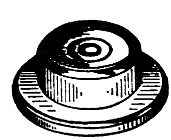


FIG. 45. Circular level.

With such a primitive instrument a difference of three or four inches on a sight of more than 100 ft. will not be extraordinary. By "double centering" the sights and bisecting the points the line will be exactly located if proper care is used. By "double centering" the error is doubled and occurs on one side of the line for the first sight and on the opposite side for the second sight. The reversal of observations and doubling of errors, to obtain a mean (average) is the underlying principle of all adjustments of instruments. This is illustrated in Fig. 46.

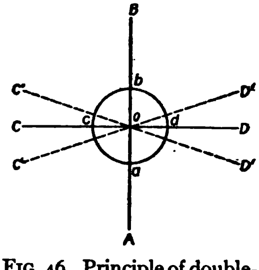


FIG. 46. Principle of double-centering.

Let A–B be a line with which line C–D is to intersect and form a right angle. The cross is set at O with a–b on the line A–B and sighting through c–d the points C' and D' are set. The line c–d may not be truly perpendicular to line a–b so the cross is turned 180 deg. and by sighting through c–d the points C'' and D'' are set. No further explanation is required to show that the true position of C is midway between C' and C'', and the true position of D is midway between D' and D''.

Assuming the board in which the slits are cut to be perpendicular to the staff the board will be level when the staff is truly vertical. If the board is not level all angles set off by using it will be too small. With such an instrument (one which may be turned 180 deg. only by turning the staff) no correction is possible for errors caused by lack of horizontality of the board. With transits or compasses, having a spindle on which the instrument is turned, lack of verticality in the spindle is corrected by "double centering."

A surveyors' cross made by a scientific instrument maker sometimes has graduations by means of which angles of 30 deg., 45 deg., 60 deg. and 90 deg. may be set off. A compass or transit, however, should be used for angles other than right angles, although a true right angle is the most difficult angle to turn.

CHAIN SURVEYS

1. *To survey a triangular field.* — On the left-hand page of the field book set down the lengths of the lines and on the right-hand page draw a diagram. This is shown in Fig. 47.

Chains.
A–B = 20
B–C = 24
C–A = 18

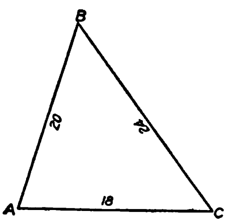


FIG. 47.

2. *To survey a field with any number of sides.* — Measure each side and then measure tie lines to divide the field into triangles. This is illustrated in Fig. 48.

Sides.	Chains.	Tie lines.	Chains.
A–B	30.60	A–C	45.0
B–C	20.40	A–D	35.0
C–D	22.40	A–E	24.20
D–E	16.20		
E–F	13.50		
F–A	28.00		

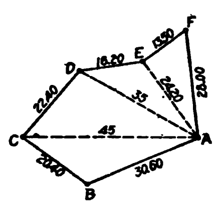


FIG. 48. Tie line survey.

3. *To survey a field with crooked sides.* — On the inside of the field close to the boundaries lay off straight lines as long as possible. From these lines measure perpendiculars to intersect the boundaries at each angle. Run tie lines on the inside to divide the interior survey into triangles. It is not necessary to measure the boundaries, the tie lines and perpendiculars sufficiently fixing the positions of the corners. This is illustrated in Fig. 49 and the following notes.

Lengths. Chains.		Offsets. Chains.
A–B	11.20	1....0.56
	at 5.40	2....1.40
	8.26	3....0.36
	11.20	4....0.36
B–C	7.96	1....0.20
	at 2.36	2....0.36
	4.28	3....0.96
	7.96	4....0.30
C–D	4.68	....
	at 4.34	.....0.30
D–E	4.20	at end 0.30
E–F	8.20	1....0.40
	at 1.04	2....0.86
	2.96	3....0.33
	5.88	4....1.00
	8.20	5....0.12
F–G	7.96	1....1.20
	at 2.0	2....0.24
	7.96	3....0.16
G–A	6.48	....
	at 3.80	1....0.80
	6.48	2....0.40

Tie lines.	Chains.
B–G	9.71
C–G	9.93
D–G	7.45
D–F	8.68

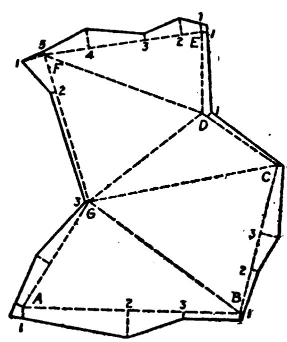


FIG. 49. Tie line and offset survey.

If the adjoining owners do not object, time and labor may sometimes be saved by running lines across boundaries and measuring offsets to the right and left. Care must be taken in recording the notes. This method applied to the last field is illustrated in Fig. 50.

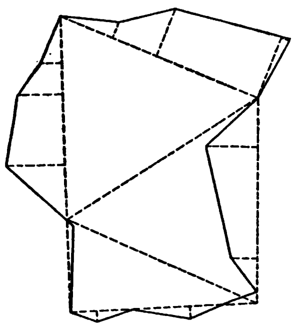


FIG. 50. Tie line and offset survey.

LOCATING OBJECTS

To locate an object on line.—The line is $A–B$, Fig. 51, and the object, assumed here to be a house, is to be shown on the map.

The line is carried past the house and marks placed at f and g where it meets the walls. From some point a on line measure the distances $a–c$, $f–c$ and $a–f$. On the other side of the house from the point b measure $b–d$, and $b–e$. When the map is drawn points a and b, f and g are marked on the line $A–B$. From a with radius $a–c$ describe an arc and from f with arc $f–c$ describe an arc intersecting the first at c. Join c to f and prolong the line through e.

From b with radius $b–e$ describe an arc cutting the line $c–f–e$ at e. From b with radius $b–d$ describe an arc through d. From e draw a line through g to d. Three corners *cde* of the building are now located and the remaining corner is at the intersection of lines parallel with $c–e$ and $d–e$, passing through c and d.

(*Note.*—When a survey is made for the purpose of drawing a plat or map and objects are to be located the method of route surveys should be followed. The first station should be marked 0, and pegs set at the end of each tape length. If the surveyors' unit of measure is used the stations will be 66 ft. long and if the engineers' unit of measure is used the stations will be 100 ft. long. Each station contains 100 units so a portion will be designated by a decimal, as 4.37 chains, or by a + sign, as 4 + 37.)

When the line is marked in stations all objects are located by reference to the nearest stations or plus station.

A sketch is drawn on the right-hand page of the book showing the shape of the object and all the tie lines. Houses are usually rectangular, which simplifies sketching.

Fences crossing a line. — Note the station where a fence crosses, and from points on line 25 ft. each side of the fence measure to points on the fence 25 ft. from the line, as shown

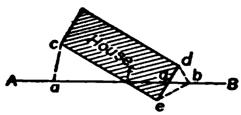


FIG. 51.

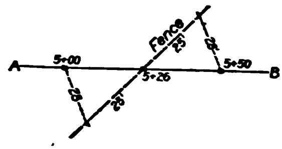


FIG. 52.

in Fig. 52. When platting this sketch *A–B* is the line. From Sta. 5 + 00 describe an arc of 28 ft. on the right of the line and from Sta. 5 + 50 describe an arc of 26 ft. on the left. From Sta. 5 + 26 describe arcs with radius of 25 ft. intersecting these two. A line drawn through these two points of intersection and through Sta. 5 + 26 represents the fence in position and direction.

Objects not on line. — In Fig. 53 a house is shown at one side of the line. At *a* a stake is set by intersection, to show where the line of the side of the house produced strikes the survey line *A–B*. At *b* and *c* perpendiculars are erected and the distances *a–b*, *b–d* and *c–e* measured. The points are easily platted, following the instructions previously given.

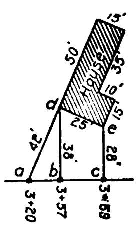


FIG. 53.

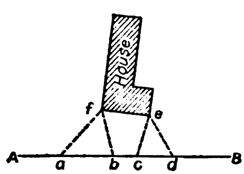


FIG. 54.

In Fig. 54 a more simple method is shown. Let *a*, *b*, *c* and *d* be four stakes on the line from which measurements are made to *e* and *f*.

By following the methods given, and such variations as will present themselves with experience, a complete survey of any farm or tract of land may be executed with a satisfactory degree of accuracy. Maps can be made and areas found without the use of compass or transit.

MAKING MAPS (OR PLATS) OF CHAIN SURVEYS

To plat the survey of the triangular field, first draw to any scale the line *A–B*, 20 chains long. From *B* with a radius of 24 chains describe an arc passing through *C*. From *A* with a radius of 18 chains describe an arc intersecting the first arc at *C*. Draw lines connecting the first arc at *C*. Draw lines connecting *A* and *C* and *B* and *C*.

To plat the second field draw the line *A–B*. From *A* with radius *A–C* describe an arc intercepting an arc described from *B* with radius *B–C*. From *A* with radius *A–D* describe an arc intercepting an arc described from *C* with radius *C–D*. From *A* with radius *A–E* describe an arc intercepting an arc described from *D* with radius *D–E*. From *E* with radius *E–F* and from *A* with radius *A–F* describe arcs intercepting at *F*.

To plat the third field draw the line *A–B* and fix the points *C*, *G*, *D*, *E* and *F* by intersection. Measure on each line the distances marked and set off the perpendicular offset lines. Connect the ends of the lines. At the corners the outside lines must be produced to intersect.

The tie lines are measured to divide fields into triangles in order to plat them as described. If the maps are drawn to a large scale they may be divided into triangles or quadrilaterals, using fine-pointed hard pencils. The lengths of all the lines are then measured with a scale and the area of each interior figure found. The sum of the areas will be the area of the whole field.

PLANE MENSURATION

A rectangle is a four-sided figure with the opposite sides parallel and each angle a right angle, Fig. 55.

A rectangle with all sides equal is called a square.

Area of a rectangle = length × breadth, or $A = lb$.

Area of a right-angled triangle.

In Fig. 56 two right-angled triangles equal in size are placed as shown, thus forming a rectangle. Since the area of a rectangle $= lb$ then the area of a right-angled triangle $=$

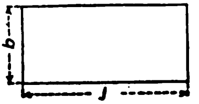


FIG. 55.

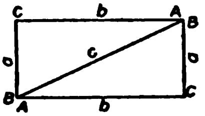


FIG. 56.

$\frac{1}{2}\,lb$, or, using the letters commonly denoting the base and altitude of a triangle, $A = \frac{1}{2}\,ab = \frac{ab}{2}$.

Area of any triangle, Fig. 57.

(*a*) Area ABC = Area ABD + Area BDC

$$= \frac{AD \times h}{2} + \frac{DC \times h}{2}$$

$$= \frac{(AD + DC)\,h}{2} = \frac{AC \times h}{2}.$$

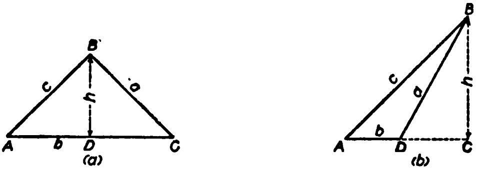


FIG. 57.

(*b*) Area ABD = Area ABC − Area BCD

$$= \frac{AC \times h}{2} - \frac{CD \times h}{2}$$

$$= \frac{(AC - CD)\,h}{2} = \frac{AD \times h}{2}.$$

Therefore the area of any triangle $= \frac{base \times height}{2}$, the height being measured on a line normal to the base.

When the perpendicular height cannot be readily obtained the area may be found by means of the expression

$$A = \sqrt{S(S-a)(S-b)(S-c)},$$

where S is half the sum of the three sides $= \frac{a+b+c}{2}$.

Example. — In a triangular field the three sides are as follows:

$$A\text{–}B = 20 \text{ chains},$$
$$B\text{–}C = 24 \text{ chains},$$
$$C\text{–}A = 18 \text{ chains}.$$

Find the area

$$S = \frac{24+20+18}{2} = 31.$$

$$S-a = 31-24 = 7.$$
$$S-b = 31-20 = 11.$$
$$S-c = 31-18 = 13.$$
$$S(S-a)(S-b)(S-c) = 31 \times 7 \times 11 \times 13 = 31{,}031.$$
$$\sqrt{31{,}031} = 176.16 \text{ sq. chains} = 17.616 \text{ acres}.$$

PROBLEMS

Compute the area of the field in Fig. 48. Find the area of each triangle into which the field is divided by tie lines, add these areas and the sum will be the area of the field.

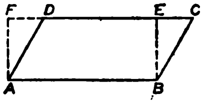


FIG. 58.

Area of a parallelogram. — A parallelogram is a four-sided figure with opposite sides parallel but none of the angles are right angles.

Let A, B, C, D be a parallelogram and from B erect a perpendicular touching C–D at E. From A erect a perpendicular meeting C–D produced to F.

Then $ABEF$ form a rectangle and the Area $= AF \times AB$.

Since AD and BC are parallel to each other $FD = EC$. Since AB and CD are parallel to each other $AF = BE$. The $\triangle AFD = \triangle BEC$, therefore the area of the rectangle $ABEF$ = area of the parallelogram $ABCD$.

The area of a parallelogram = base × perpendicular height.

Area of a trapezium. — A trapezium is a quadrilateral (four-sided figure) with no parallel sides.

Divide the trapezium into triangles and find the sum of the areas of the triangles.

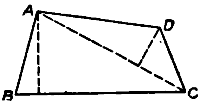


FIG. 59.

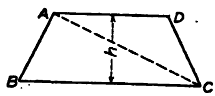


FIG. 60.

Area of a trapezoid. — A trapezoid is a quadrilateral with two sides parallel. It may, or may not, contain two right angles at the base.

1ST CASE. — No included right angles. Divide the trapezium $ABCD$ into two triangles ABC and ADC, then

$$\begin{aligned}\text{Area } ABCD &= \text{Area } (ABC) + \text{Area } (ADC)\\ &= \frac{AD \times h}{2} + \frac{BC \times h}{2}\\ &= \frac{(AD + BC)\, h}{2}.\end{aligned}$$

2ND CASE. — The two parallel sides are perpendicular to the base.

$$\text{Area} = AB \times \frac{AD + BC}{2} = AB \times EF.$$

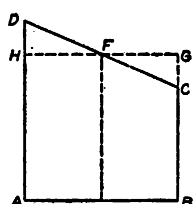


FIG. 61.

The area of a trapezoid = half the sum of the parallel sides × the perpendicular distance between them.

The area of an irregular field bounded by straight lines may be obtained by reducing the platted figure to a single equivalent triangle.

Let the field be represented by the figure $ABCD$, Fig. 62. Connect BD with a light pencil line (using a hard pencil with chisel-edge point). Draw CF parallel to BD, producing AD to F.

Draw the line BF. $\triangle BDC = \triangle BDF$ for they have a common base BD, and a common altitude, between the parallel lines BD and CF.

$\triangle BGD$ is common to $\triangle BDC$ and $\triangle BDF$ and when sub-

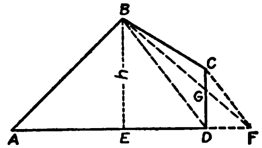


FIG. 62.

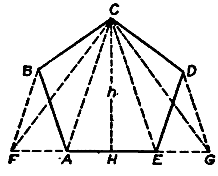


FIG. 63.

tracted leaves $\triangle DFG$ which is added to the figure to replace $\triangle BCG$ of equal area which is subtracted from it.

$$\triangle ABF = \textit{Trapezium } ABCD.$$

Draw the perpendicular BE.

$$\text{Area} = \frac{AF \times BE}{2}.$$

Fig. 63 shows the principle extended to obtain the area of a five-sided field.

$$\text{Area} = \frac{FG \times CH}{2}.$$

This method is general (applicable to fields having any number of sides).

Problem.—Plat the field shown in Fig. 49. Compute the areas of the triangles formed by the tie lines. The irregular pieces around the edges are to be computed as triangles or as trapezoids, the sum of the areas of all the small divisions being the area of the field.

FIG. 64.

Another method is to carefully plat the survey of the field and draw averaging lines through the boundaries, thus reducing the field to a trapezoidal shape (Fig. 64).

The small pieces on either side of the averaging lines must balance in area.

Area of a field bounded by a curved line.

Trapezoidal rule. — From a base line erect perpendiculars dividing the field into an even number of strips, the intervals between the perpendiculars (ordinates) being equal. The accuracy of the computation of area is fixed by the number of ordinates. The general rule is to have them so close together that each section of the intercepted curve may be considered to be practically straight.

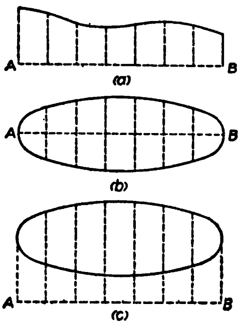


FIG. 65.

The sum of the lengths of the ordinates, divided by the number of ordinates = mean ordinate.

Area = mean ordinate × length.

The operation is shortened by adding to half the sum of the lengths of the two end ordinates the sum of the lengths of the interior ordinates and multiplying by the width of one strip.

This rule may be expressed as follows:

Let A = area,
S = width of strip,
h = ordinate (height),
$h_1 h_2 \ldots h_n$ = each ordinate, the subscript denoting the particular ordinate, the subscript n indicating the last ordinate,
$n - 1$ = number of spaces when n = number of ordinates,

then

$$A = \frac{S}{n-1}\left[\frac{h_1 - h_n}{2} + (h_2 + h_3 + h_4 + \ldots h_{n-1})\right].$$

(*Note.* — When the curve touches the base at one end or at both ends the length of the end ordinate = o.)

Assume $h_1 = o$ and $h_n = 6$ then $\frac{h_1 + h_n}{2} = \frac{o + 6}{2} = 3.$

Assume $h_1 = o$ and $h_n = o$ then $\frac{o + o}{2} = o.$

To pass a circle through three points not lying in a straight line.

Let A, B, C be the points. Join A to B and B to C. Erect a perpendicular from the middle of AB and also from the middle of BC, the perpendiculars intersecting at some point O. From O as a center with a radius $= OA$ describe a circle which will pass through A, B and C.

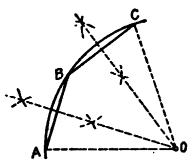


FIG. 66.

Thomas Simpson, Professor of Mathematics, Royal Military Academy, Woolwich, England; born Aug. 10, 1710; died May 14, 1761; was the author of a rule for finding the area of an irregular figure, used in ship-building calculations. The mathematical proof can only be followed by students who have completed a college course in mathematics, but it is not necessary to study the demonstration in order to use the rule. It is based on the assumption that through the ends of three equidistant ordinates we can draw arcs of parabolas, approximating to the curve between these ordinates, and a series of arcs can be thus drawn to fit the boundary of any irregular figure with a curved outline.

Simpson's Rule. — The figure is divided by an odd number of perpendiculars into an even number of strips of equal width.

To the sum of the lengths of the first and last ordinate add twice the sum of the lengths of the remaining odd ordinates and four times the sum of the even ordinates. Multiply by one-third the width of one strip.

Using the notation given for the trapezoidal rule, the area by Simpson's rule may be expressed as follows:

$$A = \frac{S}{3}\Big[h_1 + h_n + 2\,(h_3 + h_5 + h_7\ .\ .\ .\ + h_{n-2}) \\ + 4\,(h_2 + h_4 + h_6\ .\ .\ .\ + h_{n-1})\Big].$$

Examples. — Find the areas of the following irregular surfaces by the trapezoidal rule and by Simpson's rule. Simpson's rule is the more accurate.

(*Note.* — If there are nine ordinates, the first, third, fifth, seventh and ninth ordinates are the odd, and the

second, fourth, sixth and eighth are the even, ordinates referred to in the rule.)

1. Length of base 20 feet. Ordinates = 10, 16, 14, 11, 16 feet.

2. Length of base 325 links. Ordinates 0, 25, 38, 51, 64, 70, 83, 96 links. Here we have a curve cutting the base at one end.

3. Length of base 252 links. Ordinates = 0, 24, 36, 42, 54, 67, 76, 58, 49, 33, 19, 0. Here the curve cuts the base at both ends.

4. Length of base 1260 links. Ordinates = 364, 396, 418, 453, 512, 554, 578 links.

5. Length of base 2364 links. Ordinates = 0, 335, 417, 432, 524, 587, 642, 758 links.

Area by weighing. — Plat the field accurately on a thick piece of paper and cut it out carefully with a sharp knife. On the same sheet draw a square or rectangle of predetermined area to the same scale and cut out. The more nearly the area of the field and regular figure agree the more accurate the method.

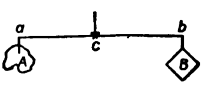


FIG. 67.

On a light rod hang the cut-out field (A) at one end and the regular figure (B) at the other end. Suspend the rod on a thin wire or over a pin and move to right or left until it remains in a horizontal position, showing that A balances B.

$$\textit{The area of } A = \frac{\textit{Area of } B \times \textit{length } bc}{\textit{length } ac}.$$

Area by planimeters. — A planimeter is an instrument used for obtaining the areas of irregular figures. There are several forms and full instructions for use are given with each one sold. Pictures with descriptions are given in the catalogues of instrument dealers.

A form of planimeter that the student can make is known as the "hatchet" planimeter, because one end slightly resembles a hatchet blade. Some writers call it the "jack-knife" planimeter because it resembles a knife with a projecting blade at each end. For demonstrating purposes a pocket knife is a fair substitute for a properly made instrument of this type.

A piece of wire is bent at A and B. On one leg is fastened a weight F and the lower end is flattened to a shape which gives the instrument the name of "hatchet" planimeter. At E on the hatchet blade end make a mark. The other

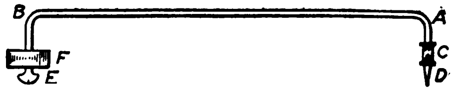


FIG. 68. Hatchet, or jack-knife, planimeter.

leg has a sleeve c, in which the wire may move freely but not too loosely. The lower end of the leg AD is brought to a point.

The distance between D and E may be any length but is usually some even number of inches, preferably a decimal division such as 5, 10, etc. Some planimeters of this type have an adjustable arm AB so the length may be altered, enabling the instrument to be used for comparatively large, as well as for small, drawings.

Estimate as closely as possible the position of the center of the area of the drawing, Fig. 69, and mark it o, drawing through it the lines AB and CD normal to each other. The accuracy of the work depends upon the accuracy with which the point o fixes the center of area (center of gravity). Place the point D of the planimeter at O and the mark E on the line CD, pressing the hatchet edge into the paper slightly to fix the position of the mark. Holding the sleeve C move the point D along the line AB to A. Then go to the right carefully tracing the perimeter with the point until A is reached, and the point again follows the line AB from A to O. The hatchet edge will be close to the starting point on the line CD and should be pressed into the paper to mark the new position of the point E. Measure the distance between the original and final positions of E normal to line CD and call this a_1.

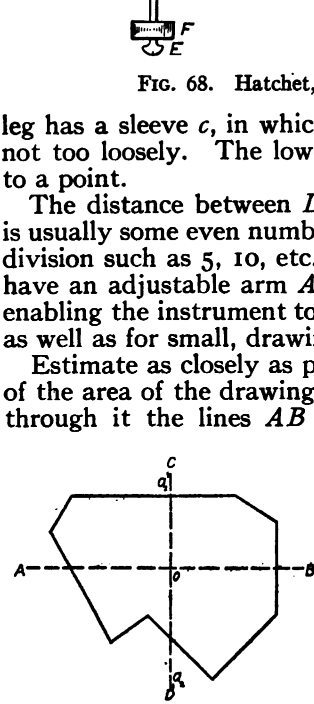


FIG. 69.

2nd. Place D at O and E on the line CD near D, which is equivalent to changing the position of drawing 180 deg. Move from O to A and move the point around the perimeter to the *left*. Call the distance between the first and final positions of E on this second operation a_2.

$$Area = length\ of\ DE \times \frac{a_1 + a_2}{2}.$$

3rd. Closer results may be obtained by next placing the planimeter on the line AB with E near A. Move D along OD towards D and trace the perimeter to the *right*. Call the distance between the first and final positions of E on this third operation a_3.

4th. Place D at O with E on the line AB near B. Move towards D on line OD to the perimeter and trace the perimeter to the *left*. Call the distance between the first and final position of E, on this fourth operation a_4.

$$Area = length\ of\ DE \times \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

To become expert in the use of a planimeter the beginner should check plats of regular figures, the areas of which may be readily found by common methods.

The Use of Squared Paper

Figures may be platted on squared paper and the included squares counted. The sum multiplied by the area of one square = area of figure.

Circles

The perimeter (boundary) of a square with side $d = 4\,d$.

The perimeter (circumference) of a circle with diameter $d = \frac{22}{7} d = 3\frac{1}{7} d = 3.1416\,d$.

The area of a square with side d

$$= \frac{4\,d^2}{4} = d^2.$$

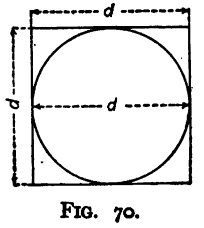


Fig. 70.

The area of a circle with diameter $d = \frac{3.1416\, d^2}{4} = 0.7854\, d^2$.

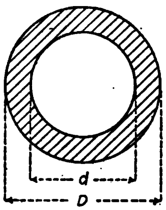


FIG. 71.

Area of a ring.

Let D = outer diameter,
d = inner diameter,
$A = 0.7854\, D^2 - 0.7854\, d^2$
$= 0.7854\, (D^2 - d^2)$.

An excellent text on mensuration is "Practical Mathematics" by Knott & Mackay ($2.00).

STAKING OUT WORK

To stake out a lot for grading.—Grade stakes on lots are generally set in 10-ft. or 25-ft. squares. At the corner of each square is set a stake about one foot long projecting 4 ins. or 6 ins. above the surface, and on this stake is marked the cut or fill.

In Fig. 72 assume that the square $ABCD$ is to be laid out in squares or rectangles. From A in any convenient way lay off the line AD, perpendicular to AB, setting the stakes and marking them Oa, Ob, Oc, etc. From B set off the line BC, perpendicular to BA, setting the stakes and marking them 8 a, 8 b, 8 c, etc.

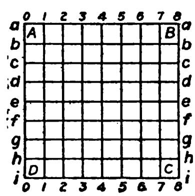


FIG. 72.

Measure from A to B, setting the stakes and marking them 1 a, 2 a, 3 a, etc.

Measure from D to C, setting the stakes and marking them 0 i, 1 i, 2 i, etc.

A 4-ft. lath, with a cloth tied near the top, is set on the line BC at each stake and also on the line DC at each stake. An observer stands at 1 a and sights towards 1 d. Another observer stands at 0 b and sights towards 8 b.

A helper with a bag of stakes and a line pole goes along the line b and is lined in by the two observers by the intersection process, the observer on the b line remaining in place, the observer on the a line moving towards B after each stake is driven.

When stake $7\ a$ is set, the observer at $0\ b$ moves to $0\ c$ and the helper goes to the c line and sets $7\ c$. He and the observer on the a line work back to the 1 line. The other observer then drops to the d line, and the work proceeds thus until all the stakes are set. In Fig. 73 a stake is shown, every stake being marked to indicate the intersection of the two lines on which it stands. The lower figures +7.3 are put on after the elevations are taken.

FIG. 73.

The + sign indicates a cut, the surface of the ground at this stake being 7.3 feet above grade. A − sign indicates a fill.

To stake out a building. — The 3, 4, 5 method for erecting perpendiculars is generally used by contractors when

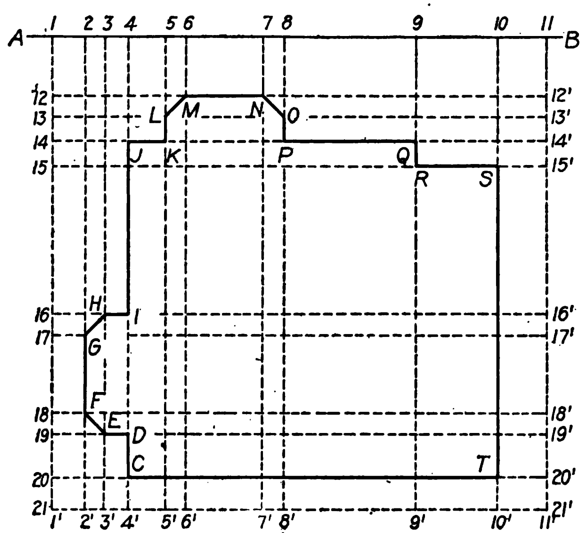


FIG. 74. Staking out a building.

staking out buildings without an instrument for turning angles.

In Fig. 74 the property line AB is assumed to be staked

and the location of the building settled. The heavy lines show the outline of the building.

At 1 and 11 stakes are set, the distance between being 10 ft. more than the width of the building, each stake being 5 ft. from the building line. A chalk line is stretched tightly from nails driven to mark the exact points in the tops of the stakes and the points 2 to 10 inclusive set and nails are driven to mark them.

At 1 and 11 perpendicular lines (1 . . . 21 and 11 to 21′) are set out, the stakes numbered 21 being 5 ft. from the building line. At 21 check measurements are made to make it certain that perfect right angles have been turned. Lines are stretched from 1 to 21 and from 11 to 21′, the intermediate points 12 to 20 inclusive measured off and set with nails marking the exact locations.

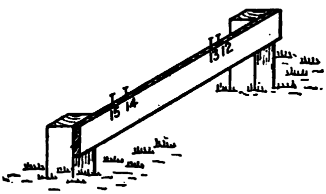


FIG. 75. Batter boards.

When a stake is used the top is usually 1 or 2 ft. above the ground and the nail projects an inch or so above the top so a cord may be tied to it. If a number of points fall within a space of 10 or 12 ft. a couple of posts are driven to which a board is spiked on the line and nails are driven in the top edge of the board.

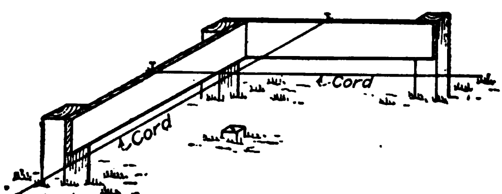


FIG. 76. Batter boards at corner.

The dotted lines represent cords stretched between nails and the points of intersection fix the building lines. The stakes being at least 5 ft. away from the building are safe from disturbance during the excavating period. It is

never necessary to have all the lines in place at one time. Two men can stretch line 2 . . . 2′ and then hold a line on 18 . . . 18′ while a third man drives a stake at *F*. Moving to 17 . . . 17′ a stake is driven at *G*. Line 2 . . . 2′ is moved to 3 . . . 3′, the cross line held on 16 . . . 16′ while *H* is driven and on 19 . . . 19′ while *E* is driven.

The corner line guides are often arranged as shown in Fig. 76, and the cords are tied around the boards, as the pull on a long line will bend a small nail, for all lines must be taut.

The Use of a Table of Squares

Surveyors should become accustomed to the use of labor-saving tables and diagrams. The table here given is very old and of great value.

Find the square of 138.

The square of $138 = 138 \times 138 = 138^2$. Looking in the column headed No. find 138. In the column headed Square find 19,044 opposite 138.

Then $138^2 = 19{,}044$.

Find the cube of 138.

The cube of $138 = 138 \times 138 \times 138 = 138^3$. Looking in the column headed No. find 138. In the column headed Cube find 2,628,072 opposite 138. Then $138^3 = 2{,}628{,}072$.

Proceeding in a similar manner the square root of $138 = \sqrt{138} = 11.747344$ and the cube root of $138 = \sqrt[3]{138} = 5.167649$.

The square root of any number is one of *two equal* factors which multiplied together will produce that number. Thus 5 is the square root of 25, for $25 = 5 \times 5$.

The cube root of any number is one of *three equal* factors which multiplied together produce that number. Thus 5 is the cube root of 125, for $125 = 5 \times 5 \times 5$.

The fourth root of any number is one of *four equal* factors which multiplied together produce that number. Thus 5 is the fourth root of 625, for $625 = 5 \times 5 \times 5 \times 5$.

The sixth root of any number is one of *six equal* factors which multiplied together produce that number. Thus 5 is the sixth root of 1,953,125, for $1{,}953{,}125 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$.

Let a = any number, then

$$a^2 = a \times a,$$

and $a^4 = a \times a \times a \times a.$

$$\therefore a^4 = a^2 \times a^2 = a^{2+2}.$$

$$a^3 = a \times a \times a,$$

and $a^6 = a \times a \times a \times a \times a \times a.$

$$\therefore a^6 = a^3 \times a^3 = a^{3+3}.$$

The exponent = the sum of the times the number is used. It follows that:

$$a^5 = a \times a \times a \times a \times a,$$
$$= a^2 \times a^3 = a^{2+3},$$

and $a^7 = a^2 \times a^2 \times a^3 = a^4 \times a^3 = a^{4+3}.$

A knowledge of the law of exponents permits the use of the table to find the fourth and higher powers.

Thus to find the fourth power of any number, first find the square. Using the square as a number find its square. Find the fourth power of 30.

$$30^2 = 900,$$
$$900^2 = 810{,}000 = 30^4 = 30^2 \times 30^2 = 30 \times 30 \times 30 \times 30.$$

The sixth power is found by using the cube. Find the sixth power of 7.

$$7^3 = 343,$$
$$343^3 = 40{,}353{,}607 = 7^6 = 7^3 \times 7^3 = 7 \times 7 \times 7 \times 7 \times 7 \times 7.$$

To find the fifth power of a number find the square and the cube of the number and multiply.

Find the fifth power of 2.

$$2^2 = 2 \times 2 = 4,$$
$$2^3 = 2 \times 2 \times 2 = 8,$$
$$2^5 = 2^2 \times 2^3 = 4 \times 8 = 32 = 2 \times 2 \times 2 \times 2 \times 2.$$

To find the seventh power of a number multiply the third power by the fourth power.

Find the seventh power of 2.

$$2^3 = 2 \times 2 \times 2 = 8,$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16,$$
$$2^7 = 2^3 \times 2^4 = 8 \times 16 = 128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2.$$

In Fig. 77 $Aa = \frac{AB}{2}$ and $ABCD$ form a square. $\therefore\ Aa' = a'e = ed = Ad$. The square $ABCD$ has an area four times that of the square $Aa'ed$, having sides one-half the length

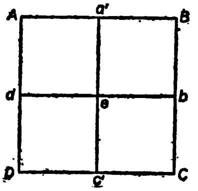


FIG. 77.

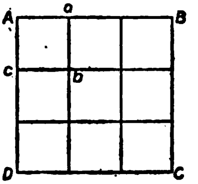


FIG. 78.

of the larger square. Similarly in Fig. 78 the square $ABCD$ has an area nine times as large as the area of the square $Aabc$, having sides one-third the length of the larger square.

The table contains no numbers larger than 1000, therefore the principle just illustrated must be used when the square of a large number is wanted.

Rule. — Divide by a number giving a quotient containing less than four figures. Multiply the square of the quotient by the square of the divisor.

Example. — Find the square of 1220.

$$\frac{1220}{10} = 122.$$
$$122^2 = 14{,}884.$$
$$1220^2 = 122^2 \times 10^2 = 14{,}884 \times 100 = 1{,}488{,}400.$$

Find 90^4.

$$90^2 = 8100.$$
$$\frac{8100}{10} = 810.$$
$$810^2 = 656{,}100.$$
$$90^4 = 810^2 \times 10^2 = 656{,}100 \times 100 = 65{,}610{,}000.$$

Find 95^4.

$$95^2 = 9025.$$
$$\frac{9025}{10} = 902.5.$$

$$\frac{902.5}{25} = 36.1 = \frac{9025}{250}.$$
$$36.1^2 = 1303.21.$$
$$250^2 = 62,500.$$
$$95^4 = 36.1^2 \times 250^2 = 1303.21 \times 62,500 = 81,450,625.$$

In the last example the method of handling squares of decimal numbers is plainly illustrated.

The square of 361 = 130,321.
The square of 36.1 = 1303.21.
The square of 3.61 = 13.0321.

To extract the square root of any number. — The square root of each number less than 1000 is found in the column headed Square Root. If the number is greater than 1000 find it in the column headed Square and the square root will be found in the column headed No.

To extract the cube root of any number. — The cube root of each number less than 1000 is found in the column headed Cube Root. If the number is greater than 1000 find it in the column headed Cube and the cube root will be found in the column headed No.

When the cube is wanted of a number larger than 1000 it will be best to use logarithms.

Barlow's Table Book contains squares, square roots, cubes and cube roots for all numbers from 1 to 10,000, and is sold by all dealers in scientific books. No books are in general circulation containing squares of higher numbers because of the small demand, although tables of squares of all numbers up to 100,000 were once printed.

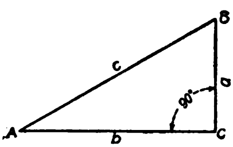


FIG. 79.

a = altitude,
b = base,
c = hypothenuse,

$$c = \sqrt{a^2 + b^2} \quad \text{or} \quad c^2 = a^2 + b^2,$$
$$b = \sqrt{(c+a) \times (c-a)} = \sqrt{c^2 - a^2},$$
$$a = \sqrt{(c+b) \times (c-b)} = \sqrt{c^2 - b^2}.$$

The above formulas should be memorized for they are very useful to surveyors.

The formulas for finding the base and altitude of a right-angled triangle illustrate the algebraic theorem

The product of the sum and difference of two quantities = the difference of their squares.

Examples. — Use the table of squares.

(1)

$$\begin{aligned} a &= 3. \\ b &= 4. \quad \text{Find } c. \\ a^2 &= 3^2 = 9 \\ b^2 &= 4^2 = \underline{16} \\ & 25 \qquad c = \sqrt{25} = 5 \end{aligned}$$

In the column headed Squares find 25, and in column headed No. find 5.

Another method is to find 25 in the column headed No. and in the column headed Square Root find 5.

(2)

$$\begin{aligned} c &= 35. \\ b &= 28. \\ c^2 &= 35^2 = 1225 \\ b^2 &= 28^2 = \underline{784} \\ & 441 \qquad a = \sqrt{441} = 21. \end{aligned}$$

When either number contains more than three figures both numbers must be divided by a number that will reduce them to less than four figure values.

Algebraically and geometrically we can prove

The value of a ratio is not altered when both terms are multiplied or divided by the same quantity.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{3 \times 5}{4 \times 5} = \frac{3 \times 7}{4 \times 7}, \text{ etc.}$$

Using the new values proceed as before. When the square root is found multiply by the number used as a divisor. The result will be the same as though the original values had been used.

We can prove by geometry

Triangles which have two angles equal each to each have their sides proportional.

Example.

$$a = 117.28 \text{ ft.}$$
$$b = 92.20 \text{ ft.} \quad \text{Find } c.$$

Divide by a common divisor.

$$\begin{array}{r|ll}
4 & 117.28 & 92.20 \\ \hline
4 & 29.32 & 23.04 \\ \hline
 & 7.33 & 5.76+
\end{array} \qquad \text{Divisor} = 4 \times 4 = 16$$

$$\begin{array}{rl}
7.33^2 = & 53.7289 \\
5.76^2 = & \underline{33.1776} \\
 & 86.9065 \\
\text{From table} & \underline{86.8624} = 9.32 \\
 & 0.0441 \quad \underline{16} \\
 & 5592 \\
 & \underline{932} \\
 & 149.12 \\
\text{add} & \underline{0.05} \\
 & 149.17 = c.
\end{array}$$

Squaring the lengths of the two sides and extracting the square root of the sum by arithmetic the length of $c = 149.182$. The difference is closer than ordinary work in the field.

No rational reason can be given for adding the remainder 0.0442 (0.05) but we know the root lies between 9.32 and 9.33, and experience has shown that when the second significant figure in the remainder is increased by 1 and the remainder then added, the final error is very small. With a table of squares of numbers up to 10,000 more exact results can be obtained.

SLIDE RULE

The slide rule has become an indispensable tool for engineers, and should be used by every surveyor. The principle of the slide rule is simple although some very complicated forms are manufactured. Full instructions for use accompany each instrument. Purchase only from

a firm of high reputation and test the graduations carefully to see that the *A* scale coincides with the *B* scale, and the *C* scale with the *D* scale.

The best form for the use of a surveyor has the ordinary Mannheim graduations with a reciprocal scale so three numbers can be multiplied at one setting. For office use a 16-in. slide rule is best. For ordinary use either an 8-in. or 10-in. rule will be found satisfactory. The man who is a slave to the slide rule carries one 5 ins. long in his pocket.

MULTIPLYING TABLES

Crelle's Multiplying and Dividing Tables ($5.00) should be in the office of every man who has to do much figuring. On a large survey the time saved will pay for the book, and mistakes can occur only through grave carelessness.

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
1	1	1	1.000	1.000	51	26 01	132 651	7.141	3.708
2	4	8	1.414	1.260	52	27 04	140 608	7.211	3.733
3	9	27	1.732	1.442	53	28 09	148 877	7.280	3.756
4	16	64	2.000	1.587	54	29 16	157 464	7.349	3.780
5	25	125	2.236	1.710	55	30 25	166 375	7.416	3.803
6	36	216	2.449	1.817	56	31 36	175 616	7.483	3.826
7	49	343	2.646	1.913	57	32 49	185 193	7.550	3.849
8	64	512	2.828	2.000	58	33 64	195 112	7.616	3.871
9	81	729	3.000	2.080	59	34 81	205 379	7.681	3.893
10	1 00	1 000	3.162	2.154	60	36 00	216 000	7.746	3.915
11	1 21	1 331	3.317	2.224	61	37 21	226 981	7.810	3.937
12	1 44	1 728	3.464	2.289	62	38 44	238 328	7.874	3.958
13	1 69	2 197	3.606	2.351	63	39 69	250 047	7.937	3.979
14	1 96	2 744	3.742	2.410	64	40 96	262 144	8.000	4.000
15	2 25	3 375	3.873	2.466	65	42 25	274 625	8.062	4.021
16	2 56	4 096	4.000	2.520	66	43 56	287 496	8.124	4.041
17	2 89	4 913	4.123	2.571	67	44 89	300 763	8.185	4.062
18	3 24	5 832	4.243	2.621	68	46 24	314 432	8.246	4.082
19	3 61	6 859	4.359	2.668	69	47 61	328 509	8.307	4.102
20	4 00	8 000	4.472	2.714	70	49 00	343 000	8.367	4.121
21	4 41	9 261	4.583	2.759	71	50 41	357 911	8.426	4.141
22	4 84	10 648	4.690	2.802	72	51 84	373 248	8.485	4.160
23	5 29	12 167	4.796	2.844	73	53 29	389 017	8.544	4.179
24	5 76	13 824	4.899	2.885	74	54 76	405 224	8.602	4.198
25	6 25	15 625	5.000	2.924	75	56 25	421 875	8.660	4.217
26	6 76	17 576	5.099	2.963	76	57 76	438 976	8.718	4.236
27	7 29	19 683	5.196	3.000	77	59 29	456 533	8.775	4.254
28	7 84	21 952	5.292	3.037	78	60 84	474 552	8.832	4.273
29	8 41	24 389	5.385	3.072	79	62 41	493 039	8.888	4.291
30	9 00	27 000	5.477	3.107	80	64 00	512 000	8.944	4.309
31	9 61	29 791	5.568	3.141	81	65 61	531 441	9.000	4.327
32	10 24	32 768	5.657	3.175	82	67 24	551 368	9.055	4.345
33	10 89	35 937	5.745	3.208	83	68 89	571 787	9.110	4.362
34	11 56	39 304	5.831	3.240	84	70 56	592 704	9.165	4.380
35	12 25	42 875	5.916	3.271	85	72 25	614 125	9.220	4.397
36	12 96	46 656	6.000	3.302	86	73 96	636 056	9.274	4.414
37	13 69	50 653	6.083	3.332	87	75 69	658 503	9.327	4.431
38	14 44	54 872	6.164	3.362	88	77 44	681 472	9.381	4.448
39	15 21	59 319	6.245	3.391	89	79 21	704 969	9.434	4.465
40	16 00	64 000	6.325	3.420	90	81 00	729 000	9.487	4.481
41	16 81	68 921	6.403	3.448	91	82 81	753 571	9.539	4.498
42	17 64	74 088	6.481	3.476	92	84 64	778 688	9.592	4.514
43	18 49	79 507	6.557	3.503	93	86 49	804 357	9.644	4.531
44	19 36	85 184	6.633	3.530	94	88 36	830 584	9.695	4.547
45	20 25	91 125	6.708	3.557	95	90 25	857 375	9.747	4.563
46	21 16	97 336	6.782	3.583	96	92 16	884 736	9.798	4.579
47	22 09	103 823	6.856	3.609	97	94 09	912 673	9.849	4.595
48	23 04	110 592	6.928	3.634	98	96 04	941 192	9.900	4.610
49	24 01	117 649	7.000	3.659	99	98 01	970 299	9 950	4.626
50	25 00	125 000	7.071	3.684	100	1 00 00	1 000 000	10.000	4.642

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
101	1 02 01	1 030 301	10.0499	4.6570	151	2 28 01	3 442 951	12.2882	5.3251
102	1 04 04	1 061 208	10.0995	4.6723	152	2 31 04	3 511 808	12.3288	5.3368
103	1 06 09	1 092 727	10.1489	4.6875	153	2 34 09	3 581 577	12.3693	5.3485
104	1 08 16	1 124 864	10.1980	4.7027	154	2 37 16	3 652 264	12.4097	5.3601
105	1 10 25	1 157 625	10.2470	4.7177	155	2 40 25	3 723 875	12.4499	5.3717
106	1 12 36	1 191 016	10.2956	4.7326	156	2 43 36	3 796 416	12.4900	5.3832
107	1 14 49	1 225 043	10.3441	4.7475	157	2 46 49	3 869 893	12.5300	5.3947
108	1 16 64	1 259 712	10.3923	4.7622	158	2 49 64	3 944 312	12.5698	5.4061
109	1 18 81	1 295 029	10.4403	4.7769	159	2 52 81	4 019 679	12.6095	5.4175
110	1 21 00	1 331 000	10.4881	4.7914	160	2 56 00	4 096 000	12.6491	5.4288
111	1 23 21	1 367 631	10.5357	4.8059	161	2 59 21	4 173 281	12.6886	5.4401
112	1 25 44	1 404 928	10.5830	4.8203	162	2 62 44	4 251 528	12.7279	5.4514
113	1 27 69	1 442 897	10.6301	4.8346	163	2 65 69	4 330 747	12.7671	5.4626
114	1 29 96	1 481 544	10.6771	4.8488	164	2 68 96	4 410 944	12.8062	5.4737
115	1 32 25	1 520 875	10.7238	4.8629	165	2 72 25	4 492 125	12.8452	5.4848
116	1 34 56	1 560 896	10.7703	4.8770	166	2 75 56	4 574 296	12.8841	5.4959
117	1 36 89	1 601 613	10.8167	4.8910	167	2 78 89	4 657 463	12.9228	5.5069
118	1 39 24	1 643 032	10.8628	4.9049	168	2 82 24	4 741 632	12.9615	5.5178
119	1 41 61	1 685 159	10.9087	4.9187	169	2 85 61	4 826 809	13.0000	5.5288
120	1 44 00	1 728 000	10.9545	4.9324	170	2 89 00	4 913 000	13.0384	5.5397
121	1 46 41	1 771 561	11.0000	4.9461	171	2 92 41	5 000 211	13.0767	5.5505
122	1 48 84	1 815 848	11.0454	4.9597	172	2 95 84	5 088 448	13.1149	5.5613
123	1 51 29	1 860 867	11.0905	4.9732	173	2 99 29	5 177 717	13.1529	5.5721
124	1 53 76	1 906 624	11.1355	4.9866	174	3 02 76	5 268 024	13.1909	5.5828
125	1 56 25	1 953 125	11.1803	5.0000	175	3 06 25	5 359 375	13.2288	5.5934
126	1 58 76	2 000 376	11.2250	5.0133	176	3 09 76	5 451 776	13.2665	5.6041
127	1 61 29	2 048 383	11.2694	5.0265	177	3 13 29	5 545 233	13.3041	5.6147
128	1 63 84	2 097 152	11.3137	5.0397	178	3 16 84	5 639 752	13.3417	5.6252
129	1 66 41	2 146 689	11.3578	5.0528	179	3 20 41	5 735 339	13.3791	5.6357
130	1 69 00	2 197 000	11.4018	5.0658	180	3 24 00	5 832 000	13.4164	5.6462
131	1 71 60	2 248 091	11.4455	5.0788	181	3 27 61	5 929 741	13.4536	5.6567
132	1 74 24	2 299 968	11.4891	5.0916	182	3 31 24	6 028 568	13.4907	5.6671
133	1 76 89	2 352 637	11.5326	5.1045	183	3 34 89	6 128 487	13.5277	5.6774
134	1 79 56	2 406 104	11.5758	5.1172	184	3 38 56	6 229 504	13.5647	5.6877
135	1 82 25	2 460 375	11.6190	5.1299	185	3 42 25	6 331 625	13.6015	5.6980
136	1 84 96	2 515 456	11.6619	5.1426	186	3 45 96	6 434 856	13.6382	5.7083
137	1 87 69	2 571 353	11.7047	5.1551	187	3 49 69	6 539 203	13.6748	5.7185
138	1 90 44	2 628 072	11.7473	5.1676	188	3 53 44	6 644 672	13.7113	5.7287
139	1 93 21	2 685 619	11.7898	5.1801	189	3 57 21	6 751 269	13.7477	5.7388
140	1 96 00	2 744 000	11.8322	5.1925	190	3 61 00	6 859 000	13.7840	5.7489
141	1 98 81	2 803 221	11.8743	5.2048	191	3 64 81	6 967 871	13.8203	5.7590
142	2 01 64	2 863 288	11.9164	5.2171	192	3 68 64	7 077 888	13.8564	5.7690
143	2 04 49	2 924 207	11.9583	5.2293	193	3 72 49	7 189 057	13.8924	5.7790
144	2 07 36	2 985 984	12.0000	5.2415	194	3 76 36	7 301 384	13.9284	5.7890
145	2 10 25	3 048 625	12.0416	5.2536	195	3 80 25	7 414 875	13.9642	5.7989
146	2 13 16	3 112 136	12.0830	5.2656	196	3 84 16	7 529 536	14.0000	5.8088
147	2 16 09	3 176 523	12.1244	5.2776	197	3 88 09	7 645 373	14.0357	5.8186
148	2 19 04	3 241 792	12.1655	5.2896	198	3 92 04	7 762 392	14.0712	5.8285
149	2 22 01	3 307 949	12.2066	5.3015	199	3 96 01	7 880 599	14.1067	5.8383
150	2 25 00	3 375 000	12.2474	5.3133	200	4 00 00	8 000 000	14.1421	5.8480

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
201	4 04 01	8 120 601	14.1774	5.8578	251	6 30 01	15 813 251	15.8430	6.3080
202	4 08 04	8 242 408	14.2127	5.8675	252	6 35 04	16 003 008	15.8745	6.3164
203	4 12 09	8 365 427	14.2478	5.8771	253	6 40 09	16 194 277	15.9060	6.3247
204	4 16 16	8 489 664	14.2829	5.8868	254	6 45 16	16 387 064	15.9374	6.3330
205	4 20 25	8 615 125	14.3178	5.8964	255	6 50 25	16 581 375	15.9687	6.3413
206	4 24 36	8 741 816	14.3527	5.9059	256	6 55 36	16 777 216	16.0000	6.3496
207	4 28 49	8 869 743	14.3875	5.9155	257	6 60 49	16 974 593	16.0312	6.3579
208	4 32 64	8 998 912	14.4222	5.9250	258	6 65 64	17 173 512	16.0624	6.3661
209	4 36 81	9 129 329	14.4568	5.9345	259	6 70 81	17 373 979	16.0935	6.3743
210	4 41 00	9 261 000	14.4914	5.9439	260	6 76 00	17 576 000	16.1245	6.3825
211	4 45 21	9 393 931	14.5258	5.9533	261	6 81 21	17 779 581	16.1555	6.3907
212	4 49 44	9 528 128	14.5602	5.9627	262	6 86 44	17 984 728	16.1864	9.3988
213	4 53 69	9 663 597	14.5945	5.9721	263	6 91 69	18 191 447	16.2173	6.4070
214	4 57 96	9 800 344	14.6287	5.9814	264	6 96 96	18 399 744	16.2481	6.4151
215	4 62 25	9 938 375	14.6629	5.9907	265	7 02 25	18 609 625	16.2788	6.4232
216	4 66 56	10 077 696	14.6969	6.0000	266	7 07 56	18 821 096	16.3095	6.4312
217	4 70 89	10 218 313	14.7309	6.0092	267	7 12 89	19 034 163	16.3401	6.4393
218	4 75 24	10 360 232	14.7648	6.0185	268	7 18 24	19 248 832	16.3707	6.4473
219	4 79 61	10 503 459	14.7986	6.0277	269	7 23 61	19 465 109	16.4012	6.4553
220	4 84 00	10 648 000	14.8324	6.0368	270	7 29 00	19 683 000	16.4317	6.4633
221	4 88 41	10 793 861	14.8661	6.0459	271	7 34 41	19 902 511	16.4621	6.4713
222	4 92 84	10 941 048	14.8997	6.0550	272	7 39 84	20 123 648	16.4924	6.4792
223	4 97 29	11 089 567	14.9332	6.0641	273	7 45 29	20 346 417	16.5227	6.4872
224	5 01 76	11 239 424	14.9666	6.0732	274	7 50 76	20 570 824	16.5529	6.4951
225	5 06 25	11 390 625	15.0000	6.0822	275	7 56 25	20 796 875	16.5831	6.5030
226	5 10 76	11 543 176	15.0333	6.0912	276	7 61 76	21 024 576	16.6132	6.5108
227	5 15 29	11 697 083	15.0665	6.1002	277	7 67 29	21 253 933	16.6433	6.5187
228	5 19 84	11 852 352	15.0997	6.1091	278	7 72 84	21 484 952	16.6733	6.5265
229	5 24 41	12 008 989	15.1327	6.1180	279	7 78 41	21 717 639	16.7033	6.5343
230	5 29 00	12 167 000	15.1658	6.1269	280	7 84 00	21 952 000	16.7332	6.5421
231	5 33 61	12 326 391	15.1987	6.1358	281	7 89 61	22 188 041	16.7631	6.5499
232	5 38 24	12 487 168	15.2315	6.1446	282	7 95 24	22 425 768	16.7929	6.5577
233	5 42 89	12 649 337	15.2643	6.1534	283	8 00 89	22 665 187	16.8226	6.5654
234	5 47 56	12 812 904	15.2971	6.1622	284	8 06 56	22 906 304	16.8523	6.5731
235	5 52 25	12 977 875	15.3297	6.1710	285	8 12 25	23 149 125	16.8819	6.5808
236	5 56 96	13 144 256	15.3623	6.1797	286	8 17 96	23 393 656	16.9115	6.5885
237	5 61 69	13 312 053	15.3948	6.1885	287	8 23 69	23 639 903	16.9411	6.5962
238	5 66 44	13 481 272	15.4272	6.1972	288	8 29 44	23 887 872	16.9706	6.6039
239	5 71 21	13 651 919	15.4596	6.2058	289	8 35 21	24 137 569	17.0000	6.6115
240	5 76 00	13 824 000	15.4919	6.2145	290	8 41 00	24 389 000	17.0294	6.6191
241	5 80 81	13 997 521	15.5242	6.2231	291	8 46 81	24 642 171	17.0587	6.6267
242	5 85 64	14 172 488	15.5563	6.2317	292	8 52 64	24 897 088	17.0880	6.6343
243	5 90 49	14 348 907	15.5885	6.2403	293	8 58 49	25 153 757	17.1172	6.6419
244	5 95 36	14 526 784	15.6205	6.2488	294	8 64 36	25 412 184	17.1464	6.6494
245	6 00 25	14 706 125	15.6525	6.2573	295	8 70 25	25 672 375	17.1756	6.6569
246	6 05 16	14 886 936	15.6844	6.2658	296	8 76 16	25 934 336	17.2047	6.6644
247	6 10 09	15 069 223	15.7162	6.2743	297	8 82 09	26 198 073	17.2337	6.6719
248	6 15 04	15 252 992	15.7480	6.2828	298	8 88 04	26 463 592	17.2627	6.6794
249	6 20 01	15 438 249	15.7797	6.2912	299	8 94 01	26 730 899	17.2916	6.6869
250	6 25 00	15 625 000	15.8114	6.2996	300	9 00 00	27 000 000	17.3205	6.6943

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
301	9 06 01	27 270 901	17.3494	6.7018	351	12 32 01	43 243 551	18.7350	7.0540
302	9 12 04	27 543 608	17.3781	6.7092	352	12 39 04	43 614 208	18.7617	7.0607
303	9 18 09	27 818 127	17.4069	6.7166	353	12 46 09	43 986 977	18.7883	7.0674
304	9 24 16	28 094 464	17.4356	6.7240	354	12 53 16	44 361 864	18.8149	7.0740
305	9 30 25	28 372 625	17.4642	6.7313	355	12 60 25	44 738 875	18.8414	7.0807
306	9 36 36	28 652 616	17.4929	6.7387	356	12 67 36	45 118 016	18.8680	7.0873
307	9 42 49	28 934 443	17.5214	6.7460	357	12 74 49	45 499 293	18.8944	7.0940
308	9 48 64	29 218 112	17.5499	6.7533	358	12 81 64	45 882 712	18.9209	7.1006
309	9 54 81	29 503 629	17.5784	6.7606	359	12 88 81	46 268 279	18.9473	7.1072
310	9 61 00	29 791 000	17.6068	6.7679	360	12 96 00	46 656 000	18.9737	7.1138
311	9 67 21	30 080 231	17.6352	6.7752	361	13 03 21	47 045 881	19.0000	7.1204
312	9 73 44	30 371 328	17.6635	6.7824	362	13 10 44	47 437 928	19.0263	7.1269
313	9 79 69	30 664 297	17.6918	6.7897	363	13 17 69	47 832 147	19.0526	7.1335
314	9 85 96	30 959 144	17.7200	6.7969	364	13 24 96	48 228 544	19.0788	7.1400
315	9 92 25	31 255 875	17.7482	6.8041	365	13 32 25	48 627 125	19.1050	7.1466
316	9 98 56	31 554 496	17.7764	6.8113	366	13 39 56	49 027 896	19.1311	7.1531
317	10 04 89	31 855 013	17.8045	6.8185	367	13 46 89	49 430 863	19.1572	7.1596
318	10 11 24	32 157 432	17.8326	6.8256	368	13 54 24	49 836 032	19.1833	7.1661
319	10 17 61	32 461 759	17.8606	6.8328	369	13 61 61	50 243 409	19.2094	7.1726
320	10 24 00	32 768 000	17.8885	6.8399	370	13 69 00	50 653 000	19.2354	7.1791
321	10 30 41	33 076 161	17.9165	6.8470	371	13 76 41	51 064 811	19.2614	7.1855
322	10 36 84	33 386 248	17.9444	6.8541	372	13 83 84	51 478 848	19.2873	7.1920
323	10 43 29	33 698 267	17.9722	6.8612	373	13 91 29	51 895 117	19.3132	7.1984
324	10 49 76	34 012 224	18.0000	6.8683	374	13 98 76	52 313 624	19.3391	7.2048
325	10 56 25	34 328 125	18.0278	6.8753	375	14 06 25	52 734 375	19.3649	7.2112
326	10 62 76	34 645 976	18.0555	6.8824	376	14 13 76	53 157 376	19.3907	7.2177
327	10 69 29	34 965 783	18.0831	6.8894	377	14 21 29	53 582 633	19.4165	7.2240
328	10 75 84	35 287 552	18.1108	6.8964	378	14 28 84	54 010 152	19.4422	7.2304
329	10 82 41	35 611 289	18.1384	6.9034	379	14 36 41	54 439 939	19.4679	7.2368
330	10 89 00	35 937 000	18.1659	6.9104	380	14 44 00	54 872 000	19.4936	7.2432
331	10 95 61	36 264 691	18.1934	6.9174	381	14 51 61	55 306 341	19.5192	7.2495
332	11 02 24	36 594 368	18.2209	6.9244	382	14 59 24	55 742 968	19.5448	7.2558
333	11 08 89	36 926 037	18.2483	6.9313	383	14 66 89	56 181 887	19.5704	7.2622
334	11 15 56	37 259 704	18.2757	6.9382	384	14 74 56	56 623 104	19.5959	7.2685
335	11 22 25	37 595 375	18.3030	6.9451	385	14 82 25	57 066 625	19.6214	7.2748
336	11 28 96	37 933 056	18.3303	6.9521	386	14 89 96	57 512 456	19.6469	7.2811
337	11 35 69	38 272 753	18.3576	6.9589	387	14 97 69	57 960 603	19.6723	7.2874
338	11 42 44	38 614 472	18.3848	6.9658	388	15 05 44	58 411 072	19.6977	7.2936
339	11 49 21	38 958 219	18.4120	6.9727	389	15 13 21	58 863 869	19.7231	7.2999
340	11 56 00	39 304 000	18.4391	6.9795	390	15 21 00	59 319 000	19.7484	7.3061
341	11 62 81	39 651 821	18.4662	6.9864	391	15 28 81	59 776 471	19.7737	7.3124
342	11 69 64	40 001 688	18.4932	6.9932	392	15 36 64	60 236 288	19.7990	7.3186
343	11 76 49	40 353 607	18.5203	7.0000	393	15 44 49	60 698 457	19.8242	7.3248
344	11 83 36	40 707 584	18.5472	7.0068	394	15 52 36	61 162 984	19.8494	7.3310
345	11 90 25	41 063 625	18.5742	7.0136	395	15 60 25	61 629 875	19.8746	7.3372
346	11 97 16	41 421 736	18.6011	7.0203	396	15 68 16	62 099 136	19.8997	7.3434
347	12 04 09	41 781 923	18.6279	7.0271	397	15 76 09	62 570 773	18.9249	7.3496
348	12 11 04	42 144 192	18.6548	7.0338	398	15 84 04	63 044 792	19.9499	7.3558
349	12 18 01	42 508 549	18.6815	7.0406	399	15 92 01	63 521 199	19.9750	7.3619
350	12 25 00	42 875 000	18.7083	7.0473	400	16 00 00	64 000 000	20.0000	7.3681

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
401	16 08 01	64 481 201	20.0250	7.3742	451	20 34 01	91 733 851	21.2368	7.6688
402	16 16 04	64 964 808	20.0499	7.3803	452	20 43 04	92 345 408	21.2603	7.6744
403	16 24 09	65 450 827	20.0749	7.3864	453	20 52 09	92 959 677	21.2838	7.6801
404	16 32 16	65 939 264	20.0998	7.3925	454	20 61 16	93 576 664	21.3073	7.6857
405	16 40 25	66 430 125	20.1246	7.3986	455	20 70 25	94 196 375	21.3307	7.6914
406	16 48 36	66 923 416	20.1494	7.4047	456	20 79 36	94 818 816	21.3542	7.6970
407	16 56 49	67 419 143	20.1742	7.4108	457	20 88 49	95 443 993	21.3776	7.7026
408	16 64 64	67 917 312	20.1990	7.4169	458	20 97 64	96 071 912	21.4009	7.7082
409	16 72 81	68 417 929	20.2237	7.4229	459	21 06 81	96 702 579	21.4243	7.7138
410	16 81 00	68 921 000	20.2485	7.4290	460	21 16 00	97 336 000	21.4476	7.7194
411	16 89 21	69 426 531	20.2731	7.4350	461	21 25 21	97 972 181	21.4709	7.7250
412	16 97 44	69 934 528	20.2978	7.4410	462	21 34 44	98 611 128	21.4942	7.7306
413	17 05 69	70 444 997	20.3224	7.4470	463	21 43 69	99 252 847	21.5174	7.7362
414	17 13 96	70 957 944	20.3470	7.4530	464	21 52 96	99 897 344	21.5407	7.7418
415	17 22 25	71 473 375	20.3715	7.4590	465	21 62 25	100 544 625	21.5639	7.7473
416	17 30 56	71 991 296	20.3961	7.4650	466	21 71 56	101 194 696	21.5870	7.7529
417	17 38 89	72 511 713	20.4206	7.4710	467	21 80 89	101 847 563	21.6102	7.7584
418	17 47 24	73 034 632	20.4450	7.4770	468	21 90 24	102 503 232	21.6333	7.7639
419	17 55 61	73 560 059	20.4695	7.4829	469	21 99 61	103 161 709	21.6564	7.7695
420	17 64 00	74 088 000	20.4939	7.4889	470	22 09 00	103 823 000	21.6795	7.7750
421	17 72 41	74 618 461	20.5183	7.4948	471	22 18 41	104 487 111	21.7025	7.7805
422	17 80 84	75 151 448	20.5426	7.5007	472	22 27 84	105 154 048	21.7256	7.7860
423	17 89 29	75 686 967	20.5670	7.5067	473	22 37 29	105 823 817	21.7486	7.7915
424	17 97 76	76 225 024	20.5913	7.5126	474	22 46 76	106 496 424	21.7715	7.7970
425	18 06 25	76 765 625	20.6155	7.5185	475	22 56 25	107 171 875	21.7945	7.8025
426	18 14 76	77 308 776	20.6398	7.5244	476	22 65 76	107 850 176	21.8174	7.8079
427	18 23 29	77 854 483	20.6640	7.5302	477	22 75 29	108 531 333	21.8403	7.8134
428	18 31 84	78 402 752	20.6882	7.5361	478	22 84 84	109 215 352	21.8632	7.8188
429	18 40 41	78 953 589	20.7123	7.5420	479	22 94 41	109 902 239	21.8861	7.8243
430	18 49 00	79 507 000	20.7364	7.5478	480	23 04 00	110 592 000	21.9089	7.8297
431	18 57 61	80 062 991	20.7605	7.5537	481	23 13 61	111 284 641	21.9317	7.8352
432	18 66 24	80 621 568	20.7846	7.5595	482	23 23 24	111 980 168	21.9545	7.8406
433	18 74 89	81 182 737	20.8087	7.5654	483	23 32 89	112 678 587	21.9773	7.8460
434	18 83 56	81 746 504	20.8327	7.5712	484	23 42 56	113 379 904	22.0000	7.8514
435	18 92 25	82 312 875	20.8567	7.5770	485	23 52 25	114 084 125	22.0227	7.8568
436	19 00 96	82 881 856	20.8806	7.5828	486	23 61 96	114 791 256	22.0454	7.8622
437	19 09 69	83 453 453	20.9045	7.5886	487	23 71 69	115 501 303	22.0681	7.8676
438	19 18 44	84 027 672	20.9284	7.5944	488	23 81 44	116 214 272	22.0907	7.8730
439	19 27 21	84 604 519	20.9523	7.6001	489	23 91 21	116 930 169	22.1133	7.8784
440	19 36 00	85 184 000	20.9762	7.6059	490	24 01 00	117 649 000	22.1359	7.8837
441	19 44 81	85 766 121	21.0000	7.6117	491	24 10 81	118 370 771	22.1585	7.8891
442	19 53 64	86 350 888	21.0238	7.6174	492	24 20 64	119 095 488	22.1811	7.8944
443	19 62 49	86 938 307	21.0476	7.6232	493	24 30 49	119 823 157	22.2036	7.8998
444	19 71 36	87 528 384	21.0713	7.6289	494	24 40 36	120 553 784	22.2261	7.9051
445	19 80 25	88 121 125	21.0950	7.6346	495	24 50 25	121 287 375	22.2486	7.9105
446	19 89 16	88 716 536	21.1187	7.6403	496	24 60 16	122 023 936	22.2711	7.9158
447	19 98 09	89 314 623	21.1424	7.6460	497	24 70 09	122 763 473	22.2935	7.9211
448	20 07 04	89 915 392	21.1660	7.6517	498	24 80 04	123 505 992	22.3159	7.9264
449	20 16 01	90 518 849	21.1896	7.6574	499	24 90 01	124 251 499	22.3383	7.9317
450	20 25 00	91 125 000	21.2132	7.6631	500	25 00 00	125 000 000	22.3607	7.9370

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
501	25 10 01	125 751 501	22.3830	7.9423	551	30 36 01	167 284 151	23.4734	8.1982
502	25 20 04	126 506 008	22.4054	7.9476	552	30 47 04	168 196 608	23.4947	8.2031
503	25 30 09	127 263 527	22.4277	7.9528	553	30 58 09	169 112 377	23.5160	8.2081
504	25 40 16	128 024 064	22.4499	7.9581	554	30 69 16	170 031 464	23.5372	8.2130
505	25 50 25	128 787 625	22.4722	7.9634	555	30 80 25	170 953 875	23.5584	8.2180
506	25 60 36	129 554 216	22.4944	7.9686	556	30 91 36	171 879 616	23.5797	8.2229
507	25 70 49	130 323 843	22.5167	7.9739	557	31 02 49	172 808 693	23.6008	8.2278
508	25 80 64	131 096 512	22.5389	7.9791	558	31 13 64	173 741 112	23.6220	8.2327
509	25 90 81	131 872 229	22.5610	7.9843	559	31 24 81	174 676 879	23.6432	8.2377
510	26 01 00	132 651 000	22.5832	7.9896	560	31 36 00	175 616 000	23.6643	8.2426
511	26 11 21	133 432 831	22.6053	7.9948	561	31 47 21	176 558 481	23.6854	8.2475
512	26 21 44	134 217 728	22.6274	8.0000	562	31 58 44	177 504 328	23.7065	8.2524
513	26 31 69	135 005 697	22.6495	8.0052	563	31 69 69	178 453 547	23.7276	8.2573
514	26 41 96	135 796 744	22.6716	8.0104	564	31 80 96	179 406 144	23.7487	8.2621
515	26 52 25	136 590 875	22.6936	8.0156	565	31 92 25	180 362 125	23.7697	8.2670
516	26 62 56	137 388 096	22.7156	8.0208	566	32 03 56	181 321 496	23.7908	8.2719
517	26 72 89	138 188 413	22.7376	8.0260	567	32 14 89	182 284 263	23.8118	8.2768
518	26 83 24	138 991 832	22.7596	8.0311	568	32 26 24	183 250 432	23.8328	8.2816
519	26 93 61	139 798 359	22.7816	8.0363	569	32 37 61	184 220 009	23.8537	8.2865
520	27 04 00	140 608 000	22.8035	8.0415	570	32 49 00	185 193 000	25.8747	8.2913
521	27 14 41	141 420 761	22.8254	8.0466	571	32 60 41	186 169 411	23.8956	8.2962
522	27 24 84	142 236 648	22.8473	8.0517	572	32 71 84	187 149 248	23.9165	8.3010
523	27 35 29	143 055 667	22.8692	8.0569	573	32 83 29	188 132 517	23.9374	8.3059
524	27 45 76	143 877 824	22.8910	8.0620	574	32 94 76	189 119 224	23.9583	8.3107
525	27 56 25	144 703 125	22.9129	8.0671	575	33 06 25	190 109 375	23.9792	8.3155
526	27 66 76	145 531 576	22.9347	8.0723	576	33 17 76	191 102 976	24.0000	8.3203
527	27 77 29	146 363 183	22.9565	8.0774	577	33 29 29	192 100 033	24.0208	8.3251
528	27 87 84	147 197 952	22.9783	8.0825	578	33 40 84	193 100 552	24.0416	8.3300
529	27 98 41	148 035 889	23.0000	8.0876	579	33 52 41	194 104 539	24.0624	8.3348
530	28 09 00	148 877 000	23.0217	8.0927	580	33 64 00	195 112 000	24.0832	8.3396
531	28 19 61	149 721 291	23.0434	8.0978	581	33 75 61	196 122 941	24.1039	8.3443
532	28 30 24	150 568 768	23.0651	8.1028	582	33 87 24	197 137 368	24.1247	8.3491
533	28 40 89	151 419 437	23.0868	8.1079	583	33 98 89	198 155 287	24.1454	8.3539
534	28 51 56	152 273 304	23.1084	8.1130	584	34 10 56	199 176 704	24.1661	8.3587
535	28 62 25	153 130 375	23.1301	8.1180	585	34 22 25	200 201 625	24.1868	8.3634
536	28 72 96	153 990 656	23.1517	8.1231	586	34 33 96	201 230 056	24.2074	8.3682
537	28 83 69	154 854 153	23.1733	8.1281	587	34 45 69	202 262 003	24.2281	8.3730
538	28 94 44	155 720 872	23.1948	8.1332	588	34 57 44	203 297 472	24.2487	8.3777
539	29 05 21	156 590 819	23.2164	8.1382	589	34 69 21	204 336 469	24.2693	8.3825
540	29 16 00	157 464 000	23.2379	8.1433	590	34 81 00	205 379 000	24.2899	8.3872
541	29 26 81	158 340 421	23.2594	8.1483	591	34 92 81	206 425 071	24.3105	8.3919
542	29 37 64	159 220 088	23.2809	8.1533	592	35 04 64	207 474 688	24.3311	8.3967
543	29 48 49	160 103 007	23.3024	8.1583	593	35 16 49	208 527 857	24.3516	8.4014
544	29 59 36	160 989 184	23.3238	8.1633	594	35 28 36	209 584 584	24.3721	8.4061
545	29 70 25	161 878 625	23.3452	8.1683	595	35 40 25	210 644 875	24.3926	8.4108
546	29 81 16	162 771 336	23.3666	8.1733	596	35 52 16	211 708 736	24.4131	8.4155
547	29 92 09	163 667 323	23.3880	8.1783	597	35 64 09	212 776 173	24.4336	8.4202
548	30 03 04	164 566 592	23.4094	8.1833	598	35 76 04	213 847 192	24.4540	8.4249
549	30 14 01	165 469 149	23.4307	8.1882	599	35 88 01	214 921 799	24.4745	8.4296
550	30 25 00	166 375 000	23.4521	8.1932	600	36 00 00	216 000 000	24.4949	8.4343

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
601	36 12 01	217 081 801	24.5153	8.4390	651	42 38 01	275 894 451	25.5147	8.6668
602	36 24 04	218 167 208	24.5357	8.4437	652	42 51 04	277 167 808	25.5343	8.6713
603	36 36 09	219 256 227	24.5561	8.4484	653	42 64 09	278 445 077	25.5539	8.6757
604	36 48 16	220 348 864	24.5764	8.4530	654	42 77 16	279 726 264	25.5734	8.6801
605	36 60 25	221 445 125	24.5967	8.4577	655	42 90 25	281 011 375	25.5930	8.6845
606	36 72 36	222 545 016	24.6171	8.4623	656	43 03 36	282 300 416	25.6125	8.6890
607	36 84 49	223 648 543	24.6374	8.4670	657	43 16 49	283 593 393	25.6320	8.6934
608	36 96 64	224 755 712	24.6577	8.4716	658	43 29 64	284 890 312	25.6515	8.6978
609	37 08 81	225 866 529	24.6779	8.4763	659	43 42 81	286 191 179	25.6710	8.7022
610	37 21 00	226 981 000	24.6982	8.4809	660	43 56 00	287 496 000	25.6905	8.7066
611	37 33 21	228 099 131	24.7184	8.4856	661	43 69 21	288 804 781	25.7099	8.7110
612	37 45 44	229 220 928	24.7386	8.4902	662	43 82 44	290 117 528	25.7294	8.7154
613	37 57 69	230 346 397	24.7588	8.4948	663	43 95 69	291 434 247	25.7488	8.7198
614	37 69 96	231 475 544	24.7790	8.4994	664	44 08 96	292 754 944	25.7682	8.7241
615	37 82 25	232 608 375	24.7992	8.5040	665	44 22 25	294 079 625	25.7876	8.7285
616	37 94 56	333 744 896	24.8193	8.5086	666	44 35 56	295 408 296	25 8070	8.7329
617	38 06 89	234 885 113	24.8395	8.5132	667	44 48 89	296 740 963	25.8263	8.7373
618	38 19 24	236 029 032	24.8596	8.5178	668	44 62 24	298 077 632	25.8457	8.7416
619	38 31 61	237 176 659	24.8797	8.5224	669	44 75 61	299 418 309	25.8650	8.7460
620	38 44 00	238 328 000	24.8998	8.5270	670	44 89 00	300 763 000	25.8844	8.7503
621	38 56 41	239 483 061	24.9199	8.5316	671	45 02 41	302 111 711	25.9037	8.7547
622	38 68 84	240 641 848	24.9399	8.5362	672	45 15 84	303 464 448	25.9230	8.7590
623	38 81 29	241 804 367	24.9600	8.5408	673	45 29 29	304 821 217	25.9422	8.7634
624	38 93 76	242 970 624	24.9800	8.5453	674	45 42 76	306 182 024	25.9615	8.7677
625	39 06 25	244 140 625	25.0000	8.5499	675	45 56 25	307 546 875	25.9808	8.7721
626	39 18 76	245 314 376	25.0200	8.5544	676	45 69 76	308 915 776	26.0000	8.7764
627	39 31 29	246 491 883	25.0400	8.5590	677	45 83 29	310 288 733	26.0192	8.7807
628	39 43 84	247 673 152	25.0599	8.5635	678	45 96 84	311 665 752	26.0384	8.7850
629	39 56 41	248 858 189	25.0799	8.5681	679	46 10 41	313 046 839	26.0576	8.7893
630	39 69 00	250 047 000	25.0998	8.5726	680	46 24 00	314 432 000	26.0768	8.7937
631	39 81 61	251 239 591	25.1197	8.5772	681	46 37 61	315 821 241	26.0960	8.7980
632	39 94 24	252 435 968	25.1396	8.5817	682	46 51 24	317 214 568	26.1151	8.8023
633	40 06 89	253 636 137	25.1595	8.5862	683	46 64 89	318 611 987	26.1343	8.8066
634	40 19 56	254 840 104	25.1794	8.5907	684	46 78 56	320 013 504	26.1534	8.8109
635	40 32 25	256 047 875	25.1992	8.5952	685	46 92 25	321 419 125	26.1725	8.8152
636	40 44 96	257 259 456	25.2190	8.5997	686	47 05 96	322 828 856	26.1916	8.8194
637	40 57 69	258 474 853	25.2389	8.6043	687	47 19 69	324 242 703	26.2107	8.8237
638	40 70 44	259 694 072	25.2587	8.6088	688	47 33 44	325 660 672	26.2298	8.8280
639	40 83 21	260 917 119	25.2784	8.6132	689	47 47 21	327 082 769	26.2488	8.8323
640	40 96 00	262 144 000	25.2982	8.6177	690	47 61 00	328 509 000	26.2679	8.8366
641	41 08 81	263 374 721	25.3180	8.6222	691	47 74 81	329 939 371	26.2869	8.8408
642	41 21 64	264 609 288	25.3377	8.6267	692	47 88 64	331 373 888	26.3059	8.8451
643	41 34 49	265 847 707	25.3574	8.6312	693	48 02 49	332 812 557	26.3249	8.8493
644	41 47 36	267 089 984	25.3772	8.6357	694	48 16 36	334 255 384	26.3439	8.8536
645	41 60 25	268 336 125	25.3969	8.6401	695	48 30 25	335 702 375	26.3629	8.8578
646	41 73 16	269 586 136	25.4165	8.6446	696	48 44 16	337 153 536	26.3818	8.8621
647	41 86 09	270 840 023	25.4362	8.6490	697	48 58 09	338 608 873	26.4008	8.8663
648	41 99 04	272 097 792	25.4558	8.6535	698	48 72 04	340 068 392	26.4197	8.8706
649	42 12 01	273 359 449	25.4755	8.6579	699	48 86 01	341 532 099	26.4386	8.8748
650	42 25 00	274 625 000	25.4951	8.6624	700	49 00 00	343 000 000	26.4575	8.8790

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
701	49 14 01	344 472 101	26.4764	8.8833	751	56 40 01	423 564 751	27.4044	9.0896
702	49 28 04	345 948 408	26.4953	8.8875	752	56 55 04	425 259 008	27.4226	9.0937
703	49 42 09	347 428 927	26.5141	8.8917	753	56 70 09	426 957 777	27.4408	9.0977
704	49 56 16	348 913 664	26.5330	8.8959	754	56 85 16	428 661 064	27.4591	9.1017
705	49 70 25	350 402 625	26.5518	8.9001	755	57 00 25	430 368 875	27.4773	9.1057
706	49 84 36	351 895 816	26.5707	8.9043	756	57 15 36	432 081 216	27.4955	9.1098
707	49 98 49	353 393 243	26.5895	8.9085	757	57 30 49	433 798 093	27.5136	9.1138
708	50 12 64	354 894 912	26.6083	8.9127	758	57 45 64	435 519 512	27.5318	9.1178
709	50 26 81	356 400 829	26.6271	8.9169	759	57 60 81	437 245 479	27.5500	9.1218
710	50 41 00	357 911 000	26.6458	8.9211	760	57 76 00	438 976 000	27.5681	9.1258
711	50 55 21	359 425 431	26.6646	8.9253	761	57 91 21	440 711 081	27.5862	9.1298
712	50 69 44	360 944 128	26.6833	8.9295	762	58 06 44	442 450 728	27.6043	9.1338
713	50 83 69	362 467 097	26.7021	8.9337	763	58 21 69	444 194 947	27.6225	9.1378
714	50 97 96	363 994 344	26.7208	8.9378	764	58 36 96	445 943 744	27.6405	9.1418
715	51 12 25	365 525 875	26.7395	8.9420	765	58 52 25	447 697 125	27.6586	9.1458
716	51 26 56	367 061 696	26.7582	8.9462	766	58 67 56	449 455 096	27.6767	9.1498
717	51 40 89	368 601 813	26.7769	8.9503	767	58 82 89	451 217 663	27.6948	9.1537
718	51 55 24	370 146 232	26.7955	8.9545	768	58 98 24	452 984 832	27.7128	9.1577
719	51 69 61	371 694 959	26.8142	8.9587	769	59 13 61	454 756 609	27.7308	9.1617
720	51 84 00	373 248 000	26.8328	8.9628	770	59 29 00	456 533 000	27.7489	9.1657
721	51 98 41	374 805 361	26.8514	8.9670	771	59 44 41	458 314 011	27.7669	9.1696
722	52 12 84	376 367 048	26.8701	8.9711	772	59 59 84	460 099 648	27.7849	9.1736
723	52 27 29	377 933 067	26.8887	8.9752	773	59 75 29	461 889 917	27.8029	9.1775
724	52 41 76	379 503 424	26.9072	8.9794	774	59 90 76	463 684 824	27.8209	9.1815
725	52 56 25	381 078 125	26.9258	8.9835	775	60 06 25	465 484 375	27.8388	9.1855
726	52 70 76	382 657 176	26.9444	8.9876	776	60 21 76	467 288 576	27.8568	9.1894
727	52 85 29	384 240 583	26.9629	8.9918	777	60 37 29	469 097 433	27.8747	9.1933
728	52 99 84	385 828 352	26.9815	8.9959	778	60 52 84	470 910 952	27.8927	9.1973
729	53 14 41	387 420 489	27.0000	9.0000	779	60 68 41	472 729 139	27.9106	9.2012
730	53 29 00	389 017 000	27.0185	9.0041	780	60 84 00	474 552 000	27.9285	9.2052
731	53 43 61	390 617 891	27.0370	9.0082	781	60 99 61	476 379 541	27.9464	9.2091
732	53 58 24	392 223 168	27.0555	9.0123	782	61 15 24	478 211 768	27.9643	9.2130
733	53 72 89	393 832 837	27.0740	9.0164	783	61 30 89	480 048 687	27.9821	9.2170
734	53 87 56	395 446 904	27.0924	9.0205	784	61 46 56	481 890 304	28.0000	9.2209
735	54 02 25	397 065 375	27.1109	9.0246	785	61 62 25	483 736 625	28.0179	9.2248
736	54 16 96	398 688 256	27.1293	9.0287	786	61 77 96	485 587 656	28.0357	9.2287
737	54 31 69	400 315 553	27.1477	9.0328	787	61 93 69	487 443 403	28.0535	9.2326
738	54 46 44	401 947 272	27.1662	9.0369	788	62 09 44	489 303 872	28.0713	9.2365
739	54 61 21	403 583 419	27.1846	9.0410	789	62 25 21	491 169 069	28.0891	9.2404
740	54 76 00	405 224 000	27.2029	9.0450	790	62 41 00	493 039 000	28.1069	9.2443
741	54 90 81	406 869 021	27.2213	9.0491	791	62 56 81	494 913 671	28.1247	9.2482
742	55 05 64	408 518 488	27.2397	9.0532	792	62 72 64	496 793 088	28.1425	9.2521
743	55 20 49	410 172 407	27.2580	9.0572	793	62 88 49	498 677 257	28.1603	9.2560
744	55 35 36	411 830 784	27.2764	9.0613	794	63 04 36	500 566 184	28.1780	9.2599
745	55 50 25	413 493 625	27.2947	9.0654	795	63 20 25	502 459 875	28.1957	9.2638
746	55 65 16	415 160 936	27.3130	9.0694	796	63 36 16	504 358 336	28.2135	9.2677
747	55 80 09	416 832 723	27.3313	9.0735	797	63 52 09	506 261 573	28.2312	9.2716
748	55 95 04	418 508 992	27.3496	9.0775	798	63 68 04	508 169 592	28.2489	9.2754
749	56 10 01	420 189 749	27.3679	9.0816	799	63 84 01	510 082 399	28.2666	9.2793
750	56 25 00	421 875 000	27.3861	9.0856	800	64 00 00	512 000 000	28.2843	9.2832

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
801	64 16 01	513 922 401	28.3019	9.2870	851	72 42 01	616 295 051	29.1719	9.4764
802	64 32 04	515 849 608	28.3196	9.2909	852	72 59 04	618 470 208	29.1890	9.4801
803	64 48 09	517 781 627	28.3373	9.2948	853	72 76 09	620 650 477	29.2062	9.4838
804	64 64 16	519 718 464	28.3549	9.2986	854	72 93 16	622 835 864	29.2233	9.4875
805	64 80 25	521 660 125	28.3725	9.3025	855	73 10 25	625 026 375	29.2404	9.4912
806	64 96 36	523 606 616	28.3901	9.3063	856	73 27 36	627 222 016	29.2575	9.4949
807	65 12 49	525 557 943	28.4077	9.3102	857	73 44 49	629 422 793	29.2746	9.4986
808	65 28 64	527 514 112	28.4253	9.3140	858	73 61 64	631 628 712	29.2916	9.5023
809	65 44 81	529 475 129	28.4429	9.3179	859	73 78 81	633 839 779	29.3087	9.5060
810	65 61 00	531 441 000	28.4605	9.3217	860	73 96 00	636 056 000	29.3258	9.5097
811	65 77 21	533 411 731	28.4781	9.3255	861	74 13 21	638 277 381	29.3428	9.5134
812	65 93 44	535 387 328	28.4956	9.3294	862	74 30 44	640 503 928	29.3598	9.5171
813	66 09 69	537 367 797	28.5132	9.3332	863	74 47 69	642 735 647	29.3769	9.5207
814	66 25 96	539 353 144	28.5307	9.3370	864	74 64 96	644 972 544	29.3939	9.5244
815	66 42 25	541 343 375	28.5482	9.3408	865	74 82 25	647 214 625	29.4109	9.5281
816	66 58 56	543 338 496	28.5657	9.3447	866	74 99 56	649 461 896	29.4279	9.5317
817	66 74 89	545 338 513	28.5832	9.3485	867	75 16 89	651 714 363	29.4449	9.5354
818	66 91 24	547 343 432	28.6007	9.3523	868	75 34 24	653 972 032	29.4618	9.5391
819	67 07 61	549 353 259	28.6182	9.3561	869	75 51 61	656 234 909	29.4788	9.5427
820	67 24 00	551 368 000	28.6356	9.3599	870	75 69 00	658 503 000	29.4958	9.5464
821	67 40 41	553 387 661	28.6531	9.3637	871	75 86 41	660 776 311	29.5127	9.5501
822	67 56 84	555 412 248	28.6705	9.3675	872	76 03 84	663 054 848	29.5296	9.5537
823	67 73 29	557 441 767	28.6880	9.3713	873	76 21 29	665 338 617	29.5466	9.5574
824	67 89 76	559 476 224	28.7054	9.3751	874	76 38 76	667 627 624	29.5635	9.5610
825	68 06 25	561 515 625	28.7228	9.3789	875	76 56 25	669 921 875	29.5804	9.5647
826	68 22 76	563 559 976	28.7402	9.3827	876	76 73 76	672 221 376	29.5973	9.5683
827	68 39 29	565 609 283	28.7576	9.3865	877	76 91 29	674 526 133	29.6142	9.5719
828	68 55 84	567 663 552	28.7750	9.3902	878	77 08 84	676 836 152	29.6311	9.5756
829	68 72 41	569 722 789	28.7924	9.3940	879	77 26 41	679 151 439	29.6479	9.5792
830	68 89 00	571 787 000	28.8097	9.3978	880	77 44 00	681 472 000	29.6648	9.5828
831	69 05 61	573 856 191	28.8271	9.4016	881	77 61 61	683 797 841	29.6816	9.5865
832	69 22 24	575 930 368	28.8444	9.4053	882	77 79 24	686 128 968	29.6985	9.5901
833	69 38 89	578 009 537	28.8617	9.4091	883	77 96 89	688 465 387	29.7153	9.5937
834	69 55 56	580 093 704	28.8791	9.4129	884	78 14 56	690 807 104	29.7321	9.5973
835	69 72 25	582 182 875	28.8964	9.4166	885	78 32 25	693 154 125	29.7489	9.6010
836	69 88 96	584 277 056	28.9137	9.4204	886	78 49 96	695 506 456	29.7658	9.6046
837	70 05 69	586 376 253	28.9310	9.4241	887	78 67 69	697 864 103	29.7825	9.6082
838	70 22 44	588 480 472	28.9482	9.4279	888	78 85 44	700 227 072	29.7993	9.6118
839	70 39 21	590 589 719	28.9655	9.4316	889	79 03 21	702 595 369	29.8161	9.6154
840	70 56 00	592 704 000	28.9828	9.4354	890	79 21 00	704 969 000	29.8329	9.6190
841	70 72 81	594 823 321	29.0000	9.4391	891	79 38 81	707 347 971	29.8496	9.6226
842	70 89 64	596 947 688	29.0172	9.4429	892	79 56 64	709 732 288	29.8664	9.6262
843	71 06 49	599 077 107	29.0345	9.4466	893	79 74 49	712 121 957	29.8831	9.6298
844	71 23 36	601 211 584	29.0517	9.4503	894	79 92 36	714 516 984	29.8998	9.6334
845	71 40 25	603 351 125	29.0689	9.4541	895	80 10 25	716 917 375	29.9166	9.6370
846	71 57 16	605 495 736	29.0861	9.4578	896	80 28 16	719 323 136	29.9333	9.6406
847	71 74 09	607 645 423	29.1033	9.4615	897	80 46 09	721 734 273	29.9500	9.6442
848	71 91 04	609 800 192	29.1204	9.4652	898	80 64 04	724 150 792	29.9666	9.6477
849	72 08 01	611 960 049	29.1376	9.4690	899	80 82 01	726 572 699	29.9833	9.6513
850	72 25 00	614 125 000	29.1548	9.4727	900	81 00 00	729 000 000	30.0000	9.6549

SQUARES, CUBES, SQUARE ROOTS AND CUBE ROOTS (*Continued*)

Nos.	Squares.	Cubes.	Square root.	Cube root.	Nos.	Squares.	Cubes.	Square root.	Cube root.
901	81 18 01	731 432 701	30.0167	9.6585	951	90 44 01	860 085 351	30.8383	9.8339
902	81 36 04	733 870 808	30.0333	9.6620	952	90 63 04	862 801 408	30.8545	9.8374
903	81 54 09	736 314 327	30.0500	9.6656	953	90 82 09	865 523 177	30.8707	9.8408
904	81 72 16	738 763 264	30.0666	9.6692	954	91 01 16	868 250 664	30.8869	9.8443
905	81 90 25	741 217 625	30.0832	9.6727	955	91 20 25	870 983 875	30.9031	9.8477
906	82 08 36	743 677 416	30.0998	9.6763	956	91 39 36	873 722 816	30.9192	9.8511
907	82 26 49	746 142 643	30.1164	9.6799	957	91 58 49	876 467 493	30.9354	9.8546
908	82 44 64	748 613 312	30.1330	9.6834	958	91 77 64	879 217 912	30.9516	9.8580
909	82 62 81	751 089 429	30.1496	9.6870	959	91 96 81	881 974 079	30.9677	9.8614
910	82 81 00	753 571 000	30.1662	9.6905	960	92 16 00	884 736 000	30.9839	9.8648
911	82 99 21	756 058 031	30.1828	9.6941	961	92 35 21	887 503 681	31.0000	9.8683
912	83 17 44	758 550 528	30.1993	9.6976	962	92 54 44	890 277 128	31.0161	9.8717
913	83 35 69	761 048 497	30.2159	9.7012	963	92 73 69	893 056 347	31.0322	9.8751
914	83 53 96	763 551 944	30.2324	9.7047	964	92 92 96	895 841 344	31.0483	9.8785
915	83 72 25	766 060 875	30.2490	9.7082	965	93 12 25	898 632 125	31.0644	9.8819
916	83 90 56	768 575 296	30.2655	9.7118	966	93 31 56	901 428 696	31.0805	9.8854
917	84 08 89	771 095 213	30.2820	9.7153	967	93 50 89	904 231 063	31.0966	9.8888
918	84 27 24	773 620 632	30.2985	9.7188	968	93 70 24	907 039 232	31.1127	9.8922
919	84 45 61	776 151 559	30.3150	9.7224	969	93 89 61	909 853 209	31.1288	8.8956
920	84 64 00	778 688 000	30.3315	9.7259	970	94 09 00	912 673 000	31.1448	9.8990
921	84 82 41	781 229 961	30.3480	9.7294	971	94 28 41	915 498 611	31.1609	9.9024
922	85 00 84	783 777 448	30.3645	9.7329	972	94 47 84	918 330 048	31.1769	9.9058
923	85 19 29	786 330 467	30.3809	9.7364	973	94 67 29	921 167 317	31.1929	9.9092
924	85 37 76	788 889 024	30.3974	0.7400	974	94 86 76	924 010 424	31.2090	9.9126
925	85 56 25	791 453 125	30.4138	9.7435	975	95 06 25	926 859 375	31.2250	9.9160
926	85 74 76	794 022 776	30.4302	9.7470	976	95 25 76	929 714 176	31.2410	9.9194
927	85 93 29	796 597 983	30.4467	9.7505	977	95 45 29	932 574 833	31.2570	9.9227
928	86 11 84	799 178 752	30.4631	9.7540	978	95 64 84	935 441 352	31.2730	9.9261
929	86 30 41	801 765 089	30.4795	9.7575	979	95 84 41	938 313 739	31.2890	9.9295
930	86 49 00	804 357 000	30.4959	9.7610	980	96 04 00	941 192 000	31.3050	9,9329
931	86 67 61	806 954 491	30.5123	9.7645	981	96 23 61	944 076 141	31.3209	9.9363
932	86 86 24	809 557 568	30.5287	9.7680	982	96 43 24	946 966 168	31.3369	9.9396
933	87 04 89	812 166 237	30.5450	9.7715	983	96 62 89	949 862 087	31.3528	9.9430
934	87 23 56	814 780 504	30.5614	9.7750	984	96 82 56	952 763 904	31.3688	9.9464
935	87 42 25	817 400 375	30.5778	9.7785	985	97 02 25	955 671 625	31.3847	9.9497
936	87 60 96	820 025 856	30.5941	9.7819	986	97 21 96	958 585 256	31.4006	9.9531
937	87 79 69	822 656 953	30.6105	9.7854	987	97 41 69	961 504 803	31.4166	9.9565
938	87 98 44	825 293 672	30.6268	9.7889	988	97 61 44	964 430 272	31.4325	9.9598
939	88 17 21	827 936 019	30.6431	9.7924	989	97 81 21	967 361 669	31.4484	9.9632
940	88 36 00	830 584 000	30.6594	9.7959	990	98 01 00	970 299 000	31.4643	9.9666
941	88 54 81	833 237 621	30.6757	9.7993	991	98 20 81	973 242 271	31.4802	9.9699
942	88 73 64	835 896 888	30.6920	9.8028	992	98 40 64	976 191 488	31.4960	9.9733
943	88 92 49	838 561 807	30.7083	9.8063	993	98 60 49	979 146 657	31.5119	9.9766
944	89 11 36	841 232 384	30.7246	9.8097	994	98 80 36	982 107 784	31.5278	9.9800
945	89 30 25	843 908 625	30.7409	9.8132	995	99 00 25	985 074 875	31.5436	9.9833
946	89 49 16	846 590 536	30.7571	9.8167	996	99 20 16	988 047 936	31.5595	9.9866
947	89 68 09	849 278 123	30.7734	9.8201	997	99 40 09	991 026 973	31.5753	9.9900
948	89 87 04	851 971 392	30.7896	9.8236	998	99 60 04	994 011 992	31.5911	9.9933
949	90 06 01	854 670 349	30.8058	9.8270	999	99 80 01	997 002 999	31.6070	9.9967
950	90 25 00	857 375 000	30.8221	9.8305	1000	1 00 00 00	1 000 000 000	31.6228	10.0000

Measures of Length and Area.

12 inches . . . . 1 foot = 0.3047973 meter.
3 feet . . . . 1 yard = 36 ins. = 0.9143919 meter.
$5\frac{1}{2}$ yards . . . . 1 rod, pole or perch = $16\frac{1}{2}$ ft. = 198 ins.
40 rods . . . . 1 furlong = $\frac{1}{8}$ mile = 220 yds. = 660 ft.
8 furlongs . . . . 1 statute mile = 320 rods = 1760 yds. = 5280 ft.
3 miles . . . . 1 league = 24 furlongs = 960 rods = 5280 yds. = 15,840 ft.

Gunter's chain for surveyors = 66 ft. = 4 rods = 100 links.

1 link = 0.66 ft. = 7.92 ins.

1 section of land = 1 sq. mile = 640 acres.

1 acre contains 43,560 sq. ft. and measures 208.71 × 208.71 ft. = 10 sq. Gunter's chains.

The vara is an old Spanish measure of length. It is used in Mexico and in some of the western states. The legal vara in California = 33.372 ins. In San Francisco, Cal., the vara = 33 ins. The vara of Castile = 32.8748 ins.

The metric system is decimal and is based on the meter (39.370428 ins.). The decimeter = $\frac{1}{10}$ m., centimeter = $\frac{1}{100}$ m., millimeter = $\frac{1}{1000}$ m. The dekameter = 10 m., hectometer = 100 m., kilometer = 1000 m., myriameter = 10,000 m. The metric square measures have the word "square" prefixed to the measures of length except the square hectometer which is known as the hectare = 2.4711 acres.

CHAPTER III

LEVELING

The object of leveling is to determine the difference in elevation between two or more points. To do this work requires no mathematical knowledge beyond the ability to add and subtract.

A vertical line points to the center of the earth and a line perpendicular to a vertical line is a horizontal line.

A level line is parallel with the surface of still water and each point marks an equal distance from the center of the earth. In plane surveying the distances between points are so short that for all practical purposes a horizontal line is considered to be a level line.

A level line in plane surveying is one having the same elevation throughout the length and a horizontal line is actually only a line of *apparent* level.

In Fig. 80 let BE represent an arc of the earth's surface. AD represents an arc about the height of the eye of an observer parallel to BE. When C is seen from A the points A and D are on the same *true* level and the points A and C are on the same *apparent* level. In a distance of one mile the difference CD is practically 8 ins.

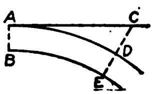


FIG. 80.

Let D = distance in miles,

h = difference in feet between true and apparent level,

then $h = \dfrac{2\,D^2}{3}.$

When D = distance in feet, $h = 0.000{,}000{,}024\,D^2$.

Refraction causes objects near the horizon to appear higher than they are actually. For very long sights,

especially when taken early or late in the day, a correction must be made for refraction. The formulas then become,

when D = distance in miles,

$$h = \frac{5D^2}{9},$$

and, when D = distance in feet,

$$h = 0.000,000,021\, D^2.$$

With instruments ordinarily used and with usual lengths of sights curvature of the earth and refraction cannot affect the work. If a backsight, however, is very short and a foresight is very long both the aforementioned factors may have a slight effect, and if the instrument is not in good adjustment errors will be multiplied in the proportion the length of backsight bears to the length of foresight. The instrument should be set as nearly as possible equidistant between points on which the rod is held. This is important.

Very simple leveling instruments were used by surveyors and engineers in early times, and are just as useful today when well-made modern instruments are not available.

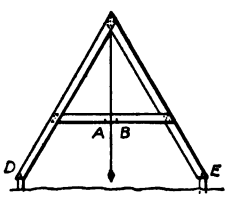


FIG. 81. Miner's triangle.

The miner's triangle. — The early miners in California set grade pegs on hundreds of miles of ditches and roads with this primitive instrument. When the grade pegs were set at intervals of one rod ($16\frac{1}{2}$ ft.) the distance from D to E = 1 rod. When this made the triangle inconveniently large the distance was $8\frac{1}{4}$ ft., or 10 ft.

To adjust the triangle two pegs were driven so the ends of the triangle could rest on them, the tops of the pegs being, as nearly as the eye could judge, at the same elevation. From the apex of the triangle a plumb-bob was suspended by a thread or fine cord and the feet of the triangle placed on the pegs.

The point where the plumb line crossed the brace was then marked (say at A). The ends of the triangle were then reversed, and the place where the line crossed on this trial was marked (B). Halfway between A and B a

mark *C* was cut on the brace. Whenever the triangle was held so the vertical plumb line crossed the brace at *C*, the ends *D* and *E* were at the same elevation.

To run a grade line with a miner's triangle. — Suppose the grade is to be $\frac{1}{4}$ in. in one rod. At *E* drive a nail with the head projecting $\frac{1}{4}$ in. When the first grade peg is driven rest the leg *D* on it and swing the triangle until *E* rests on the ground and the plumb line crosses the mark at *C*. Swing *E* far enough to one side to permit a peg to be driven where the end *E* had touched the ground. Now place *E* so the nail head rests on the peg and drive the peg until the plumb line crosses *C*. The top of the peg under *E* will be $\frac{1}{4}$ in. lower than the top of the peg under *D*.

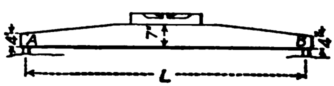


FIG. 82. Straight-edge and level.

To run a grade with a carpenter's level and straight-edge. — A straight-edge is more convenient to use than a miner's triangle, and is used in the same manner on smooth land and in ditches. When the ground is covered with rocks or vegetation the triangle is better although clumsy. A straight-edge is usually made from a 2-in. plank. In the middle for a length of 2 or 3 ft. the depth is about 7 ins. and at the ends about 3 ins. The bottom is first made perfectly straight after which the top is made parallel with it. A carpenter's level is used on top, this being more convenient than a plumb-bob and line. The grade nail is driven in the bottom at one end when the edges have been made truly parallel.

To adjust a straight-edge so the top and bottom will be parallel. — First test the level and be certain the bubble is in correct adjustment. There being no grade nail in the bottom of the straight-edge, drive a peg at each end and with the carpenter's level held on top drive the pegs until the bubble indicates the tops of the pegs to be at the same elevation.

Mark the glass at the end of the bubble and holding the level in position reverse the ends of the straight-edge, resting them on the pegs. The bubble will move and the new position of the end is to be marked. Midway between the two marks make a third mark. Holding the

level in place and the straight-edge on the pegs drive down the high peg gently until the end of the bubble moves to the middle mark.

The tops of the pegs are now at exactly the same elevation. Holding the straight-edge in place reverse the level and plane down the high end of the top of the straight-edge until the end of the bubble touches the middle mark. The bubble in the carpenter's level should remain in the middle no matter how the level is placed on top of the straight-edge.

Road percenter. — This instrument was in common use for several centuries and the author once used one in laying out a road for a mining company when no better instrument was available. The same principle is seen today in the straight-edges used by brick masons for plumbing walls.

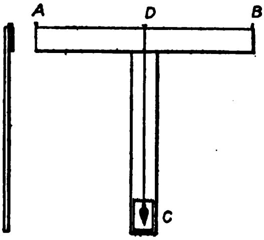


FIG. 83. Road percenter.

A piece of wood 2 ins. by 4 ins. is made perfectly straight on top, with pieces of tin at A and B for sights. Through each sight at points the same height above the top of the wood a hole $\frac{1}{32}$ in. in diameter is made. At D a 1-in. hole is bored through for the upper part of a Jacob staff. A $\frac{5}{8}$-in. by 3-in. piece 15 ins. long is attached to the side so that a line scratched down the middle is exactly perpendicular to the top. The space C is cut out and a 12-oz. plumb-bob hung by a silk thread one foot long attached near the top swings in this space.

Theoretically when the instrument is placed on the Jacob staff and the silk plumb line covers the vertical scratch the line of sight through the sighting holes is horizontal. A level rod is used with this instrument. It is termed a percenter because a graduated arc is sometimes placed above the opening in which the plumb-bob swings. When the eye end is lowered by tipping the Jacob staff until the plumb line crosses the 2 per cent mark the line of sight instead of being horizontal is inclined 2 per cent to the horizontal. Road grades are expressed in rise or fall per 100 ft., that is in per cent of rise per foot and the

angle corresponding to any per cent of rise being known the line of sight may be set at the proper angle and the grade stakes set. The percenter in theory is good but practically is of little service except on straight lines. Used as a level fairly good results may be obtained with careful work.

Water-tube level. — This level consists of a metal tube two or three feet long bent up at the ends with glass vials projecting above the ends and open at the top. In the middle a socket for the head of a Jacob staff or a tripod is fastened. The tube is filled with colored liquid and the line of sight across the top of the liquid is level.

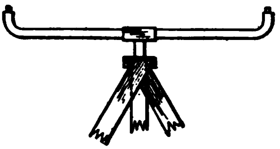


Fig. 84. Water tube level.

Leveling with rubber hose. — For leveling line shafts in mills where it is often difficult to use levels and rods the use of small rubber tubes is common. One end of the tube is fastened to a tank containing water with the surface at nearly the elevation of the shafting. In the other end is a graduated glass tube corked to prevent the loss of water while the tube is being carried. Holding the glass tube against a post or beam on which a mark is to be placed and raising it until the top of the water is a few inches below the end, the cork is removed. The surface of the water when the cork is removed, so the compression of enclosed air will have no effect, will be level with the surface of the water in the tank and the height can be marked by a cut or by driving a nail. A number of points are placed, from which the millwright can measure to level the shafting.

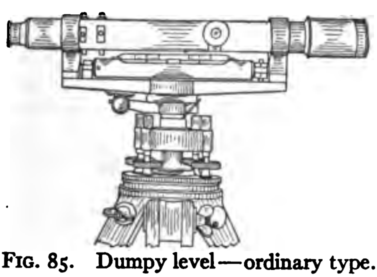


Fig. 85. Dumpy level—ordinary type.

A modern level consists of a telescope mounted in a rigid frame together with a long level tube containing a sensitive bubble. By means of right-and-left-threaded screws the bubble is brought to the middle of the tube. When the instrument is in adjustment the bubble remains

stationary while the frame is revolved on the vertical axis so the observer may look in any direction through the telescope.

A ring inside the telescope tube carries cross-hairs, or fine wires, to define the line of sight. The wires are focussed, or brought into the field of view, by moving the

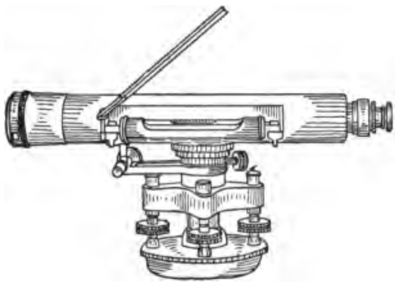


FIG. 86. Dumpy level — Bausch and Lomb type.

eyepiece. The line of sight passes through the center of the eyepiece and the intersection of the wires. When the instrument is in adjustment the line of sight is horizontal when the level bubble is in the middle of the glass tube, which is graduated so the position of the bubble can be located.

In Fig. 87 the usual arrangement of wires is shown at (*a*). The vertical wire is of some assistance in enabling the leveler to tell when the rod is vertical one way while waving the rod insures verticality the other way. To assist in obtaining equal backsights and foresights two additional wires are sometimes used as shown at (*b*) and (*c*). The horizontal line is caught with the middle wire (*b*) and the upper and lower wires are spaced so they will intercept one foot on the rod at a distance of 100 ft., two feet at a distance of 200 ft., etc. Three horizontal wires are used by some men to prevent mistakes in reading the rod, by taking readings on the three wires at each turning point or bench mark. When a level has three horizontal

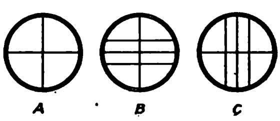


FIG. 87.

wires and one only is read mistakes often occur, so the style shown at (*c*) is sometimes used. With this arrangement the rodman holds his rod horizontally for the distance to be read, after which he sets the turning point and holds the rod vertically on it.

Three kinds of levels are in common use for the best work and they are known as the wye (Y), dumpy and precision level. The latter is used only for the highest grade of government work and is made as a form of wye level by some makers and as a form of dumpy level by others. The latest style of precision level is of the dumpy type and is known as the United States Coast and Geodetic Survey level.

The wye level for some unexplained reason obtained a strong footing in the United States nearly a century ago. It is comparatively easy to adjust, but the adjustments are those of the shop rather than the field, so the instrument requires frequent adjustment. European engineers generally express surprise when they first learn that the wye is so extensively used in this country. The author sold his wye nearly twenty years ago and purchased a dumpy, since which time he has been a strong advocate of the latter type. A description of the wye level and its adjustments is given in every instrument maker's catalogue. A wye level may be adjusted in the same manner as a dumpy.

The dumpy was so named because the earlier ones had inverting telescopes (that is all objects were seen upside down) and the omission of one set of lenses called for short (dumpy) tubes. Inverting telescopes gather more light than erecting telescopes and are used for government work requiring the highest precision. For most of the work done by engineers and surveyors nothing is gained by using an inverting telescope, and the amount of training necessary to become accustomed to viewing objects in an unnatural position is trying.

The dumpy weighs less than a wye of the same power; costs less; retains adjustments longer and stands rough usage better.

The bubble should be sensitive. The sensitiveness of a bubble is a test of workmanship. No maker will put a sensitive bubble on a poorly made instrument and a sluggish bubble is a warning to the prospective purchaser.

TO ADJUST A DUMPY LEVEL

1*st adjustment. The level must be perpendicular to the vertical axis.*

Set the level up by spreading the tripod legs so the telescope will be about five feet above the ground. Push each leg into the ground firmly. The two plates should first be made parallel by means of the leveling screws and when the instrument is set up the plates should be as nearly horizontal as possible.

Turn the telescope so it is over two screws. Then turn the screws together and bring the bubble to the middle of the tube. The thumbs move towards or from each other but never move in the same direction. The bubble travels in the direction in which the right thumb moves. The screws should move easily without binding but never loosely, for a firm seating of the ends is necessary. If screwed tight the instrument becomes strained and the slightest touch disturbs the bubble. Making the screws tight also injures the threads. After the bubble is brought to the middle over one pair of screws revolve the telescope ninety degrees and repeat the leveling process over the other pair of screws.

The foregoing instructions are general and apply to the use of the level as well as when adjustments are being tested.

The level having been brought to the middle of the tube over each pair of screws turn the telescope end for end over one pair and if the bubble remains in the middle the level is perpendicular to the vertical axis. If it moves away from the middle bring it halfway back by means of one pair of screws and the rest of the way by turning the capstan head screws, on one end of the level tube, with an adjusting pin. Then level it over the other pair and test.

2nd adjustment. The horizontal wire must be perpendicular to the vertical axis.

The instrument maker fixes the wires in the reticule perpendicular to each other. Hang a heavy plumb-bob in a sheltered place so the plumb line will not move. After adjusting the bubble sight on the plumb line. If the vertical wire covers it the horizontal wire is perpendicular to the vertical axis.

If the wire and plumb line form a small angle loosen the capstan head screws on the sides of the telescope, holding the reticule, and tap *lightly* with the finger until the reticule is shifted enough to bring the wire into a vertical position. Then tighten the screws. Fig. 88.

3rd *adjustment. The line of sight must be perpendicular to the vertical axis.*

After the instrument has been tried for the first and second adjustments, and the adjustments made, drive two

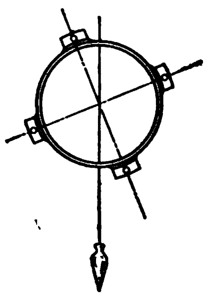


FIG. 88.

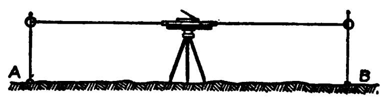


FIG. 89.

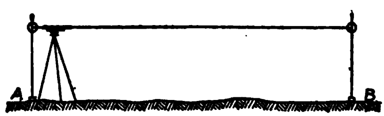


FIG. 90.

stakes from 200 to 300 ft. apart and set the level exactly midway.

Level it carefully and read the rod held on A. The rod is next held on B and a reading taken with the instrument level, that is with the bubble in the middle of the tube. The difference of the readings is the difference in elevation between A and B, no matter how badly the instrument may be out of adjustment.

The instrument is then set as close as possible to a rod held vertically on one stake with the eyepiece next to the rod. The bubble is brought to the middle of the tube and the observer looking through the object glass focuses the telescope until he can read the rod, which appears to be at a great distance. The cross-hairs cannot be seen so the elevation is taken in the center of the field of view, a lead pencil or pointed stick marking the point.

Assuming, for example, that the instrument is placed in front of stake A, and that B is lower than A, *add* the differ-

ence to the rod reading. If B is higher than A *subtract* the difference in elevation from the rod reading. Set the target at the height thus obtained and have the rodman hold the rod on B.

The effect of the curvature of the earth and refraction, assuming the distance of B from A to be 250 ft., will amount to $0.000,000,021 \times 250^2 = 0.001$ ft., by which amount the target should be lowered if the correction is applied. Errors in observation will probably offset this small correction which may therefore be disregarded.

The rod being held vertically on B the telescope is pointed towards the target and the leveling screws turned until the bubble is exactly in the middle of the tube. By means of the capstan screws on the top and bottom of the telescope the reticule is raised or lowered until the horizontal wire intercepts the center of the target. This completes the adjustment. This adjustment on a wye level makes the wye adjustment unnecessary.

THE DATUM

It has been shown that a level is merely an instrument by means of which a horizontal line may be determined.

The horizontal line passing through the center of the eyepiece and the intersection of the cross wires is a base, but for convenience in platting and computing, this base is assumed to be at some definite height above a parallel base termed a "datum," or "datum plane." An "arbitrary datum" is one arbitrarily chosen for a particular piece of work and is often assumed as being 100 ft. below the line of sight at the starting point.

In city work the datum is usually assumed to be 100 ft. below the lowest point on the streets of the city. Whenever any Government bench marks are close by the surveyor uses the elevation marked on the nearest one, the Government datum (0) being the mean of low tides. In some seacoast cities the mean of lower-low tides is used as datum. If it is believed that excavations will be made below the Government datum it is best to use an elevation of 100 ft. for the datum instead of 0 to avoid mistakes likely to arise when minus elevations are used. This in effect

fixes an arbitrary datum one hundred feet below the mean of low tides.

If a hole is dug ten feet below Government datum then the elevation, referred to datum, is -10. A wall 10-ft. high with the bottom at datum will have an elevation on top of $+10$. The use of the $+$ and $-$ signs has been a fruitful source of error and the simple expedient of assuming the zero datum below the lowest point an excavation may reach stops all trouble.

In the example just cited assume that an arbitrary datum has been selected 100 ft. below the Government datum. The bottom of the hole will have an elevation of 90 ft. above datum and the top of the wall will have an elevation of 110 ft. above datum.

LEVEL RODS

There are several types of level rods known as Boston, New York, Philadelphia, etc. A description of each rod is not necessary for this information is contained in the catalogues of instrument makers.

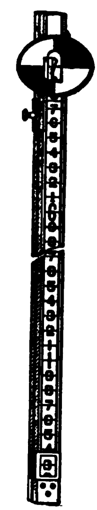


FIG. 91. Philadelphia self-reading rod.

Rods graduated with thin black lines on varnished wood on which a target is necessary are not much used today. The favorite type is some form of Philadelphia rod. This is known as "self-reading" because a target is not necessary. The face of the rod is painted white and originally was graduated in feet and tenths of a foot. When closer readings were wanted a scale on the sliding target was used by means of which half-hundredths could be read.

The rod was later made with graduations of one-hundredth of a foot stenciled on the face, the marks being alternately black and white. A target with scale was attached to the rod so that readings could be taken to the nearest half-hundredth. An improvement was made when the graduations were cut into the white face of the rod with the alternate graduations painted black. Another form has the graduations cut

without the alternate black and white spaces. The author, and probably the majority of engineers and surveyors, prefers the broad marks, as the reading of the rod does not tax the eyes and it is "self-reading" at long distances. A target with a vernier reading to a thousandth of a foot is used on all rods except those with a stenciled face.

The vernier is a scale attached to the target. It is divided into ten equal divisions with the zero on the center line of the target. The ten divisions on the vernier cover nine divisions (nine one-hundredths) on the rod, each division on the vernier being thus one-tenth of one-hundredth, or one-thousandth, of a foot less than the smallest graduation on the rod.

To read less than one-hundredth of a foot, read upward on the rod until the hundredth below the zero of the vernier is reached. Then read upwards from zero on the vernier until a line is reached that coincides with some line on the rod. The vernier in Fig. 92 reads 6 feet, 1 tenth, 2 hundredths and 4 thousandths (6.124) and is read "six point one, two, four."

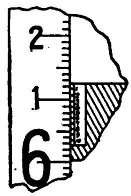


FIG. 92. Vernier for level rod target.

A level rod is merely a rod graduated in feet, tenths and hundredths of a foot; or in feet, inches and eighths of an inch, the division in inches being used only by architects and building mechanics. The decimally divided foot is used by engineers and surveyors.

The graduations proceed upward, the foot of the rod being zero. It is customary to mark the feet in red, the figures being "one-tenth" high and the tenths in black, the figures being a half-tenth high. Each five-hundredth mark projects slightly beyond the marks denoting hundredths.

When a level is set up it may be turned in any direction and a rod is used to determine the height of the horizontal line above the ground. The rod must be held perfectly vertical, or plumb, and wherever the horizontal line of sight strikes it a "reading" is obtained.

A rodman must always stand directly back of the rod and face the leveler, so the latter will see the face of the rod fully and not at an angle. The bottom of the rod must

rest firmly on the point on which it is held. The hands should be about the height of the chin to support the rod properly and the fingers should grasp the sides, never encircling the rod, for they will cover some of the graduations and may interfere with a sight if placed across the face.

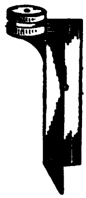


FIG. 93. Rod level.

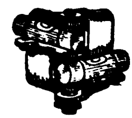


FIG. 94. Folding rod level.

For holding the rod in a vertical position several forms of rod levels are on the market. Some men use a light plumb-bob, generally a nuisance in a wind. A time-honored method is "waving the rod."

In Fig. 95 A–B is the horizontal line and C–D the rod. When the rod is held at C and allowed to lean towards D' or fall back towards D'', the horizontal line gives a greater reading than when C–D is perpendicular to A–B. When the rodman has no level or other means for insuring a vertical position of the rod and holds the rod on a point on which a close reading is desired, the rod is waved towards and from the leveler until he obtains the shortest reading possible.

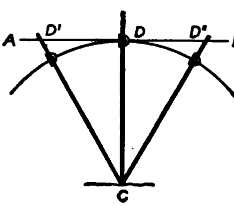


FIG. 95.

The "waving" must be done very slowly. Some instrument makers furnish targets made in the shape of an angle and the horizontal division line on the target cannot be completely covered by the cross wire unless the rod is vertical. This necessitates signaling by the leveler and the author believes waving a rod on particular points cannot be improved upon. If a rodman is not experienced he should have a rod level for intermediate sights.

A rod is read with target to the nearest "thousandth" on bench marks; to the nearest "half-hundredth" on

turning points, and the nearest "tenth" for intermediate points. The use of the target on turning points is optional but is customary so the reading by the leveler can be checked by the rodman. The target is never used on intermediate sights or for "stations" except when grade pegs are set. In setting grades it is very convenient and easy on the eyes to use a target.

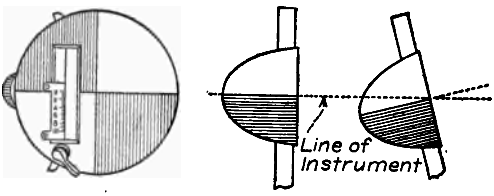


FIG. 96. Angle target.

Checking a reading is done as follows: The leveler reads the rod and records the reading. He then motions to the rodman to set the target and directs him in moving it up or down until it is practically right. Then the rod is waved and the target moved until it is set, when the rodman clamps it. He then reads the rod and compares his reading with that of the leveler as the latter goes past to a new set up. When the backsight is taken he gives his reading to the leveler as a check before going ahead on line.

When the leveler wishes to have the target used he raises his right arm vertically as high as possible and describes a small horizontal circle. This means "turning point." The rodman selects a good point, holds the rod on it, slides the rod to where he thinks the line of sight will intersect it and awaits orders. The target is moved up when the leveler puts his right hand out horizontally to the side. It is moved down when he holds out his left hand. He thrusts out both arms when it is right. When the rod is to be waved he holds up his right arm and waves it forward and back.

For readings on line at stations the rod is held on the ground. Turning points must be hard and so firm they

will not move when the rod is held on top. In stony country the tops of stones make excellent turning points and in timbered land exposed roots and notches in trees are used. In some sections where the soil is light or sandy the rodman carries pegs to drive in the ground for use as turning points.

A convenient turning point consists of an iron pin attached by a light chain to the rodman's belt. The pin is pushed into the ground and the rod held on top of the ring. The chain is used to keep the rodman from going off without the pin after holding the rod on it.

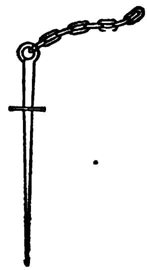


FIG. 97.

To find the difference in elevation between two points not far apart.

Assume the points to be so close together that they may both be seen from an intermediate point. Let A and B be the points on the surface of the earth and C and D be points vertically above them on a true level with F, on the line of sight.

If F (position of level) is midway between C and D the difference between true level and apparent level will be the same. That is

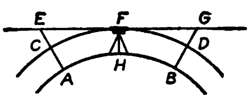


FIG. 98.

$$\frac{CA}{BD} = \frac{EA}{BG}.$$

If the level is not midway between A and B it will be necessary to measure the distances AH and BH and use the formulas already given to determine the difference between the true level on the arc CFD and the apparent level on the horizontal line EFG. The difference in elevation is obtained by setting the instrument up at some point H between A and B and leveling it so the line of sight EFG is horizontal. A graduated rod is held vertically at A and the reading taken where the line of sight intersects the face of the rod. The rod is then held vertically at B and a reading taken. The difference between these readings gives the difference in elevation between the two points, provided they differ in elevation.

PROBLEMS

1. Let AH = 240 ft. and BH = 300 ft. AE = 10 ft., BG = 6 ft. What is the *true difference* in elevation between A and B? If AH = BH what will be the true difference in elevation?

2. Let AH = 1040 ft. and BH = 1820 ft. AE = 5 ft., BG = 6 ft. Find true and apparent difference in elevations.

3. Let AH = 792 ft. and BH = 1188 ft. AE = 3.17 ft., BG = 5.67 ft. Find true and apparent difference in elevations.

DIFFERENTIAL LEVELING

To find the difference in elevation between two points far apart. The line $ABCD$ is a profile of the ground. The difference in elevation between A and D is wanted and the points are assumed to be about 2000 ft. apart. Since sights should not be taken more than 400 ft. and the level should be as nearly as possible equidistant from turning points the level must be set up three times.

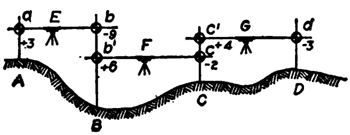


FIG. 99.

The leveler starts from A and walks in the direction of D, counting his paces until he is about 100 paces from A. Going to one side of the line until he believes a horizontal line will be above the ground at A, he sets up the level and levels the telescope. The elevation of A is assumed to be some definite distance above datum — if the actual elevation is not known — and the rodman holds his rod on A.

Sta.	+	−
A	3	0
B	6	9
C	4	2
D	0	3
	13	14
		13
		1

The leveler reads the rod and puts the rod reading (a) in his field book in a column headed by a + sign. The rodman now paces from A to E and an equal number of paces beyond E to B where he selects a good turning point and holds his rod. The leveler reads the rod and puts the rod reading (b) in the column headed by a − sign. He now goes to F and levels the instrument, the rodman holding the rod on the turning point (or turn, as it is commonly called) but turning it so the leveler can

see the face. The leveler takes the reading (b^1) and records it in the + column while the rodman paces to F and an equal distance beyond and holds the rod on a turn at C. The leveler now reads the rod and puts the reading (c) in the − column, then sets the level at G, judging as closely as possible with the eye, for no great accuracy is required, that the distance from C to $G = G$ to D. He then reads the rod and puts the reading (c') in the + column. The rodman then holds the rod on D and the leveler places the reading (d) in the − column.

The + readings and − readings are separately added and the difference between them is the difference in elevation between A and D. The forward point is lower than the starting point if the − readings are less than the + readings. It is of course higher if the sum of the − readings exceeds the sum of the + readings.

The rod readings given in the example are marked in the figure for identification. The profile is not drawn in the

FIG. 100. Bench mark on tree.

field book, the record of readings on the ruled page being sufficient. A "backsight" is a + reading and a "foresight" is a − reading. The intermediate points are not marked in differential leveling, the leveler entering on his book, instead of B and C, the letters "T. P." (turning point) or "Peg," the latter word being a survival from the days when every turning point was actually a peg left in the ground.

BENCH MARKS AND TURNING POINTS

A bench mark is a permanent point with a recorded elevation. In the country a stone is used when available or a tree is cut and marked. A cut is made in the side as if for a broad blaze. The lower part, however, is cut in like a

shelf and sloped each way from a ridge in the middle so there will be a point on which to hold the rod. At the edge a nail or spike is driven flush. If the head projects it may be hammered down and the elevation thus disturbed. The object in driving the spike is to furnish a hard point on which to hold the rod. The bench mark may be used for years and if the wood rots the elevation will not be preserved unless a metal point is used.

On some smooth place as close as possible to the bench mark the elevation should be marked. When trees are blazed the figures are cut into the wood with a surveyor's scribe, which is a useful tool sold by all instrument dealers. An accurate description of each bench mark is written in the level book and recorded in the office.

On route surveys bench marks are placed about one-third of a mile apart as close as possible to the line but far enough to the side to be preserved during the construction period. Turning points are merely temporary benches and no record is usually made of them. For the most accurate work the distance between turning points should be not less than 200 ft. (100 ft. from the instrument) nor more than 600 ft. For ordinary work the distance between turning points should not exceed 1200 ft. in the middle of a comfortable day. Sights should be short in the early morning and late afternoon; also on very bright, hot days. The instrument must be level, in perfect adjustment and midway between turning points. The rod must be vertical.

In cities or on construction work there cannot be too many bench marks. Corners of iron or masonry steps; curb stones; tops of hydrants, etc., are used and recorded. Always have two benches close together on such work and read on both. The records give the difference in elevation of the benches and if this is not verified then one must have been disturbed and each must be compared with some near-by bench until two are found to agree with the records. These are used and the faulty ones removed from the records.

Errors in adjustment, effect of the curvature of the earth and effect of refraction are taken out by equality of sights to turning points. A difference of one-tenth the length of

the average sight, provided the differences are about equally plus and minus, will not affect the work noticeably.

The rod may be held approximately vertical on intermediate sights for ground reading but lack of verticality on bench marks and turning points causes errors of a cumulative nature.

The sun heats the end of the level toward it and the metal expanding raises that end. The bubble always seeks the high end and this error, which is cumulative, can only be guarded against by bringing the bubble to the center *after* setting the target, and then obtaining a new reading. On very careful work the instrument should be shaded.

Errors in reading, of one foot, one-tenth, etc., are often made, but are more common with target rods than with self-reading rods. Requiring the rodman to carry a book for "peg readings" so he can check the leveler is a practice that should never be omitted. The only real check is to duplicate the work in an opposite direction, so on all route work, or "profile leveling," the leveler "checks benches" every ten miles and sometimes for shorter distances.

All persons using instruments carry their personality into their work so a recognized difference in results obtained by different individuals has been termed by scientific men the "personal equation." With increase in experience errors due to personality become small and tend to balance, thus eliminating error due to this cause. The personal equation shows up when a leveler checks on a bench set by another man before the personal errors of either have balanced.

Careful leveling is a continual balancing of small errors but the residual errors are cumulative. To duplicate a line of levels in an opposite direction does not change the sign (+ or −) of the residual error. The error increases with distance so to run a line of levels ten miles and re-run it in the opposite direction is the same as running one line twenty miles. All work should be thus checked when possible and one-half the error found at the starting point should be used as a correction to the bench elevation at the other end and proportionately to intermediate benches.

Failures to make a close check when one route survey crosses another and a leveler reads a bench set by another

man often happens because rods of different makers were used without comparing the graduations. When a number of rods are used they should be purchased under good specifications from one manufacturer and before sending to the field should be carefully tested for accuracy of the graduations with a standard steel tape.

On long lines of levels the error increases as the square of the distance, while on short lines the error is much less. In city work benches should be established if possible on the corner of each block, in pairs, and in the middle of long blocks. The limit of error in feet should never exceed the following:

$$e = a\sqrt{m},$$

in which e = error in feet,

a = error factor,

m = distance in miles between the benches.

For city work and all accurate leveling $a = 0.017$.

For ordinary route leveling $a = 0.034$.

When leveling down hill time is saved if the horizontal line is brought close to the foot of the rod for a backsight. When going up hill the horizontal line should be brought close to the top of the rod. It often happens that an attempt to read near one end of the rod results in the horizontal line striking below or above the rod, so the level must again be set up. To avoid this the leveler should have a hand level by means of which he can obtain a horizontal line at the height of his eyes. The level is then set up with the telescope at this height and leveled.

ROUTE, OR PROFILE, LEVELING

Differential leveling has been illustrated and such work is done when the only elevations wanted are those at the ends of the line, every intermediate reading being on a turning point. "Check leveling," to check elevations of bench marks, is differential leveling.

Profile leveling is for the purpose of making a profile and obtain a record of heights at all changes of elevation on the line.

The route is laid off in stations and the leveler obtains the elevation of the ground at each station, and also at

intermediate points where decided changes occur, as gullies, etc. Between adjacent T. P.'s there may be several readings and on steep ground there may be several turns in one station.

The level is set up and a reading taken on a convenient bench mark. This reading added to the elevation of the bench gives the elevation of the horizontal wire and is recorded in the column headed H. I. (Height of Instrument). Readings are then taken on the ground at each station and + station and subtracted from the H. I., the difference being the ground elevation. A reading taken to a bench mark or turning point to get the H. I. is known as a backsight and placed in the B. S. column. A reading taken on the ground or on a turning point or bench mark to obtain the elevation is known as a foresight and is placed in the F. S. column. The notes are placed on the left-hand page of the field book, as follows:

Station.	B. S.	H. I.	F. S.	Elevation.	Remarks.
B. M.	1.234	101.234	....	100.00	B. M. on oak tree 20 ins. diam.
0	....	....	2.7	98.5	11 ft. to left of Sta. 1
1	....	....	5.4	95.8	
+50	....	....	7.8	93.4	
+85	....	....	3.0	98.2	
T. P. 2 △	2.176	102.723	0.687	100.547	T. P. on hub at Sta. 2
3	....	....	1.75	101.00	
4	....	....	2.2	100.5	
Peg	....	....	8.431	94.292	

In Fig. 101 is illustrated a common feature in profile leveling. A gully runs across the surveyed line and the chainmen when setting stakes measured the + distances to the edges and bottom. When the levels are taken the rodman paces the distance from the station to the bank and a reading is obtained.

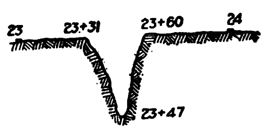


FIG. 101.

Leaving his level the leveler by means of his hand level obtains the elevation at the bottom, the process being differential leveling. The notes

are placed on the right-hand page of the level book. He returns to his level and the rodman holds on the opposite bank. After a reading is taken he paces the distance to the next station and calls it to the leveler who then writes down the + stationing. The three + distances are known to be only closely approximate but they check the distances previously measured with the tape and so identify the place. In rough country the hand level saves much time.

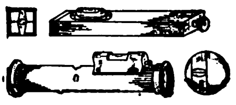


FIG. 102. Two types of hand levels.

The notes are platted on profile paper, there being two rulings in common use, plate *A* with 20 and plate *B* with 30 horizontal lines to an inch. Each plate has four vertical

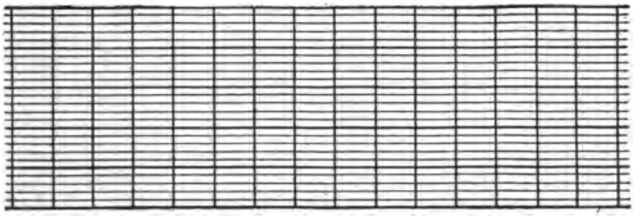


FIG. 103. Plate A profile paper.

lines to an inch. When plotting the profile each horizontal space represents a station and each vertical space represents one foot, the distorted scale showing plainly all minor irregularities in the surface. Cuts and fills and differences in elevation are easily read to the nearest half foot and estimated to the nearest quarter of a foot. For estimating earthwork quantities this is convenient but the principal value of the exaggerated vertical scale is the ease with which grade lines are selected.

The purpose in making a profile survey is usually to select the grade for a road, railway or ditch. When the profile is made a thread is stretched between the hands and held on the profile so the projections above the line are equal to those below. In common language the "cuts and fills" balance. If this gives a steeper grade than the

maximum allowed, the position of the thread must be altered. Two considerations therefore enter in the problem of fixing a grade line, one being economy in construction, the other economy in hauling loads up the grade. In irri-

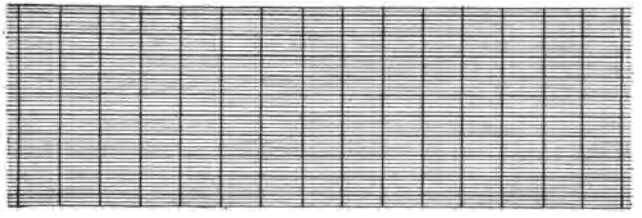


FIG. 104. Plate B profile paper.

gation ditches the grade must be light so water can be delivered to the highest point of the land yet steep enough to secure a velocity that will prevent the deposit of silt and the growth of vegetation.

In drainage ditches the grade should be as steep as possible without producing a velocity of flow great enough to scour the bottom and make a gully of the ditch.

TO SET GRADE STAKES

Contour road and ditch surveys are generally made with a target rod. The first stake is driven so that the top is on grade. The fall per station is given to the rodman. A reading is taken with the rod held on the grade peg, the target set and the reading recorded. A chainman holds one end of the tape or chain at the grade peg and the rodman holding the other end draws the tape taut and holds the rod vertically for a new peg. At each station the target is moved the amount of fall per station. If the line is going uphill the target is moved down and it is moved up if the line is going downhill. The leveler calls the correct reading to the rodman, *after* the latter has clamped the target, as a check each time.

In setting grade stakes for sewers the tops are placed at some definite height above the bottom of the sewer when

the street is already on grade. The line being straight a peg is driven to grade at each end of the block. Over one is set the level and the height from the top of the grade peg to the center of the telescope is carefully measured.

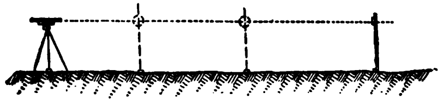


FIG. 105. Plunging a grade.

A light rod is driven in the ground at the other peg and a white card tacked to it so the middle of the card is at a height above the top of the peg equal to the H. I. above the other peg. The exact height is measured and marked on the card. By manipulating the leveling screws the telescope is pointed to the mark on the card. The line of sight is then parallel to the grade line instead of being horizontal. The target is set on the rod to a height equal to the height of the cross-hairs above the grade line. The rod being held vertically the bottom is on line with the tops of the grade pegs when the horizontal wire cuts the center of the target and stakes are driven to grade wherever desired. This method is termed "plunging a grade."

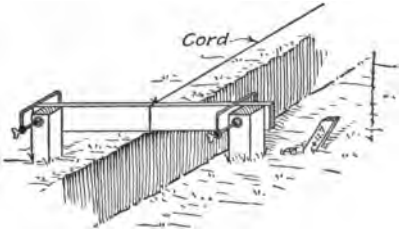


FIG. 106. Sewer grade line on street.

When the ground is irregular so that some stakes would be driven with tops below the surface and others would stand high if driven to grade, it is best to drive a peg with the top flush with the ground at each station. Beside each peg is driven a stake marked with figures indicating the distance to grade, a + sign indicating a cut and a − sign a fill.

To lay pipes to grade, all the witness stakes will be marked +, for they will all be above grade. Beside each grade peg drive a post and on the opposite side of the trench drive another. Measure up from the grade peg some distance to obtain a line a certain number of feet above the flow line and at this height nail or clamp a plank to the two posts, the plank to be perfectly level. This is done at each station and a cord stretched from plank to plank will show the grade and be out of the way of the workmen in the trench. A piece of wood about one inch square with a small bracket fastened at the lower end is used to obtain the grade at the bottom, the projecting arm resting in the bottom of the pipe when an upper mark touches the cord. The rod must be vertical and a rod level should be used.

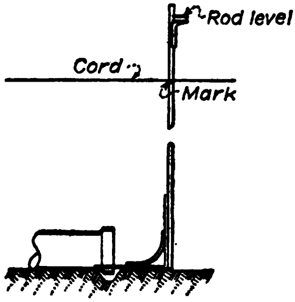


FIG. 107. Transferring surface grade to pipe in trench.

MANUAL SIGNALS FOR SURVEYORS

It is not always possible to shout instructions to assistants in the field so signals are necessary. The author mentioned his signals for telling a rodman when to move a target, and the direction in which to move it. For conveying information into which numbers enter a very old custom is to write the number with the right hand on an imaginary vertical surface. When this is done by the rodman the leveler reads the number in a reversed direction and the rodman reads it in a reversed direction when the writing is done by the leveler. Some men in making such signals stand with their back to the recipient and so manage to overcome the difficulty of reading numbers reversed.

In *Engineering News*, May 22, 1913, Mr. F. T. Darrow illustrated a set of signals in use on the Burlington Lines, these signals being shown in Fig. 108. The reader will notice that one hand is used, the other presumably holding the rod or instrument.

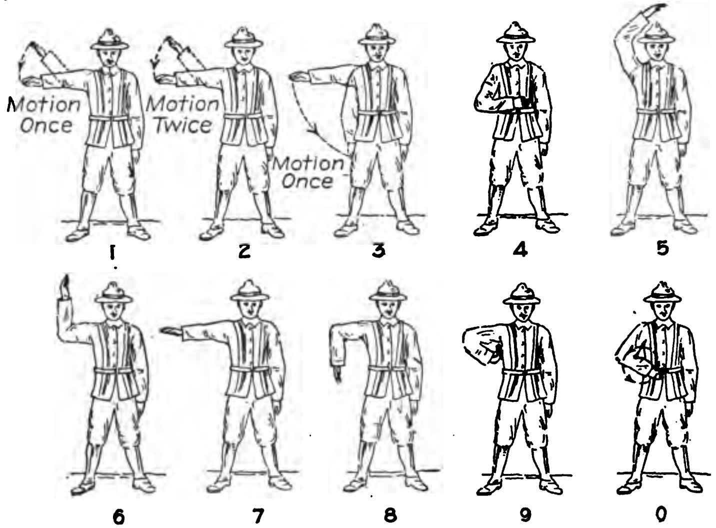


FIG. 108. Signals for surveyors' assistants.

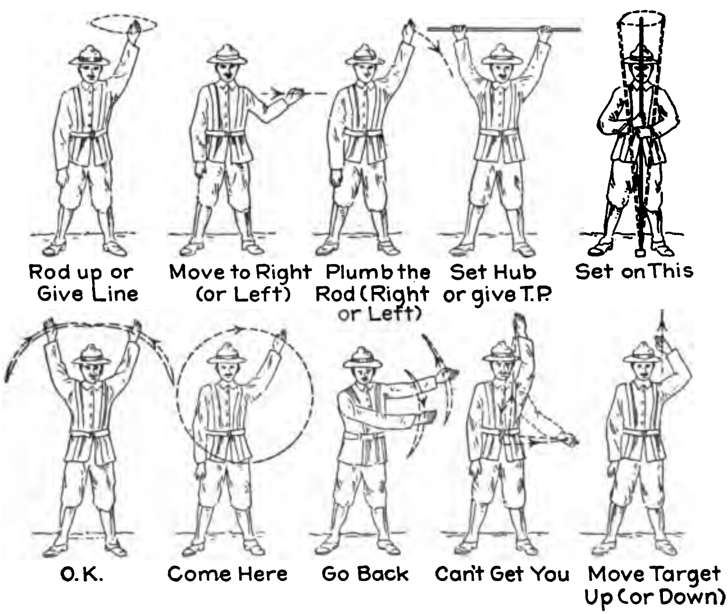


FIG. 109. Signals for surveyors' assistants.

In *Engineering News*, Aug. 21, 1913, Mr. Robert S. Beard illustrated a set of signals requiring the use of both hands. The author has omitted from Fig. 109 the signals for numbers as he prefers signals for which one hand only is used. It is necessary, however, to convey other ideas than numbers so those signals in the article by Mr. Beard which relate to common instructions are shown in Fig. 109. The signals may be used by instrument men, rodmen, chainmen or other helpers and require no explanation as the information is placed under each figure. These signals are those in common use by experienced men the world over.

CHAPTER IV

COMPASS SURVEYING

Nearly all land surveys before the middle of the last century were made with the compass. It is still used today where land is not high in price.

About 70 or 80 years ago a fairly certain method was developed for eliminating the effects of local attraction, but it was not in common use. Until about 100 years ago little attention was paid to the declination of the needle, or variation as it was commonly called. The declination is the angle between the true meridian and the meridian indicated by the compass needle. The variation is the *change* in declination. In 1836 William A. Burt patented the solar compass of which he was the inventor jointly with John Mullett, both Government surveyors in Michigan where local ore bodies attracted the needle and rendered it valueless for running lines. With the solar compass a true north and south line could be run from observations on the sun. It is now obsolete so will not be described in this book. The engineers' transit with solar attachment superseded the solar compass and today few men use a solar attachment, direct observations on the sun tending to greater accuracy and simplicity.

After 1836 all Government land surveys had to be made with a solar compass. About 50 years later a transit with solar attachment was required and now direct solar observations are permitted. The General Land Office at present prohibits employees and contracting surveyors from depending to any extent on courses derived from the needle.

The compass possesses the following merits:

1. It is light.

2. It is readily set up and therefore rapid work can be done with it.

3. When there is no local attraction an error made in

reading the needle at any station is not cumulative, but is confined to the one course. If a "back reading" is taken the error is caught and may be corrected.

4. Two points are not required from which to start, as with a transit. The needle pointing always in one direction (when there is no local attraction) any station may be used as a starting point.

5. Fewer helpers are needed than with the transit.

6. On preliminary lines and on random lines considerable time may be saved in cutting brush, for the compass can be set to one side and a parallel offset line run past an obstruction by merely setting the sight on the proper course indicated by the needle.

7. In re-tracing old surveys it is better than a more exact instrument for the work is duplicated more nearly when the methods adopted closely copy those originally used.

8. When land is very low in value or when the information wanted is only closely approximate, the compass is the best instrument to use, because men of ordinary ability can do the work and do it quickly.

Before mentioning the defects of the compass it is necessary to say that all land survey methods and terms now used were developed during the era preceding the introduction of well-made accurate instruments. The only difference is increased accuracy so that an understanding of compass work is necessary for a full knowledge of land surveying.

When the compass was invented no one knows and the name of the inventor has not been preserved. The inventor is supposed to have been a Chinese and he lived at least 1900 years ago. The first mention in print of the compass is found in a Chinese dictionary printed in 121 A.D., in which book the lodestone is defined as "a stone with which an attraction can be given to the needle." The first mention of the compass by any European writer was in 1190 but it was in common use by seamen in 1250. Its use in surveying seems to be first mentioned about 1300 and on his voyage to America in 1492 Christopher Columbus verified the fact long suspected, that the needle did not point constantly to the North Star. In 1600 Gilbert

showed that the earth is a great magnet with lines of force flowing from pole to pole in which the needle settled and that angels and demons and the stars in the heavens had no influence upon the needle.

The needle does not point to the North Pole but to a shifting magnetic pole located in Canadian territory. When a compass is set so that a line drawn through the north pole, the magnetic pole and the instrument will be straight (that is in the plane of a great circle) we have a line of no variation, an agonic line.

At all points east of the agonic line the needle points *west* of true north and has a *west* declination. At all points west of the agonic line the needle points *east* of the true north and has an *east* declination. Since all meridians pass through the earth's north pole a true north and south line is spoken of as "the meridian."

The Isogonic Chart (frontispiece) is reduced from one issued by the United States Coast and Geodetic Survey for January, 1910, in Special Publication No. 9, Terrestrial Magnetism, which should be owned by every land surveyor.

The full lines show the actual declinations for the year 1910 as determined by observations at 979 stations in the United States. The dotted lines show the annual change, for the north end of the compass needle is moving to the westward at all places east of the line of no annual change and to the eastward at all places west of that line.

To use the Isogonic Chart. — What will be the magnetic declination near Denver, Colorado, in 1917?

An examination of the chart shows the declination January 1, 1910, to be about 14° 50′ E., with an annual change of about $3\frac{1}{4}$ min. From 1910 to 1917 the change will be $7 \times 3.25 = 22.75$ min. The declination on January 1, 1917, will be $14° 50' + 22.75' = 15° 12.75'$ E.

Many old time surveyors knew little about the declination of the compass needle and cared less, so their lines do not agree with the bearings set down in the records.

The annual change (variation) of the declination was unknown to the majority of surveyors 100 years ago.

That minor variations in the pointing of the needle occur during the day is not known to all surveyors of the present time.

Many surveyors paid small attention to the probable effects of local attraction and some neglected correcting readings which were incorrect because of local attraction.

Badly made instruments, some containing small quantities of iron; bent pivots; bent needles; neglected adjustments; weakly magnetized needles; mistakes in reading bearings; bearings read only to half degrees or quarter degrees; all these things during the 400 years during which the compass was the principal instrument used by surveyors finally resulted in its abandonment as an instrument of precision.

It is not wise to purchase a second-hand compass except from a reputable maker, and then only with his certificate that it has been carefully tested and found to contain no metal liable to deflect the needle. Cheap surveying instruments are a poor investment and no instruments should be purchased from stores or agents. The needle of a surveying compass should be at least 4½ ins. long and delicately balanced so it appears to be always quivering while the ends remain in one plane. A swinging of the ends indicates local attraction which may be caused by iron or steel in the compass box, bodies of iron ore under the surface of the ground, wire fences, wire in the hat brim of the instrument man, or steel or iron on his person. When a compass is set up chains, tapes, hatchets and all tools containing metal which might attract the needle should be removed to a considerable distance.

Diurnal variation. — In addition to the secular variation of the declination the needle swings backward and forward each day through an arc sometimes as great as 20 minutes in size. This variation cannot be predicted for it alters with the time of day, seasons of the year, temperature and amount of sunshine. It is different in localities not far apart. In practical work it is ignored, except in hot weather when the sun is bright. On such days it is best to work only before 9 A.M. and after 3 P.M.

It is impossible to make all compass needles alike. All makers testify that a number of needles can be made alike in shape and of equal weight from one piece of steel, yet when magnetized and placed on pivots the readings will differ as much as 10 min.

Several men reading the bearing shown by a needle will obtain different results. This personal error taken with the differences in needles shows that any attempt to allow for the diurnal variation is generally a needless refinement.

Notwithstanding all the drawbacks mentioned, a well-trained, conscientious surveyor using a well-made modern compass can do very good work.

READING THE COMPASS

Let the line *N* . . . *S* represent the true meridian in which the needle is supposed to lie and the direction (bearing) of the line *A* . . . *B* is wanted. The compass is set at some point *O* on the line *A* . . . *B* and the needle allowed to swing freely after the instrument is leveled. When the needle comes to rest the observer looks through the sights towards point *B*. The compass circle is graduated from north to east and north to west, the north point being marked *O* and the east and west points 90. It is graduated similarly from south to east and west, this method being termed "quadrantal graduation," because the circle is divided into four quadrants, or quarters.

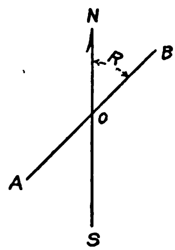


FIG. 110.

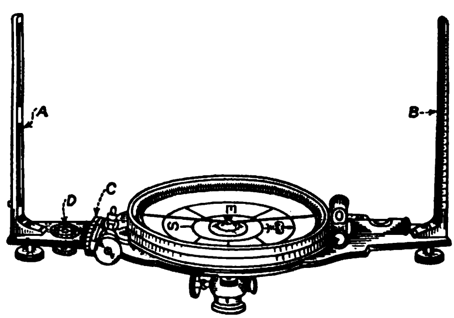


FIG. 111. Surveyor's compass with open sights.

When the sights are set on the line *A* . . . *B* the needle still points to *N*. The zero being on the line *A* . . . *B*

(in the line of sight) the end of the needle points to some number on the circle, which number indicates the angle *N, O, B*, between the lines *O . . . N* and *O . . . B*, this angle being called the bearing of the line *O–B*.

Fig. 111 illustrates a good type of compass with variation plate by means of which the declination of the needle is set off. Underneath the compass is the ball-and-socket joint by means of which the compass is leveled. Leveling screws, although a great improvement, are seldom used on compasses. Two levels at right angles show when the plate is level before clamping the joint. *A* is the sight at the south end and *B* the sight at the north end of the compass. Each sight has a very fine slit, and at the ends of the slits round openings are made so objects may be readily found. Sometimes each sight has two slits as shown. In the upper slit in one sight and the lower slit in the other sight a fine wire or horse hair is stretched to enable closer sights to be taken. The surveyor looks through the plain slit and bisects the object by means of the wire in the opposite slit.

The right edge of *B* is graduated to half degrees for angles of elevation to be read through the lower peephole on sight *A*. The left edge is graduated for angles of depression to be read through the upper peephole on sight *A*. To read an angle of elevation or depression, look through the proper peephole at a point the same height above the ground as the compass plate. Slide a card up sight *B* until the top edge strikes the line of sight. The card is held against the sight until the angle is read.

At *C* is shown a variation plate. By means of a milled screw the compass circle can be revolved about the vertical axis about 30 degrees each side of the line of sight, thus enabling the declination to be set off on the plate at *C*. The declination may be set off in one of three ways. If a well-defined line can be found of which the bearing is known, set the instrument on it and sight to a stake on the line. By means of the screw attached to the variation (declination) plate move the compass circle until the needle points to the course of the line given in the record. The declination can then be read on the plate, which should be clamped and not again touched. Another method is to

lay off a line in the true meridian, set the compass on it and turn the compass ring until the needle points to zero, when the declination may be read on the plate at *C*. For closely approximate work the declination may be obtained from the Isogonic Chart and set off on the plate.

The dial at *D* is used to keep tally in chaining. Under the compass plate is a screw by means of which the needle is raised and held against the glass cover when not in use, so it will not dull or bend the pivot when the instrument is being carried.

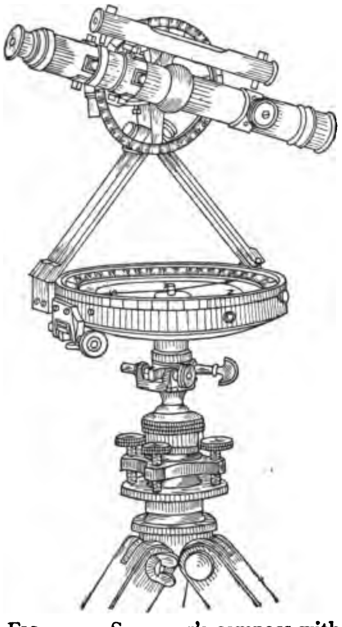


FIG. 112. Surveyor's compass with telescope.

On the compass plate the letter *E* is on the left and the letter *W* is on the right. The needle points to the north and if the line of sight is pointed in an easterly direction the needle will show the angle, as already explained, between the meridian and the line of sight. The north end of the needle will be to the left of the line of sight, so by transposing the *E* and *W* the direction and amount of divergence from the meridian are both indicated by the needle. The student must never forget that the needle indicates the line of sight but does not lie in it.

Fig. 112 shows a compass with a telescope instead of open sights, a vertical circle for reading angles of elevation and depression and a level adapter instead of ball and socket.

COMPASS ADJUSTMENTS

There are five adjustments of the compass:

1. *The compass circle must be perpendicular to the vertical axis.* — The manufacturer alone can make this adjustment.

2. *The levels must be perpendicular to the line of sight.* — This adjustment cannot be made if the compass circle is not perpendicular to the vertical axis, this fact being a check on the first adjustment. Level the compass by bringing the bubbles to the middle of their respective tubes. Turn the instrument 180 deg. and if the bubbles have moved correct *half* the difference by means of the capstan head screws holding the level tubes. Check and repeat until the bubbles remain stationary during a complete revolution of the compass on the vertical axis.

3. *The needle must be perpendicular to the vertical axis.* — This means the needle must be horizontal and also straight. The north end of the needle tends to dip towards the earth and a coil of fine wire is placed near the south end to counteract this tendency.

Before making any adjustments involving the needle it should be re-magnetized. Holding the needle in one hand hold the magnetic pole of a permanent magnet firmly with the other hand against the needle near the middle and pass the magnet to the north end of the needle. Before each pass describe a circle about one foot in diameter with the magnet in a plane with the needle. This operation should be repeated for the south end of the needle but care should be taken that the north and the south ends are applied to the opposite poles of the magnet or the work will be wasted. About twenty-five passes will generally be sufficient. This operation should be performed three or four times each year.

A weak needle is affected by the friction on the pivot so it is necessary to keep it charged in order to do good work. The method above described is not always satisfactory but in these days of electrical power plants the surveyor can readily charge his needle. Place the needle in the magnetic field of a dynamo and then test to see if the magnetism is reversed. If the needle points south instead of north put it again in the magnetic field of the dynamo in a reverse position from that used first.

When satisfied that the needle is properly charged level the compass, bringing the north end of the needle to zero at the north end. Read both ends and reverse the instrument so the north end of the needle will read the same as

the south end of the needle on the first trial. If now the south end of the needle does not read zero correct *half* the difference by bending the needle.

4. *The point of the pivot must lie in the vertical axis.* — Having performed the third adjustment, or made a test that showed the needle to be straight, read the north end of the needle at *N*, *E*, *S* and *W* (that is at points 90 deg. apart). Note the reading of the south end as each quadrant is read. If in any quadrant the south end passes over less than 90 deg. while the north end passes 90 deg., bend the pivot away from that quadrant. Test each quadrant in this manner until the needle swings freely, the two ends reading the same in each quadrant on points 180 deg. apart.

5. *The line of sight must lie in the vertical axis.* — This adjustment can be made only by an instrument maker, but the surveyor must be certain that the slits in the sighting vanes are vertical. Sight at a long plumb line and if it does not coincide with an edge of the slit, put paper under the low side or file the bottom of the high side under the foot of the sight and screw tight. This is to be done with both sights so the edges of the slits will be truly vertical when the instrument is in adjustment.

Caution. — Good work, even with an instrument in perfect adjustment, cannot be done if any unnecessary walking around the instrument is allowed. When the compass is set up and leveled it must not be disturbed.

CARE OF THE COMPASS

Treat the instrument with care and it will give good service for several generations. Avoid all shocks and jars, and handle it so there will be no danger of bending any of the parts. Carry it in the hollow of the arm and not as one carries a pail or basket. Do not use the sights as handles.

Be careful of the pivot that it will not be dulled or bent. Lower the needle gently. When the needle is let down on the pivot check the vibrations by lifting it off the point at each swing until it settles. When moving to another station lift the needle before taking up the compass and on arriving at the next station level the compass carefully,

sight backward to the station left and let the needle down gently. It will then be parallel to the position last occupied and will rest on the pivot without swinging.

The needle should always be held off the pivot when not in use. When returned to the case to remain until again required hold the plate level and let the needle down gently on the pivot until it swings in the magnetic meridian. Then raise it off the pivot.

Avoid riding in electric cars with a compass. If such cars must be used it is well to hold the compass as nearly level as possible and let the needle swing. Rubbing the glass with a cloth often causes trouble because of frictional electricity, so the surface should be slightly moistened by breathing on it after cleaning. The compass needle should not be played with by drawing the end from side to side with magnets or pieces of metal.

THE USE OF THE COMPASS

The field shown in Fig. 113 was surveyed and the first station occupied was at *O*, the work proceeding with the field on the right hand of the surveyor, "surveying with the sun," to use an old expression. There is no reason for this other than convenience. Some surveyors begin a survey at the most easterly or the most westerly corner to keep the signs either all + or all − when computing areas. When a station other than the most easterly or westerly is taken as a starting point the signs will be mixed and unless the computer is careful he may meet with difficulties in making his work close. It makes no difference which station is chosen from which to start and the lines may be

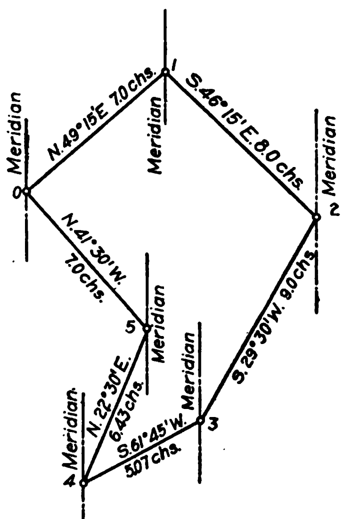


FIG. 113.

run to the right or to the left. In computing areas it is not necessary to begin the computations at the first station of the field work.

At each station a hub is set and the instrument is set over it carefully with a plumb-bob marking the center, if a tripod is used. If the compass is mounted on a Jacob staff the staff is thrust into the ground close beside the hub. The compass is carefully leveled and the sight at the *N* end placed ahead. Looking back through the sights to the station last set the needle is lowered. When it stops swinging the reading is taken, thus checking the forward reading. At station *O* no backsight is taken, the instrument being placed so the needle, as nearly as one can judge, points to the north before lowering it. The more nearly the needle lies in the meridian before being released the more quickly it comes to rest with least wear on the pivot.

A back reading (backsight) should always be taken to guard against effects of local attraction and as a check on readings, for it is easy to make a mistake of 10 degrees and sometimes letters are transposed in recording. A backsight should give the same reading as a foresight, that is, the bearing should be the same at all points on a line.

A reading is recorded by writing the letters and the included angle as shown in Fig. 113. With the instrument at *O* the forward reading was $N\,49^\circ\,15'\,E$ to 1. At 1 the bearing of the line should be $S\,49^\circ\,15'\,W$ to *O*. Beginners are often told to read the south end of the needle on backsights. Practical surveyors seldom do it for it may lead to mistakes and is often confusing. The common method is to set the instrument and while the needle is still clamped sight backward at the last station. Then lower the needle gently and it will be in the line last read. When the needle rests in this bearing sight carefully through the slits, standing at the *N* end of the compass. Then go to the *S* end and read the needle, which should give the same bearing as at the last station. Then the next bearing may be read. This procedure gives a check reading without mental computation or any changing of letters. A common error when the south end of the needle is read is to take 86 deg. to be 84 deg., 83 deg. to be 87 deg., or vice versa when the reading is almost due east or west; and, when the reading

is so close to 90° *E* is often read for *W* or *W* for *E*. It being customary to complete a survey before correcting the bearings, such errors may be serious.

The field is gone around in the manner stated until station *O* is occupied a second time, to obtain a backsight on Sta. 5, if a backsight was not taken to this station at the beginning of the work. The notes are placed on the left-hand page, the right-hand page being used for sketches and memoranda.

1	2	3	4	5	6
Station.	Corrected bearing.	Distance (chains).	F. S.	B. S.	Corr.
0	N 49° 15′ E	7.00	N 51° 15′ E		−2° 00′
				N 47° 00′ W	
1	S 46° 15′ E	8.00	S 48° 30′ E		−2° 15′
				S 45° 45′ E	
2	S 29° 30′ W	9.00	S 30° 00′ W		−0° 30′
				S 29° 30′ W	
3	S 61° 45′ W	5.07	S 61° 45′ W		±0
				S 61° 45′ W	
4	N 22° 30′ E	6.43	N 22° 30′ E		±0
				N 25° 15′ E	
5	N 41° 30′ W	7.00	N 38° 45′ W		+2° 45′
				N 39° 30′ W	

In the field the first, third, fourth and fifth columns are used. When the survey is completed the corrections are entered in the sixth column and the corrected courses placed in the second column. The *plus* and *minus* signs before the corrections refer to the difference between the corrected course and the backsight.

TO CORRECT LOCAL ATTRACTION

When sufficient care is exercised it may be assumed that all differences discovered between bearings from the two ends of a line are due to local attraction. The reference to sufficient care must not be overlooked, for surveyors often make mistakes of several degrees in reading angles.

The following method for correcting errors due to local attraction has been taken from Flint's "Survey," the first

edition of which was published in 1805. Sometime between 1832 and 1851, L. W. Meech, A.M., was employed by the publishers to revise the work, which had gone through six editions before 1835, and he is credited with being the author of the method since used by a number of men, but mentioned in few texts.

Examining the field notes it is found that the forward reading from Sta. 3 agrees with the backsight from Sta. 4. Evidently there was no local attraction at either station so the forward and back readings are assumed to be correct. Therefore the bearing of the line from 2 to 3 is $S\,29°\ 30'\ W$, from 3 to 4 is $S\,61°\ 45'\ W$ and from 4 to 5 is $N\,22°\ 30'\ E$.

Start at Sta. 4 to make corrections:

4.	Correct bearing	$N\,22°\ 30'\ E$
	Backsight	$N\,25°\ 15'\ E$
		$+\ 2°\ 45'$

The backsight is greater so the sign of the difference is *plus*.

The sign of the difference is *minus* when the backsight is less than the forward reading.

The difference is placed under the course following. If this course and that preceding are in the same or opposite quadrant a *plus* sign becomes *minus* and a *minus* sign becomes *plus*. The signs are not altered when the courses are not in the same or an opposite quadrant. When the proper sign is determined the correction is applied.

5.	Forward reading	$N\,38°\ 45'\ W$
	Correction	$+\ 2°\ 45'$
		$N\,41°\ 30'\ W$

The correction is added here, for courses 4 and 5 are in adjacent quadrants and the backsight was greater than the forward bearing.

5.	Corrected bearing	$N\,41°\ 30'\ W$
	Backsight	$N\,39°\ 30'\ W$
		$-\ 2°\ 00'$
0.	Forward reading	$N\,51°\ 15'\ E$
	Correction	$-\ 2°\ 00'$
		$N\,49°\ 15'\ E$

The correction is subtracted as the backsight was less than the forward bearing and the two bearings are in adjacent quadrants.

0. Corrected bearing........................	*N* 49° 15′ *E*
Backsight................................	*N* 47° 00′ *E*
	− 2° 15′
1. Forward reading........................	*S* 48° 30′ *E*
Correction..............................	− 2° 15′
	S 46° 15′ *E*
1. Corrected bearing........................	*S* 46° 15′ E
Backsight................................	*S* 45° 45′ E
	− 0° 30′
2. Forward reading........................	*S* 30° 00′ *W*
Correction..............................	− 0° 30′
	S 29° 30′ *W*
2. Corrected bearing........................	*S* 29° 30′ *W*
Backsight................................	*S* 29° 30′ *W*

If the last corrected bearing checks with its backsight the courses may be assumed to be correct, but such a close check is not always obtained.

This method also takes care of the diurnal declination of the needle and all errors are greatly reduced even when not entirely eliminated.

In the following example the first course is assumed to be correct. The difference between the forward and back readings gives a correction of +3 deg. but courses 1 and 2 being in the same quadrant the sign is changed.

Station.	Bearing.	Distance, chains.	F. S.	B. S.	Corr.
0	N 85° W	120.00	N 85° W	88°	±0
1	N 14° W	60.00	N 17° W	15½°	−3°
2	N 75½° E	90.00	N 74° E	73½	+1½°
3	S 27½° E	102.60	S 29½ E	27½	−2°

Correct the following field notes:

Station.	F. S.	B. S.	Station.	F. S.	B. S.
1	N 19° E	18°	1	S 25° W	25°
2	S 78° E	79°	2	S 10° W	10¼°
3	S 29° E	26½°	3	S 75¼° W	76°
4	S 52½° W	53°	4	N 11° E	7°
5	S 14½° E	15½°	5	N 2° E	5½°
			6	N 85½° E	85¼°

In correcting bearings by the foregoing method a check will not always be obtained because needle readings are taken to the nearest quarter degree with the best compasses. Closer readings are only estimates. The correction for local attraction will be found generally to be more accurate than the reading of bearings.

All compass surveys should be corrected for the effects of local attraction and thus the greatest source of error is guarded against. When several lines seem to show an agreement, as in the worked example, it is taken for granted the *true* bearings were obtained, although the local attraction may have been the same in amount and direction at certain stations, in which case only the *average probable* bearings were obtained. If local attraction is proved to be present at all stations then that course where it seems to be least may be taken as the standard.

When the object of the survey is to obtain the area of a piece of land the corrected bearings will give accurate results no matter how far wrong the selected initial bearing may be. The effect of local attraction being eliminated the correct angles between the lines are obtained.

When the survey is to be recorded and future surveyors are to be guided by the record it will be necessary to have the *true* bearings.

Set two stakes on the *true* meridian, at some place not far from the field, where it is known that no local attraction exists. The compass is set over the stake at the south end of the line and sighted to the stake at the north end. By means of the declination plate screw the compass box is revolved on the vertical axis until the needle reads zero and the declination is set off on the plate. A sight is then

taken to some corner on the survey and the bearing is read. The line is then run perfectly straight by backsights and foresights without using the needle, until the corner is reached. Setting up on the corner a backsight is taken along the line just run (the starting point of which is within an area having no local attraction) and the compass box is turned until the needle points to the correct bearing. This of course alters the declination, which makes no difference as it is not necessary to record the declination when a survey is stated to have been run from a true meridian. Having set the needle the first forward reading will be correct and all other bearings may be corrected by using it as a standard.

THE TRUE MERIDIAN

The declination of the needle may be set off on the compass plate after using the Isogonic Chart as already described. If this method is not available the true meridian may be obtained with an accuracy equal to that of good compass work, from a record of equal altitudes of the sun.

On a *level* area set a straight pole vertically. About three hours before noon drive a small stake in the ground at the end of the shadow of the pole. With a chain or tape attached to a ring around the pole strike a semicircle with a radius equal to the length of the shadow. If a cord is used a strong pull may stretch it and a wire will do instead of a chain or tape.

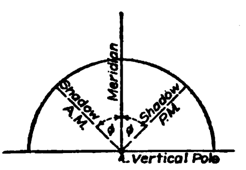


FIG. 114. True meridian from equal altitudes of sun.

In the afternoon drive a small stake on the end of the shadow when it touches the circumference of the circle. Midway between the two stakes make a mark. A line through this mark and the center of the pole will define the true meridian, which may be laid off to any length by using a chalk line stretched from the pole over the meridian point.

MAKING A COMPASS SURVEY

Field notes should be clear and the surveyor must never forget that other men may have to use his notes after he is dead. Nothing essential should be omitted, nothing non-essential should be put down.

Common custom indicates that for compass work as well as transit work it is most convenient to use the left-hand page of the book for notes and the right-hand page for sketches. The stations should begin with *O* and at the bottom of the page. One line should be used for each chain or on long lines every tenth chain, even when no stakes are set between instrument points, or stations. In this way fences, roads, buildings, etc., may be located by measurement and be shown by sketches in their relative positions. See Fig. 7.

From each instrument point bearings to objects are often taken and measurements made and the surveyor should carry a thin flexible ruler in his field book for sketching purposes, drawing lines as nearly as possible in the right direction.

A good workman is judged by the evidences of his work, and neat full notes go far towards fixing the reputation of a surveyor, even when the notes may fall into the hands of persons ignorant of surveying. The criterion by which the quality of field notes may be judged is that they may be sent to a draftsman so he can map the survey and no further

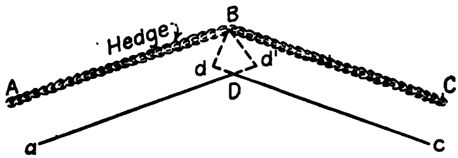


FIG. 115.

information is required by him. When an instrument man must plat his own notes or stay near a draftsman who is using them, he is incompetent.

It is seldom possible to place an instrument on a corner, for fences or hedges usually cover the lines, so surveys are made on offset lines.

Fig. 115 illustrates a method used for offsetting when using a compass. The point *D* is selected by eye to bisect the included angle between *AB* and *BC*. The instrument is set over *D* but the offset line parallel to *AB* is measured

to d', this point being, as nearly as may be estimated, perpendicular to AB opposite B. Similarly the offset line parallel to BC is measured from d.

When boundaries are crooked one line may be run. At each angle the perpendicular distance is measured from the offset line and recorded. Sometimes the surveyed line is run on one side entirely, so all measurements will be

FIG. 116.

to the right or to the left, instrument men occasionally forgetting to set down the right letter. To prevent errors sketches should be made to supplement the written notes.

When buildings, fences or other objects are to be located for the purpose of showing them on a map, bearings are read to them and the distances measured.

Sometimes the surveyor surveys an interior polygon, from the corners of which bearings are taken to the corners of the field and the distances measured as shown in Fig. 117, where the instrument was set only at the corners A, B, C, D and E with bearings and distances taken to the corners 0, 1, . . . 8 of the field. This saves time in the field, when the surveyor and his assistants are under pay, and increases the time in the office when the draftsman alone is working. The possibility of mistakes is increased because of the greater number of computations and lack of checks. A good surveyor never neglects to check every operation when possible.

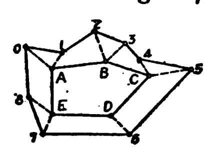


FIG. 117.

The office work in the case of a field surveyed by radiating courses from an interior polygon begins by making corrections for local attraction at each station and then correcting the radiating bearings. This method of surveying by radiation is very old.

ANGLES FROM BEARINGS

When much local attraction exists it is a good plan to mark on the map the angles at the corners after correcting the bearings. A note should call attention to the fact that these angles are to be used in preference to the bearings on re-surveys, for the cause of the local attraction may be removed after the original survey.

The angles may be deflections or included angles.

An angle is the amount of divergence between two intersecting lines in a plane.

A deflection or exterior angle is the difference in direction between two courses, shown by *d* in Fig. 118.

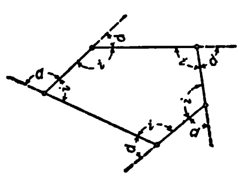


FIG. 118. Interior and deflection angles.

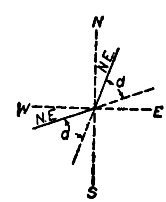


FIG. 119.

An included, or interior, angle is the supplement of the exterior angle, that is $180° - d$. In Fig. 118 the letter *i* is used to indicate the included angles.

RULES FOR OBTAINING DEFLECTION ANGLES

1. *First letters alike and last letters alike.* Fig. 119.

Rule. — The deflection is equal to the difference between the courses.

The deflection is to the *right* in the *N E* and *S W* quadrants when the following course is the greater.

$N\ 70°\ E$

$\underline{N\ 20°\ E}$

50° deflection. If course $N\ 70°\ E$ follows $N\ 20°\ E$ the deflection is $50°\ R$. If $N\ 20°\ E$ follows $N\ 70°\ E$ the deflection is $50°\ L$.

The deflection is to the *left* in the *S E* and *N W* quadrants when the following course is the greater.

N 70° *W*
N 20° *W*

50° deflection. If course *N* 70° *W* follows *N* 20° *W* the deflection is 50° *L*. If *N* 20° *W* follows *N* 70° *W* the deflection is 50° *R*.

(*Note.* — *N* 70° *E* and *S* 70° *W* describe the same line viewed from different ends.)

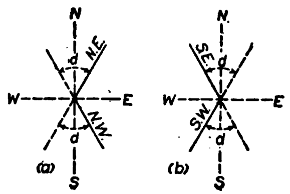


FIG. 120.

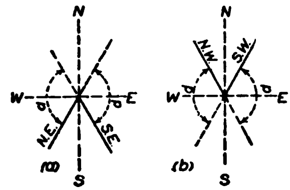


FIG. 121.

2. *First letters alike and last letters unlike.* Fig. 120.
Rule. — Add the courses.

Bearing changing from *W* to *E*, going north the deflection is to the *right;* going south the deflection is to the *left.*

Bearing changing from *E* to *W*, going north the deflection is to the *left;* going south the deflection is to the *right.*

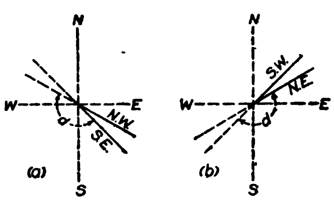


FIG. 122.

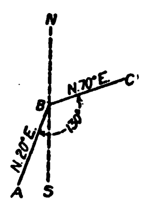


FIG. 123.

3. *First letters unlike and last letters alike.* Fig. 121.
Rule. — Subtract the sum of the courses from 180°.

Going east the deflection is to the *right* when *N* changes to *S*, and to the *left* when *S* changes to *N*.

Going west the deflection is to the *right* when *S* changes to *N* and to the *left* when *N* changes to *S*.

4. *First letters unlike and last letters unlike.* Fig. 122.

Rule.—Subtract the difference of the courses from 180 deg.

Going north if the bearing changes toward the west, or going south the bearing changes toward the east the deflection is *left.*

Going north if the bearing changes toward the east, or going south the bearing changes toward the west the deflection is *right.*

RULES FOR OBTAINING INCLUDED ANGLES

5. If the first letters are alike and the last are alike subtract the greater course from 180 deg. and add the smaller course. Fig. 123.

6. If the first letters are alike and the last are unlike subtract the sum of the courses from 180 deg. Fig. 124.

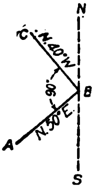


FIG. 124.

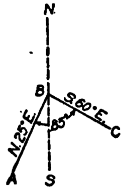


FIG. 125.

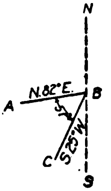


FIG. 126.

7. If the first letters are unlike and the last are alike, add the courses. Fig. 125.

8. If the first letters are unlike and the last are unlike the included angle is equal to the difference of the courses. Fig. 126.

It will be seen that the required angle in each case is *A*, *B*, *C* in the last four rules.

The student should have considerable exercise in obtaining angles from bearings. This can be done very well on a table by using a circular protractor. Procure a paper protractor 8 ins. in diameter and trim it to circle. Number the graduations each way from a meridian line from 0 deg. to 90 deg., lettering the quadrants like a compass, with *E* and *W* reversed. Tack down a sheet of drawing paper and on it draw a straight line to represent the true meridian. Lay the protractor on this line so the zero points touch it. Draw a

line normal to the meridian and set the center of the protractor at the intersection of the two lines.

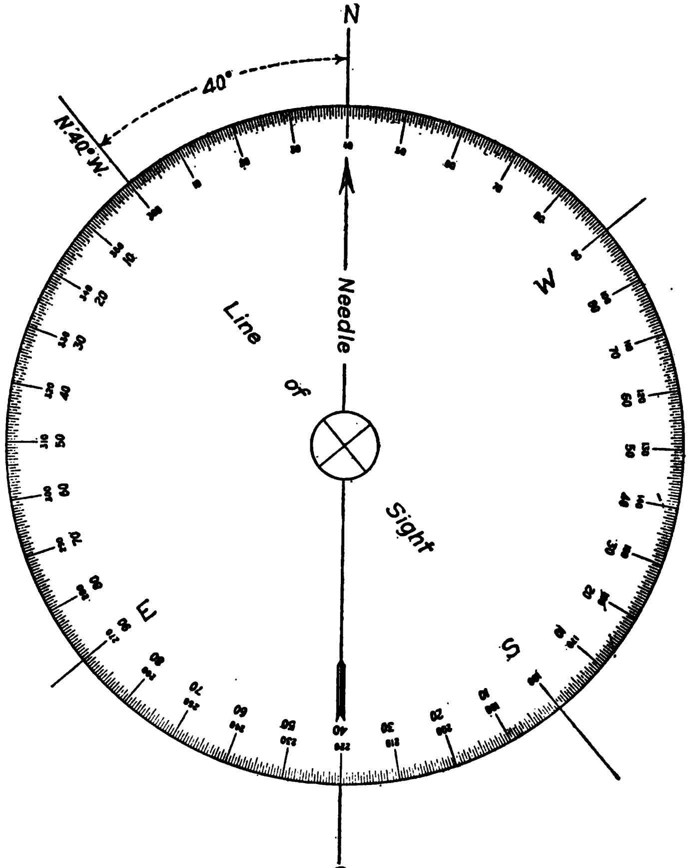


FIG. 127. Obtaining angles from bearings.

To plot a course turn the protractor until the required angle touches the meridian line. On the drawing paper mark a dot at the N zero point and opposite it write the

course. Set off the second course. Then with the N zero at one course the included angle, or the deflection angle, can be read off as a check on the calculations.

To obtain the true bearing from a random line.

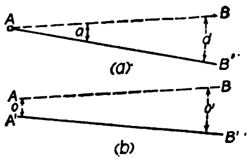


FIG. 128.

In Fig. 128 two cases are shown of random lines. In (a) the dotted line represents a boundary and the surveyor started from A to re-trace the line but when the required distance was measured found himself at B_1 instead of B. The difference d was then measured.

In (b) the case is similar but an offset line A_1B_1 was run so that instead of the offset at each end being O, it was O at A_1 and O^1 at B_1.

The difference $d = O' - O$.

Then angle $a = \dfrac{57.3 \times d}{A_1 \times B_1} = \dfrac{57.3 \times d}{A \times B_1}$.

Assume the length of the line to be 20 chains and d = 20 links = 0.20 chain.

$$\frac{57.3 \times 0.2}{20} = 0.573 \text{ deg.}$$

$$0.573 \times 60 = 34.38 \text{ min.}$$

The line AB_1 (or A_1B_1) was run on a bearing of $N\ 45°\ 00'\ E$, but as it ran to the right of the true line the bearing must be corrected to the left and becomes $N\ 45°\ 35'\ E$.

Suppose the original field notes to have called for a bearing of $N\ 45°\ E$ for line AB and the re-tracement as above showed a difference of 35 min. between the actual bearing and that given in the old field notes; this difference can be set off on the variation plate. The whole survey can then be re-traced according to the recorded bearings, provided the original surveyor made no serious errors in reading the needle.

MAKING THE MAP

In the center of a sheet of paper rule two long straight lines perpendicular to each other. The line from the bottom to the top is the true meridian and the line from left to right is an east and west line.

Graduate a paper protractor in quadrants but do not transpose the E and W as on a compass plate. On the protractor, which should be 14 ins. in diameter, draw a fine ink line joining the N and S zeros and another joining the E and W, 90°, points. At the center where the lines

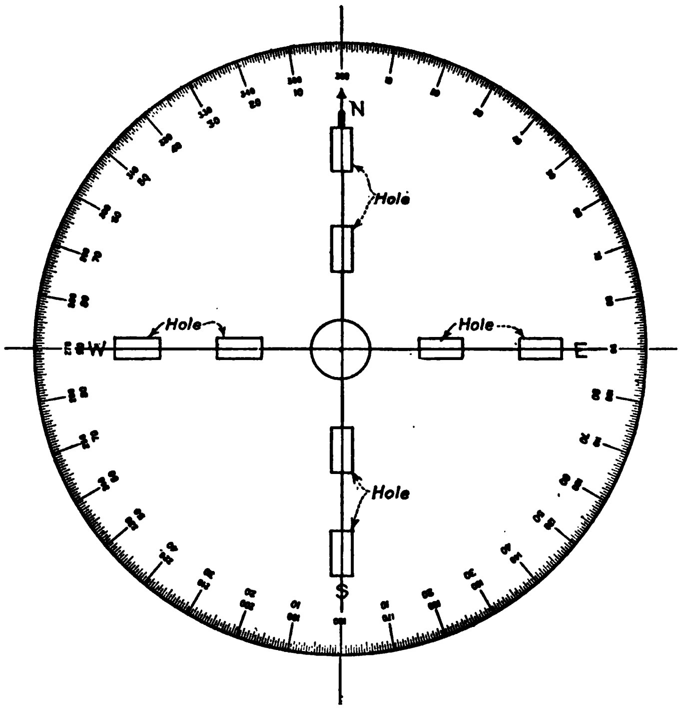


FIG. 129. Paper protractor for plotting angles.

intersect and at a couple of places on each line cut square or round holes about $\frac{1}{2}$ in. across, with clean smooth edges. Trim the protractor along the edge into circular form, for it is printed on a square sheet.

Lay the protractor on the drawing paper over the two ruled lines so they may be seen through the holes and their intersection will indicate the exact center of the protractor.

To keep it in position two thumb tacks may be used, or, if the holes will be considered objectionable in the map use paper weights. Very good paper weights are made of chamois skin bags containing two lbs. of fine shot, an outer bag of cloth being used as a cover.

The protractor being carefully set, with a sharp-pointed pencil mark all the angles on the drawing paper, numbering each dot to correspond with the number of the course, or write the course after each dot. When all the angles are marked remove the protractor and connect by fine lines each dot with the center. The result will look like Fig. 130.

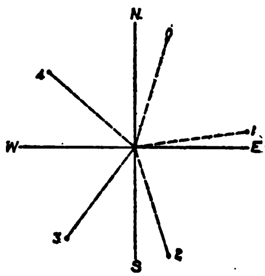


FIG. 130. Plotting bearings.

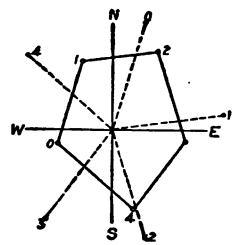


FIG. 131. Plat made by bearings.

Select a starting point and with triangles, or with triangle and straight-edge, transfer the first course to that point and draw a line parallel to the course. With the scale set off the length on the line and mark the end for Sta. 1. From this station set off the second course and proceed in this manner until the boundary is all platted. The completed result is shown in Fig. 131.

On long lines of survey the true meridian is carefully transferred by straight-edge and triangle to a convenient place so it will not be necessary to transfer the courses farther than is convenient with a triangle, thus avoiding danger of slipping and consequent error.

PLOTTING BY DEFLECTIONS

Some inexperienced, or poorly instructed, men will use small protractors and plot deflections. Each course is laid off to a considerable length and the distance marked.

The protractor is placed with center on the station and the deflection angle set off. A line is then drawn through the two points, the length of the next course marked and the

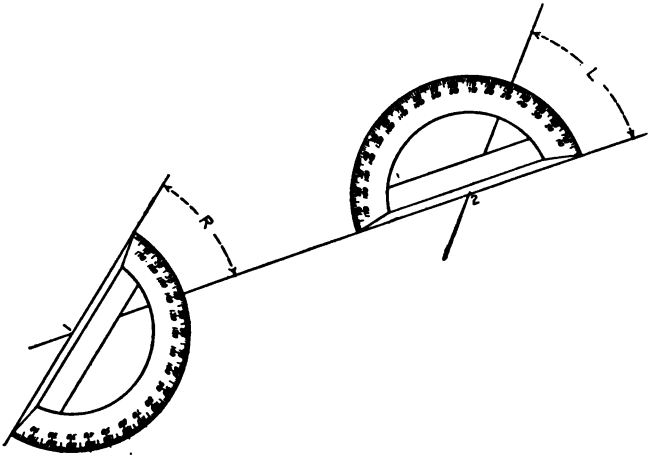


FIG. 132. Platting deflections.

operation repeated until all the courses are plotted. The method is troublesome and very inaccurate for the errors are cumulative.

Another method is to use a heavy steel straight-edge and large triangle and carry the meridian through the end of each course. A large protractor is placed on each station and the following course laid off. This is an improvement on the method of plotting deflections, but slow and not so accurate as the method first described.

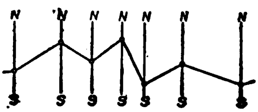


FIG. 133. Platting from meridians.

Other methods of plotting, requiring some knowledge of Trigonometry, are described in Chapter VI.

DISTRIBUTING ERRORS

Errors in surveys show up when making the map, for the last distance will carry the end past the last station (the starting point), or perhaps fall short. It will also be

to one side. A plat of a survey is apt to look like Fig. 134.

Measure the distance from O' to o and call it a. Call the sum of all the courses b. The closing error $= \frac{1}{\sqrt{a^2/b}}$.

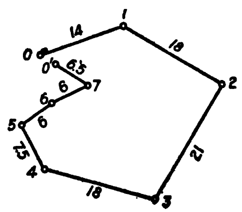


FIG. 134. Plat in which error is not distributed.

If this exceeds $\frac{1}{500}$, or any limit of error selected, then each course should be mentally examined to discover if possible whether a mistake was made. If no evidence is found it will be best to re-run the lines and bring the error within proper limits.

An error within proper limits may be distributed over the courses in proportion to the actual or weighted lengths.

To weight errors the survey is mentally reviewed and those courses on which no difficulties were encountered are given a weight of 1, those on marshy land, 1.5, those on steep land, 2, these values being here chosen merely for illustrative purposes. The values indicate the relative probability of errors occurring.

Multiply each course by the weight assigned to it and add the new lengths progressively. This new total length

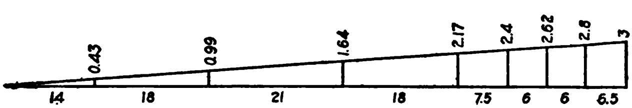


FIG. 135. Graphical method for distributing errors.

is measured off on a straight line and at one end the total error is measured off on a perpendicular line and a triangle formed. From each station point a perpendicular is erected to intersect the hypothenuse and the length of the perpendicular at any station represents the amount of error.

A line is drawn from O' to O on the map and parallel lines are transferred to each station as shown in Fig. 136. The ends are connected and the survey is closed.

Multiplying a distance by the weight assigned lengthens it and thus gives it that much greater proportion of error. This is evident from the law of proportionality of triangles. Instead of making a triangle the error may be distributed as follows:

Divide the error by the total length of all the courses, after weighting same. Multiply the quotient by each course to obtain the error for each course.

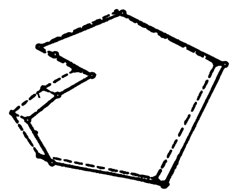


FIG. 136. Plat of field with errors distributed.

Example. — Closing error 0.19 chain.

No. of course.	Distance, chains.	Weight.	Weighted distance.	Error.
0	4.75	1	4.75	0.018
1	3.15	1	3.15	0.012
2	6.22	1	6.22	0.024
3	7.10	1.5	10.65	0.041
4	9.05	2	18.10	0.069
5	2.76	2.5	6.90	0.026
			49.77	0.190

$$\frac{0.19}{49.77} = 0.0038 + \text{ per chain.}$$

When the lines are adjusted and the survey is closed the map may be completed. From the notes and sketches in the field book plat the fences, etc. Draw meridians through

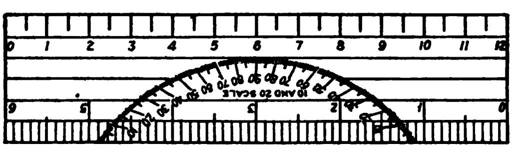


FIG. 137.

stations from which bearings were taken to objects; place a protractor on the meridians and lay off the bearings. For this work a transparent protractor with scale on edge is very convenient. It is also a useful tool to carry in the

field book to assist in making sketches. The center is placed on the station and the meridian passes through the angle. The edge then indicates the bearing to the object, the distance to which is measured by means of the graduations.

Protractors are made of horn, paper, metal, celluloid, ivory and rubber. The prices range from fifteen cents to eighty-five dollars. For ninety per cent of the work done by surveyors and engineers, 14-in. paper protractors graduated to $\frac{1}{4}$ degrees are better than the more expensive kinds.

To find the area of a surveyed field, first plot it as described and when the lines are closed divide it into quadrilaterals, triangles, etc.; obtain the area of each, and the sum of these areas is the area of the field.

OBTAINING AREAS BY COMPUTATION WITHOUT MAKING A MAP

All compass bearings are angles measured to the right or left of the true meridian.

A course may therefore be drawn, forming the hypothenuse of a right-angled triangle the base of which is on the meridian. The base is called the latitude, for latitude is measured north and south from the equator. The altitude is called the departure, for it measures the amount by which the course "departs" from a true north and south line.

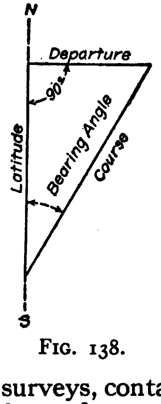


FIG. 138.

By some writers within recent years, the "departure" has been termed "longitude difference." The reasoning is that through each point a true meridian may be drawn and longitude being measured on circles normal to meridians the distance between meridians is the difference in longitude. A Traverse Table (page 142), used for compass surveys, contains the latitude and departure for all bearings from 0° to 90°, varying by quarters of a degree.

Example 1. — Find the latitude and departure for $N\ 5\frac{1}{4}°\ W$ 23.77 chains.

Looking in the column headed Course, trace down to the horizontal line opposite $5° 15'$. Arrange in columns as follows:

	Latitude.	Departure.
Dist. $2 \times 10 =$	19.916	1.83
Dist. $3 \times 1 =$	2.9874	0.2745
Dist. $7 \times 0.1 =$	0.6970	0.0645
Dist. $7 \times .01 =$	0.0697	0.0064
	23.6701	2.1754

Therefore in going $N\ 5° 15'\ W$ 23.77 chains the line runs north 23.67 chains and west 2.18 chains.

For an angle of 45° the latitude and departure are equal.

For an angle of 0° the latitude is 1 and the departure = 0.

For an angle of 90° the latitude is zero and the departure = 1.

Therefore the latitude of an angle less than 45° is equal to the departure of an angle of the same size between 45° and 90°. For example the latitude for 13° is the departure for 77° and vice versa. This fact saves much labor in the computation of tables and economizes space in the printing of tables. For all angles under 45° read the courses on the left-hand edge of the page going down, using the Lat. and Dep. columns at the top. For all angles over 45° read the courses on the right-hand edge of the page going up, using the Lat. and Dep. columns at the bottom.

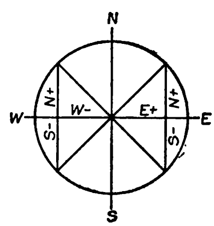


FIG. 139.

North latitude is positive (+) and south latitude is negative (−). Similarly east departure is positive (+) and west departure negative (−). This is illustrated in Fig. 139. All distances due north and due east are said to be measured in a positive sense and all distances due south and due west, the opposite of north and east respectively, are said to be measured in a negative sense. If a man starts from a certain point and measures north +10 chains, then measures back on the line south −4 chains, he ends at a point $+10 - 4 = +6$ chains north of the starting point.

Example. — A line is surveyed as follows:

N 50° *E*, 11 chains,
N 7° *W*, 12 chains,
S 14° *E*, 22 chains.

How far apart are the two ends of the line, measured by the latitude and departure of a line connecting the points?

Bearing.	Distance.	N+	S−	E+	W−
N 5° E	11	10.958	....	0.959	....
N 7° W	12	11.911	....	....	1.462
S 14° E	22	....	21.347	0.532	....
		22.869	21.347	1.491	1.462

After ruling a sheet of paper as above and filling in the bearings and distances use the traverse table and place the results in the proper columns.

N 5° E.			N 7° W.			S 14° E.		
Dist.	Lat.	Dep.	Dist.	Lat.	Dep.	Dist.	Lat.	Dep.
10	9.9619	0.8716	10	9.9255	1.2187	20	19.406	0.4838
1	0.9962	0.0872	2	1.9851	0.2437	2	1.941	0.0484
	10.9581	0.8588		11.9106	1.4624		21.347	0.5323

North (+) latitudes = 22.869
South (−) latitudes = 21.347
1.522 Diff. (+)

East (+) departures = 1.491
West (−) departures = 1.462
0.029 Diff. (+)

The last point set is 1.522 chains north and 0.029 chain east from the starting point.

In computing areas of compass surveys the process commonly followed is essentially that of reducing the figure to a triangle of equivalent area and then finding the area of the triangle. This is the method of Double Meridian

Distances. In some old American textbooks it was known as the Pennsylvania Method but the original discoverer was Thomas Burgh of Ireland in 1778, to whom it is said the Irish Parliament awarded a pension equal to the interest on twenty thousand pounds as a recognition of the great value of his work.

In the April, 1879, number of Van Nostrand's *Engineering Magazine* a new rule was proposed by Professor J. Woodbridge Davis, which is less laborious than the old rule. The author used it instead of Double Meridian Distances during those years in which much of his work was surveying and cannot understand why the old rule is not abandoned. The newer one is called the Total Latitude rule and will alone be given in this book. It is based on trapezoids instead of triangles.

Rule. — Multiply the total latitude of each station by the sum of the departures of the two adjacent courses. The algebraic half sum of these products is the area.

Algebraic sum must first be understood and the rules memorized.

To add positive (+) *and negative* (−) *quantities, subtract the lesser from the greater and prefix to the result the sign of the greater.* This has been illustrated in the use of the traverse table.

To subtract positive and negative quantities change the sign of the subtrahend and then proceed as in addition.

To multiply positive and negative quantities the rules are:

(*a*) *Like signs produce plus.*

$$(+5) \times (+6) = +30.$$
$$(-5) \times (-6) = +30.$$

(*b*) *Unlike signs produce minus.*

$$(+5) \times (-6) = -30.$$

In practical work the positive (+) sign is generally understood so is seldom written.

The following example is taken from the article of Professor Davis.

Sta.	Bearing.	Dist.	Lat. N+	Lat. S−	Dep. E+	Dep. W−	Total lat.	Adj. dep.	Double areas.
0	N 35° E	2.70	2.21	....	1.55	....	......	......	....
1	N 83½° E	1.29	0.15	....	1.28	....	2.21	2.83	6.2543
2	S 57° E	2.22	....	1.21	1.86	....	2.36	3.14	7.4104
3	S 34¼° W	3.55	....	2.93	....	2.00	1.15	−0.14	−0.1610
4	N 56½° W	3.23	1.78	....	....	2.69	−1.78	−4.69	8.3482
			4.14	4.14	4.69	4.69			2)21.8519
						Square chains			10.9259

Area 1.0926 acres.

When using the Double Meridian Distance rule certain considerable advantage is secured by starting the work of computation at the most easterly or the most westerly station. When using the Total Latitude rule any station may be used as a starting point but if that station is chosen for which the latitude is most nearly the average of all, the fewest possible figures will be used in the double area factors.

In the example the work was performed as follows:

Total latitude:

$$
\begin{array}{r}
+2.21 \\
+\underline{0.15} \\
+2.36 \\
-\underline{1.21} \\
+1.15 \\
-\underline{2.93} \\
-1.78 \text{ Check}
\end{array}
$$

Check. — The total latitude for the last station must be equal to the latitude of that station, with opposite sign. The total latitude of the first station is zero.

The adjacent departure for the first station is zero.

$$
\begin{array}{llll}
\text{2nd station.} & \text{Dep. 0} & = +1.55 & \\
 & \text{Dep. 1} & = +\underline{1.28} & +2.83 \\
 & \text{Dep. 1} & = +1.28 & \\
 & \text{Dep. 2} & = +\underline{1.86} & +3.14 \\
 & \text{Dep. 2} & = +1.86 & \\
 & \text{Dep. 3} & = -\underline{2.00} & -0.14 \\
 & \text{Dep. 3} & = -2.00 & \\
 & \text{Dep. 4} & = -\underline{2.69} & -4.69
\end{array}
$$

The only checks on the adjacent departures addition are care and re-computing.

Following the algebraic law for dealing with positive and negative signs the column of double areas is filled. For Sta. 1 and 2 the sign is + for both the total latitude and the adjacent departure, therefore the double area is positive. For Sta. 3 the first is + and the second is −, therefore the double area is negative. For Sta. 4 both signs are − and the double area is positive.

DISTRIBUTING ERRORS

When measuring with a chain, using a compass for the angles, lengths are usually taken to the nearest link. Some surveyors record the nearest half link but when this is done they attempt to read the bearing more closely than a quarter of a degree by estimating the position of the north end of the needle between graduations.

The maximum error in angle when the nearest quarter of a degree is recorded is seven minutes, corresponding to a departure of 0.2 link per chain or 1 in 500. The maximum allowable error in chaining for ordinary farming land should not exceed 1 in 500, so that the work of chainmen having some experience is about equal in accuracy to that of ordinary compass work.

The errors here referred to are the accidental errors of the work. Errors due to a chain or tape being longer or shorter than standard do not show until a re-survey is made with a standard.

Minor errors of closure are distributed proportionately to lengths in compass surveys, the effect being to change bearings and distances. The transit being an instrument by means of which angles can be read very closely, all closing errors are distributed over the lengths only, it being easily possible to check the angles and obtain exact angular closures.

The method here given for distributing errors is applicable alike to compass work or transit work since it is bad practice to alter bearings and distances when the error of closure is found to be within reasonable limits. Forcing a closure is called "fudging" and notes of a compass survey

are looked upon with suspicion when they close without error.

Example. — A field was surveyed and the following notes recorded:

		Chains
0.	*N* 35° *E*	2.73
1.	*N* 83½° *E*	1.30
2.	*S* 57° *E*	2.21
3.	*S* 34½° *W*	3.54
4.	*N* 56½° *W*	3.22

When the latitudes and departures were computed the field did not close. Before the area of a field can be computed the northings must equal the southings and the eastings must equal the westings. In other words, in measuring the boundaries of a field the surveyor goes as far north as he goes south and as far east as he goes west.

Sta.	Bearing.	Dist.	Lat.		Dep.		Balanced Lat.		Balanced Dep.	
			N+	S−	E+	W−	N+	S−	E+	W−
0	N 35° E	2.73	2.214	.....	1.550	.....	2.2132	.....	1.5456	.....
1	N 83½° E	1.30	0.147	.....	1.292	.....	0.1469	.....	1.2883	.....
2	S 57° E	2.21	.....	1.209	1.862	.....	.....	1.2094	1.8566	.....
3	S 34½° W	3.54	.....	2.926	.....	1.992	.....	2.9271	.....	1.9978
4	N 56½° W	3.22	1.777	.....	.....	2.685	1.7764	.....	.....	2.6927
		13.00	4.138	4.135	4.704	4.677	4.1365	4.1365	4.6905	4.6905
			4.135		4.677					
	Error in lat. =		0.003		0.027 = Error in dep.					

$$\text{Error of closure} = \sqrt{\frac{0.3^2 + 2.7^2}{1300}} = 0.00208,$$

$$\frac{1}{0.00208} = 1 \text{ in } 480.$$

The error, being within reasonable limits, is distributed as follows:

Latitudes $\left\{\begin{array}{l} 4.138 \\ \underline{4.135} \end{array}\right.$

$$\frac{8.273}{2} = 4.1365 \text{ correct sum.}$$

$$\begin{array}{l} 4.1380 \\ \underline{4.1365} \\ 0.0015 \end{array}$$ to be subtracted from the northings.

$$\begin{array}{l} 4.1365 \\ \underline{4.1350} \\ 0.0015 \end{array}$$ to be added to the southings.

$$1 + \frac{0.0015}{4.1365} = 0.00036 \text{ to increase southings.}$$

$$1 - \frac{0.0015}{4.1365} = 1 - 0.00036 = 0.99964 \text{ to decrease northings.}$$

	Northings.	Southings.
2.214 × 0.99964	2.2132	......
0.147 × 0.99964	0.1469	......
1.209 × 1.00036	......	1.2019
2.926 × 1.00036	......	2.9271
	4.1365	4.1365

The results are placed in the "Balanced" columns.

Departures $\left\{\begin{array}{l} 4.704 \\ \underline{4.677} \end{array}\right.$

$$\frac{9.381}{2} = 4.6905 \text{ correct sum.}$$

$$\begin{array}{l} 4.7040 \\ \underline{4.6905} \\ 0.0135 \end{array}$$ to be subtracted from eastings.

$$\begin{array}{l} 4.6905 \\ \underline{4.677\ \ } \\ 0.0135 \end{array}$$ to be added to westings.

$$1 + \frac{0.0135}{4.6905} = 1.00287 \text{ to increase westings.}$$

$$1 - \frac{0.0135}{4.6905} = 0.99713 \text{ to decrease eastings.}$$

	Eastings.	Westings.
1.55 × 0.99713	1.5456	....
1.292 × 0.99713	1.2883	....
1.862 × 0.99713	1.8566	....
1.992 × 1.00287	......	1.9978
2.685 × 1.00287	......	2.6927
	4.6905	4.6905

In actual practice few surveyors do all the figuring here shown, the corrections being made mentally and distributed in approximately proportionate amounts. The practical surveyor also pays great attention to the weighting of the various courses when his chainmen are inexperienced and their work is therefore not wholly reliable.

OMISSIONS

It is not possible always to measure each side of a field or take every bearing and the missing parts must be supplied by platting or by computation, this latter method requiring a knowledge of trigonometry. If a map is not accurately made to a very large scale omissions cannot be obtained from it within the limits of even reasonable accuracy. For the purpose, however, of *computing areas by mensuration* missing lines may be supplied by plotting. The methods to be now described should be carefully studied and worked out in full by the student, for he will then fully understand the methods of computation in the chapter following.

There are six cases in omissions and one field will be used to illustrate them.

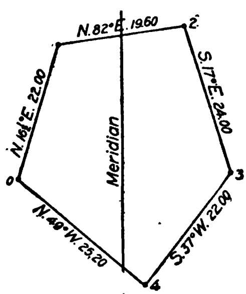


FIG. 140.

		Chains
0.	*N* 16½° *E*...............	22.00
1.	*N* 82° *E*...............	19.60
2.	*S* 17° *E*...............	24.00
3.	*S* 37° *W*..............	22.00
4.	*N* 49° *W*...............	25.20

CASE I. — Bearing and distance omitted. Fig. 141.

Assume the course from 2 to 3, $S\ 17^\circ\ E$, 24 chains to be omitted.

Plat the two courses 0 and 1, beginning at Sta. 0. Reverse the bearings for courses 3 and 4 so they may be platted beginning at Sta. 0. The line from 2 to 3 may then be drawn. A close check on the bearing may be obtained by drawing a meridian line through one end of the line and applying a protractor.

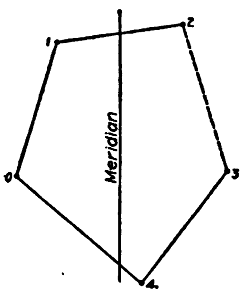


FIG. 141.

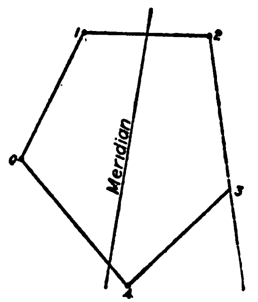


FIG. 142.

CASE II.—Bearing read but length not measured. Fig. 142.

The omitted length is for course 2. Begin at Sta. 0 and lay off the bearings and distances to Sta. 2 and from Sta. 2 lay off by the protractor a long line having the bearing $S\ 17^\circ\ E$. Reversing the bearings of courses 3 and 4 as in the first example, plot them; Sta. 3 should be on the long line drawn from Sta. 2 provided the field work and platting are correct.

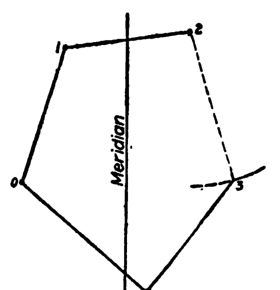


FIG. 143.

CASE III. — Length measured, bearing not read. Fig. 143.

Assume same course to be affected. Beginning at Sta. 0 plot courses 0 and 1 in a forward direction and courses 3 and 4 reversed. From Sta. 2 as a center with a radius equal to 24 chains, describe an arc. If the work was properly done the arc should pass through Sta. 3.

CASE IV. — Two bearings read but lengths not measured.

(*a*) Adjacent courses. Fig. 144.

The omitted lengths are assumed to be for courses 2 and 3. Begin at Sta. 0 and plot in a forward direction to Sta. 2, from which point draw a long line on the given bearing. Then plot the reversed courses; the intersection of the long line from Sta. 2 with that from Sta. 4 fixes the location of Sta. 3.

(*b*) Courses not adjacent. Fig. 145.

Assume the omitted lengths to be in courses 1 and 4. Begin at 0 and lay off the line from 0 to 1. The length of

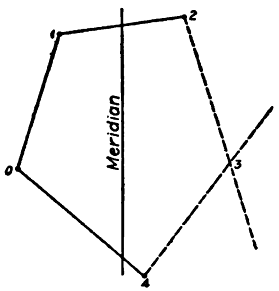


FIG. 144.

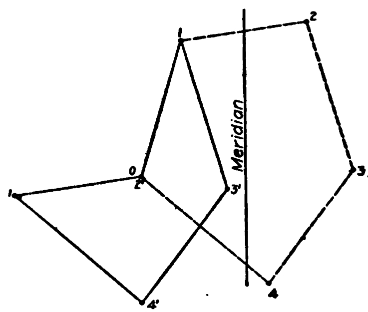


FIG. 145.

the next line is not known so this course is disregarded and from Sta. 1 lay off the bearing and distance of course 2. *S* 17° *E* 24 chains and follow that with the next course. Then from 0 lay off the bearing *S* 82° *W* (the reverse of *N* 82° *E*). From 4′ lay off the bearing *N* 49° *W*. These lines will intersect at 1′. By using triangles transfer the line 1′–2′ to 1–2 and thus locate 2. From 2 draw 2–3 equal and parallel to 1–3′ and from 3 draw line 3–4 equal and parallel to 3′–4′. Then the line 4–0 will be equal and parallel to line 4′–1′.

CASE V. — Two lengths measured but bearings not read.

(*a*) Adjacent courses. Fig. 146.

Assuming the bearings of courses 2 and 3 to have been omitted plot the other courses from Sta. 0 as before. From

Sta. 2 describe an arc with a radius equal to the length of course 2 and from Sta. 4 describe an arc with a radius equal to the length of course 3. These arcs will intersect in 3 and in 3′. It will be necessary to view the land in order to know which point is correct.

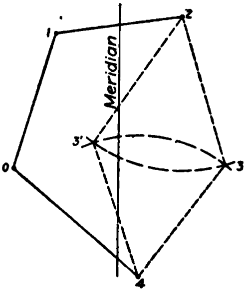


Fig. 146.

(*b*) Courses not adjacent. Fig. 147. The bearings of courses 2 and 4 were not read. The remaining three courses are plotted from 0 to 1, 1 to 2, 2 to 3′. From 0 an arc with radius equal to the length of course 4 is described and from 3′ an arc with radius equal to the length of course 2 is described. Fig. 147 (a) shows the work, (b) the figure resulting when courses 1–2 and 0–4 are omitted and (c) the figure resulting when 0–1 and 4–3 are omitted. In this case a knowledge of the shape of the field is necessary in order to know which solution is right.

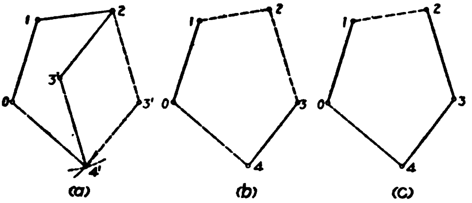


Fig. 147.

Case VI. — Bearing of one course read but length not measured and length measured but bearing omitted on another course.

(*a*) Adjacent courses. Fig. 148.

Plat as before. The bearing from Sta. 2 having been read lay off a dotted line. The length of course 3 having been measured describe an arc with this radius from Sta. 4. This gives two possible solutions so that actual knowledge of the shape of the field is necessary.

(*b*) Courses not adjacent. Fig. 149.

It is assumed that the bearing from Sta. 1 was omitted and the length of course 4 was omitted.

Plot the course 0–1. From Sta. 1 describe an arc with radius equal to the length of course 1 and from 0 lay off the line 0–4 with the bearing of course 4. On a piece of

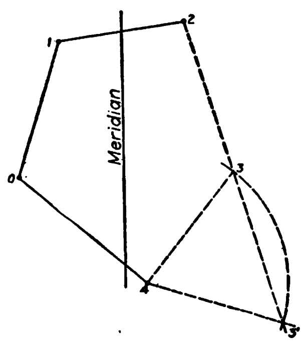


FIG. 148.

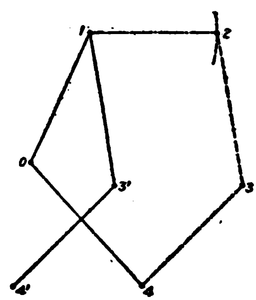


FIG. 149.

tracing paper plot the line 3–4 and from 3 the line 3–2, extending the line considerably beyond the mark at 2. From 4 plot a line having the bearing of course 4. Place the line 4–0 on the tracing paper over the same line on the drawing paper and move it along this line until the line 2–3 is tangent to the arc and the point 2 is on the arc. A needle put through at 2, 3 and 4 will mark the stations and lines may then be drawn to complete the map.

A careful examination of the problems given shows that in reality there are only four cases, in which the lost, or omitted, parts may be:

CASE I. — Bearing and length of one course.

CASE II. — Lengths of two courses.

CASE III. — Bearing of two courses.

CASE IV. — Length of one course and bearing of another.

PROBLEMS

Find by computation the areas of the following fields. Plat same carefully and check the computations by mensuration.

			Chains
1.	0.	N 16½° E	22.00
	1.	N 82° E	19.60
	2.	S 17° E	24.00
	3.	S 37° W	22.00
	4.	N 49° W	25.20

			Chains
2.	0.	N 15° E	80
	1.	N 37½° E	40
	2.	East	30
	3.	S 11° E	50
	4.	South	54
	5.	West	40
	6.	S 36½° W	40
	7.	N 38¼° W	34

			Chains
3.	0.	N 75° E	13.70
	1.	N 20½° E	10.30
	2.	East	16.20
	3.	S 33½° W	35.30
	4.	S 76° W	16.00
	5.	North	9.00
	6.	S 84° W	11.60
	7.	N 53¼° W	11.60
	8.	N 36¾° E	19.20
	9.	N 22½° E	14.00
	10.	S 76¾° E	12.00
	11.	S 15° W	10.85
	12.	S 16¾° W	10.12

Traverse Table

Course.		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.			
		Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.		
°	′											°	′
0	15	1.0000	0.0044	2.0000	0.0087	3.0000	0.0131	4.0000	0.0175	5.0000	0.0218	89	45
	30	0000	0087	1.9999	0175	2.9999	0262	3.9998	0349	4.9998	0436		30
	45	0.9999	0131	9998	0262	9997	0393	9997	0524	9996	0654		15
1	0	9998	0175	9997	0349	9995	0524	9994	0698	9992	0873	89	0
	15	9998	0218	9995	0436	9993	0654	9990	0873	9988	1091		45
	30	9997	0262	9993	0524	9990	0785	9986	1047	9983	1309		30
	45	9995	0305	9991	0611	9986	0916	9981	1222	9977	1527		15
2	0	9994	0349	9988	0698	9982	1047	9976	1396	9970	1745	88	0
	15	9992	0393	9985	0785	9977	1178	9969	1570	9961	1963		45
	30	9990	0436	9981	0872	9971	1309	9962	1745	9952	2181		30
	45	0.9988	0.0480	1.9977	0.0960	2.9965	0.1439	3.9954	0.1919	4.9942	0.2399		15
3	0	9986	0523	9973	1047	9959	1570	9945	2093	9931	2617	87	0
	15	9984	0567	9968	1134	9952	1701	9936	2268	9920	2835		45
	30	9981	0610	9963	1221	9944	1831	9925	2442	9907	3052		30
	45	9979	0654	9957	1308	9936	1962	9914	2616	9893	3270		15
4	0	9976	0698	9951	1395	9927	2093	9903	2790	9878	3488	86	0
	15	9973	0741	9945	1482	9918	2223	9890	2964	9863	3705		45
	30	9969	0785	9938	1569	9908	2354	9877	3138	9846	3923		30
	45	9966	0828	9931	1656	9897	2484	9863	3312	9828	4140		15
5	0	9962	0872	9924	1743	9886	2615	9848	3486	9810	4358	85	0
	15	0.9958	0.0915	1.9916	0.1830	2.9874	0.2745	3.9832	0.3660	4.9790	0.4575		45
	30	9954	0958	9908	1917	9862	2875	9816	3834	9770	4792		30
	45	9950	1002	9899	2004	9849	3006	9799	4008	9748	5009		15
6	0	9945	1045	9890	2091	9836	3136	9781	4181	9726	5226	84	0
	15	9941	1089	9881	2177	9822	3266	9762	4355	9703	5443		45
	30	9936	1132	9871	2264	9807	3396	9743	4528	9679	5660		30
	45	9931	1175	9861	2351	9792	3526	9723	4701	9653	5877		15
7	0	9925	1219	9851	2437	9776	3656	9702	4875	9627	6093	83	0
	15	9920	1262	9840	2524	9760	3786	9680	5048	9600	6310		45
	30	9914	1305	9829	2611	9743	3916	9658	5221	9572	6526		30
	45	0.9909	0.1349	1.9817	0.2697	2.9726	0.4046	3.9635	0.5394	4.9543	0.6743		15
8	0	9903	1392	9805	2783	9708	4175	9611	5567	9513	6959	82	0
	15	9897	1435	9793	2870	9690	4305	9586	5740	9483	7175		45
	30	9890	1478	9780	2956	9670	4434	9561	5912	9451	7390		30
	45	9884	1521	9767	3042	9651	4564	9534	6085	9418	7606		15
9	0	9877	1564	9754	3129	9631	4693	9508	6257	9384	7822	81	0
	15	9870	1607	9740	3215	9610	4822	9480	6430	9350	8037		45
	30	9863	1650	9726	3301	9589	4951	9451	6602	9314	8252		30
	45	9856	1693	9711	3387	9567	5080	9422	6774	9278	8467		15
10	0	9848	1736	9696	3473	9544	5209	9392	6946	9240	8682	80	0
	15	0.9840	0.1779	1.9681	0.3559	2.9521	0.5338	3.9362	0.7118	4.9202	0.8897		45
	30	9833	1822	9665	3645	9498	5467	9330	7289	9163	9112		30
	45	9825	1865	9649	3730	9474	5596	9298	7461	9123	9326		15
11	0	9816	1908	9633	3816	9449	5724	9265	7632	9081	9540	79	0
	15	9808	1951	9616	3902	9424	5853	9231	7804	9039	9755		45
	30	9799	1994	9598	3987	9398	5981	9197	7975	8996	9968		30
	45	9790	2036	9581	4073	9371	6109	9162	8146	8952	1.0182		15
12	0	9781	2079	9563	4158	9344	6237	9126	8316	8907	0396	78	0
	15	9772	2122	9545	4244	9317	6365	9089	8487	8862	0609		45
	30	9763	2164	9526	4329	9289	6493	9052	8658	8815	0822		30
	45	0.9753	0.2207	1.9507	0.4414	2.9260	0.6621	3.9014	0.8828	4.8767	1.1035		15
13	0	9744	2250	9487	4499	9231	6749	8975	8998	8719	1248	77	0
	15	9734	2292	9468	4584	9201	6876	8935	9168	8669	1460		45
	30	9724	2334	9447	4669	9171	7003	8895	9338	8618	1672		30
	45	9713	2377	9427	4754	9140	7131	8854	9507	8567	1884		15
14	0	9703	2419	9406	4838	9109	7258	8812	9677	8515	2096	76	0
	15	9692	2462	9385	4923	9077	7385	8769	9846	8462	2308		45
	30	9681	2504	9363	5008	9044	7511	8726	1.0015	8407	2519		30
	45	9670	2546	9341	5092	9011	7638	8682	0184	8352	2730		15
15	0	9659	2588	9319	5176	8978	7765	8637	0353	8296	2941	75	0
		Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.	
		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.			

TRAVERSE TABLE (*Continued*)

Course.		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.			
		Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.		
°	′											°	′
0	15	5.9999	0.0262	6.9999	0.0305	7.9999	0.0349	8.9999	0.0393	9.9999	0.0436	89	45
	30	9998	0524	9997	0611	9997	0698	9997	0785	9996	0873		30
	45	9995	0785	9994	0916	9993	1047	9992	1178	9991	1309		15
1	0	9991	1047	9989	1222	9988	1396	9986	1571	9985	1745	89	0
	15	9986	1309	9983	1527	9981	1745	9979	1963	9976	2181		45
	30	9979	1571	9976	1832	9973	2094	9969	2356	9966	2618		30
	45	9972	1832	9967	2138	9963	2443	9958	2748	9953	3054		15
2	0	9963	2094	9957	2443	9951	2792	9945	3141	9939	3490	88	0
	15	9954	2356	9946	2748	9938	3141	9931	3533	9923	3926		45
	30	9943	2617	9933	3053	9924	3490	9914	3926	9905	4362		30
	45	5.9931	0.2879	6.9919	0.3358	7.9908	0.3838	8.9896	0.4318	9.9885	0.4798		15
3	0	9918	3140	9904	3664	9890	4187	9877	4710	9863	5234	87	0
	15	9904	3402	9887	3968	9871	4535	9855	5102	9839	5669		45
	30	9888	3663	9869	4273	9851	4884	9832	5494	9813	6105		30
	45	9872	3924	9850	4578	9829	5232	9807	5886	9786	6540		15
4	0	9854	4185	9829	4883	9805	5581	9781	6278	9756	6976	86	0
	15	9835	4447	9808	5188	9780	5929	9753	6670	9725	7411		45
	30	9815	4708	9784	5492	9753	6277	9723	7061	9692	7846		30
	45	9794	4968	9760	5797	9725	6625	9691	7453	9657	8281		15
5	0	9772	5229	9734	6101	9696	6972	9658	7844	9619	8716	85	0
	15	5.9748	0.5490	6.9706	0.6405	7.9664	0.7320	8.9622	0.8235	9.9580	0.9150		45
	30	9724	5751	9678	6709	9632	7668	9586	8626	9540	9585		30
	45	9698	6011	9648	7013	9597	8015	9547	9017	9497	1.0019		15
6	0	9671	6272	9617	7317	9562	8362	9507	9408	9452	0453	84	0
	15	9643	6532	9584	7621	9525	8709	9465	9798	9406	0887		45
	30	9614	6792	9550	7924	9486	9056	9421	1.0188	9357	1320		30
	45	9584	7052	9515	8228	9445	9403	9376	0578	9307	1754		15
7	0	9553	7312	9478	8531	9404	9750	9329	0968	9255	2187	83	0
	15	9520	7572	9440	8834	9360	1.0096	9280	1358	9200	2620		45
	30	9487	7832	9401	9137	9316	0442	9230	1747	9144	3053		30
	45	5.9452	0.8091	6.9361	0.9440	7.9269	1.0788	8.9178	1.2137	9.9087	1.3485		15
8	0	9416	8350	9319	9742	9221	1134	9124	2526	9027	3917	82	0
	15	9379	8610	9276	1.0044	9172	1479	9069	2914	8965	4349		45
	30	9341	8869	9231	0347	9121	1825	9011	3303	8902	4781		30
	45	9302	9127	9185	0649	9069	2170	8953	3691	8836	5212		15
9	0	9261	9386	9138	0950	9015	2515	8892	4079	8769	5643	81	0
	15	9220	9645	9090	1252	8960	2859	8830	4467	8700	6074		45
	30	9177	9903	9040	1553	8903	3204	8766	4854	8629	6505		30
	45	9133	1.0161	8989	1854	8844	3548	8700	5241	8556	6935		15
10	0	9088	0419	8937	2155	8785	3892	8633	5628	8481	7365	80	0
	15	5.9042	1.0677	6.8883	1.2456	7.8723	1.4235	8.8564	1.6015	9.8404	1.7794		45
	30	8995	0943	8828	2756	8660	4579	8493	6401	8325	8224		30
	45	8947	1191	8772	3057	8596	4922	8421	6787	8245	8652		15
11	0	8898	1449	8714	3357	8530	5265	8346	7173	8163	9081	79	0
	15	8847	1705	8655	3656	8463	5607	8271	7558	8079	9509		45
	30	8795	1962	8595	3956	8394	5949	8193	7943	7992	9937		30
	45	8743	2219	8533	4255	8324	6291	8114	8328	7905	2.0364		15
12	0	8689	2475	8470	4554	8252	6633	8033	8712	7815	0791	78	0
	15	8634	2731	8406	4852	8178	6974	7951	9096	7723	1218		45
	30	8578	2986	8341	5151	8104	7315	7867	9480	7630	1644		30
	45	5.8521	1.3242	6.8274	1.5449	7.8027	1.7656	8.7781	1.9863	9.7534	2.2070		15
13	0	8462	3497	8206	5747	7950	7996	7693	2.0246	7437	2495	77	0
	15	8403	3752	8137	6044	7870	8336	7604	0628	7338	2920		45
	30	8342	4007	8066	6341	7790	8676	7513	1010	7237	3345		30
	45	8281	4261	7994	6638	7707	9015	7421	1392	7134	3769		15
14	0	8218	4515	7921	6935	7624	9354	7327	1773	7030	4192	76	0
	15	8154	4769	7846	7231	7538	9692	7231	2154	6923	4615		45
	30	8089	5023	7770	7527	7452	2.0030	7133	2534	6815	5038		30
	45	8023	5276	7693	7822	7364	0368	7034	2914	6705	5460		15
15	0	7956	5529	7615	8117	7274	0706	6933	3294	6593	5882	75	0
		Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.	
		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.			

Traverse Table (*Continued*)

Course.	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
° ′											° ′
15 15	0.9648	0.2630	1.9296	0.5261	2.8944	0.7891	3.8591	1.0521	4.8239	1.3152	74 45
30	9636	2672	9273	5345	8909	8017	8545	0690	8182	3362	30
45	9625	2714	9249	5429	8874	8143	8498	0858	8123	3572	15
16 0	9613	2756	9225	5513	8838	8269	8450	1025	8063	3782	74 0
15	9600	2798	9201	5597	8801	8395	8402	1193	8002	3991	45
30	9588	2840	9176	5680	8765	8520	8353	1361	7941	4201	30
45	9576	2882	9151	5764	8727	8646	8303	1528	7879	4410	15
17 0	9563	2924	9126	5847	8689	8771	8252	1695	7815	4619	73 0
15	9550	2965	9100	5931	8651	8896	8201	1862	7751	4827	45
30	9537	3007	9074	6014	8612	9021	8149	2028	7686	5035	30
45	0.9524	0.3049	1.9048	0.6097	2.8572	0.9146	3.8096	1.2195	4.7620	1.5243	15
18 0	9511	3090	9021	6180	8532	9271	8042	2361	7553	5451	72 0
15	9497	3132	8994	6263	8491	9395	7988	2527	7485	5658	45
30	9483	3173	8966	6346	8450	9519	7933	2692	7416	5865	30
45	9469	3214	8939	6429	8408	9643	7877	2858	7347	6072	15
19 0	9455	3256	8910	6511	8366	9767	7821	3023	7276	6278	71 0
15	9441	3297	8882	6594	8323	9891	7764	3188	7204	6485	45
30	9426	3338	8853	6676	8279	1.0014	7706	3352	7132	6690	30
45	9412	3379	8824	6758	8235	0138	7647	3517	7059	6896	15
20 0	9397	3420	8794	6840	8191	0261	7588	3681	6985	7101	70 0
15	0.9382	0.3461	1.8764	0.6922	2.8146	1.0384	3.7528	1.3845	4.6910	1.7306	45
30	9367	3502	8733	7004	8100	0506	7467	4008	6834	7510	30
45	9351	3543	8703	7086	8054	0629	7405	4172	6757	7715	15
21 0	9336	3584	8672	7167	8007	0751	7343	4335	6679	7918	69 0
15	9320	3624	8640	7249	7960	0873	7280	4498	6600	8122	45
30	9304	3665	8608	7330	7913	0995	7217	4660	6521	8325	30
45	9288	3706	8576	7411	7864	1117	7152	4822	6440	8528	15
22 0	9272	3746	8544	7492	7816	1238	7087	4984	6359	8730	68 0
15	9255	3786	8511	7573	7766	1359	7022	5146	6277	8932	45
30	9239	3827	8478	7654	7716	1481	6955	5307	6194	9134	30
45	0.9222	0.3867	1.8444	0.7734	2.7666	1.1601	3.6888	1.5468	4.6110	1.9336	15
23 0	9205	3907	8410	7815	7615	1722	6820	5629	6025	9537	67 0
15	9188	3947	8376	7895	7564	1842	6752	5790	5940	9737	45
30	9171	3987	8341	7975	7512	1962	6682	5950	5853	9937	30
45	9153	4027	8306	8055	7459	2082	6612	6110	5766	2.0137	15
24 0	9135	4067	8271	8135	7406	2202	6542	6269	5677	0337	66 0
15	9118	4107	8235	8214	7353	2322	6470	6429	5588	0536	45
30	9100	4147	8199	8294	7299	2441	6398	6588	5498	0735	30
45	9081	4187	8163	8373	7244	2560	6326	6746	5407	0933	15
25 0	9063	4226	8126	8452	7189	2679	6252	6905	5315	1131	65 0
15	0.9045	0.4266	1.8089	0.8531	2.7134	1.2797	3.6178	1.7063	4.5223	2.1328	45
30	9026	4305	8052	8610	7078	2915	6103	7220	5129	1526	30
45	9007	4344	8014	8689	7021	3033	6028	7378	5035	1722	15
26 0	8988	4384	7976	8767	6964	3151	5952	7535	4940	1919	64 0
15	8969	4423	7937	8846	6906	3269	5875	7692	4844	2114	45
30	8949	4462	7899	8924	6848	3386	5797	7848	4747	2310	30
45	8930	4501	7860	9002	6789	3503	5719	8004	4649	2505	15
27 0	8910	4540	7820	9080	6730	3620	5640	8160	4550	2700	63 0
15	8890	4579	7780	9157	6671	3736	5561	8315	4451	2894	45
30	8870	4617	7740	9235	6610	3852	5480	8470	4351	3087	30
45	0.8850	0.4656	1.7700	0.9312	2.6550	1.3968	3.5400	1.8625	4.4249	2.3281	15
28 0	8829	4695	7659	9389	6488	4084	5318	8779	4147	3474	62 0
15	8809	4733	7618	9466	6427	4200	5236	8933	4045	3666	45
30	8788	4772	7576	9543	6365	4315	5153	9086	3941	3858	30
45	8767	4810	7535	9620	6302	4430	5069	9240	3836	4049	15
29 0	8746	4848	7492	9696	6239	4544	4985	9392	3731	4240	61 0
15	8725	4886	7450	9772	6175	4659	4900	9545	3625	4431	45
30	8704	4924	7407	9848	6111	4773	4814	9697	3518	4621	30
45	8682	4962	7364	9924	6046	4886	4728	9849	3410	4811	15
30 0	8660	5000	7321	1.0000	5981	5000	4641	2.0000	3301	5000	60 0
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.
	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		

Traverse Table (*Continued*)

Course.	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
° ′											° ′
15 15	5.7887	1.5782	6.7535	1.8412	7.7183	2.1042	8.6831	2.3673	9.6479	2.6303	74 45
30	7818	6034	7454	8707	7090	1379	6727	4051	6363	6724	30
45	7747	6286	7372	9001	6996	1715	6621	4430	6246	7144	15
16 0	7676	6538	7288	9295	6901	2051	6514	4807	6126	7564	74 0
15	7603	6790	7203	9588	6804	2386	6404	5185	6005	7983	45
30	7529	7041	7117	9881	6706	2721	6294	5561	5882	8402	30
45	7454	7292	7030	2.0174	6606	3056	6181	5938	5757	8820	15
17 0	7378	7542	6941	0466	6504	3390	6067	6313	5630	9237	73 0
15	7301	7792	6851	0758	6402	3723	5952	6689	5502	9654	45
30	7223	8042	6760	1049	6297	4056	5835	7064	5372	3.0071	30
45	5.7144	1.8292	6.6668	2.1341	7.6192	2.4389	8.5716	2.7438	9.5240	3.0486	15
18 0	7063	8541	6574	1631	6085	4721	5595	7812	5106	0902	72 0
15	6982	8790	6479	1921	5976	5053	5473	8185	4970	1316	45
30	6899	9038	6383	2211	5866	5384	5349	8557	4832	1730	30
45	6816	9286	6285	2501	5754	5715	5224	8930	4693	2144	15
19 0	6731	9534	6186	2790	5641	6045	5097	9301	4552	2557	71 0
15	6645	9781	6086	3078	5527	6375	4968	9672	4409	2969	45
30	6558	2.0028	5985	3366	5411	6705	4838	3.0043	4264	3381	30
45	6471	0275	5882	3654	5294	7033	4706	0413	4118	3792	15
20 0	6382	0521	5778	3941	5175	7362	4572	0782	3969	4202	70 0
15	5.6291	2.0767	6.5673	2.4228	7.5055	2.7689	8.4437	3.1151	9.3819	3.4612	45
30	6200	1012	5567	4515	4934	8017	4300	1519	3667	5021	30
45	6108	1257	5459	4800	4811	8343	4162	1886	3514	5429	15
21 0	6015	1502	5351	5086	4686	8669	4022	2253	3358	5837	69 0
15	5920	1746	5241	5371	4561	8995	3881	2619	3201	6244	45
30	5825	1990	5129	5655	4433	9320	3738	2985	3042	6650	30
45	5729	2233	5017	5939	4305	9645	3593	3350	2881	7056	15
22 0	5631	2476	4903	6222	4175	9969	3447	3715	2718	7461	68 0
15	5532	2719	4788	6505	4043	3.0292	3299	4078	2554	7865	45
30	5433	2961	4672	6788	3910	0615	3149	4442	2388	8268	30
45	5.5332	2.3203	6.4554	2.7070	7.3776	3.0937	8.2998	3.4804	9.2220	3.8671	15
23 0	5230	3444	4435	7351	3640	1258	2845	5166	2050	9073	67 0
15	5127	3685	4315	7632	3503	1580	2691	5527	1879	9474	45
30	5024	3925	4194	7912	3365	1900	2535	5887	1706	9875	30
45	4919	4165	4072	8192	3225	2220	2378	6247	1531	4.0275	15
24 0	4813	4404	3948	8472	3084	2539	2219	6606	1355	0674	66 0
15	4706	4643	3823	8750	2941	2858	2059	6965	1176	1072	45
30	4598	4882	3697	9029	2797	3175	1897	7322	0996	1469	30
45	4489	5120	3570	9306	2651	3493	1733	7679	0814	1866	15
25 0	4378	5357	3442	9583	2505	3809	1568	8036	0631	2202	65 0
15	5.4267	2.5594	6.3312	2.9860	7.2356	3.4125	8.1401	3.8391	9.0446	4.2657	45
30	4155	5831	3181	3.0136	2207	4441	1233	8746	0259	3051	30
45	4042	6067	3049	0411	2056	4756	1063	9100	0070	3445	15
26 0	3928	6302	2916	0686	1904	5070	0891	9453	8.9879	3837	64 0
15	3812	6537	2781	0960	1750	5383	0719	9806	9687	4229	45
30	3696	6772	2645	1234	1595	5696	0544	4.0158	9493	4620	30
45	3579	7006	2509	1507	1438	6008	0368	0509	9298	5010	15
27 0	3460	7239	2370	1779	1281	6319	0191	0859	9101	5399	63 0
15	3341	7472	2231	2051	1121	6630	0012	1209	8902	5787	45
30	3221	7705	2091	2322	0961	6940	7.9831	1557	8701	6175	30
45	5.3099	2.7937	6.1949	3.2593	7.0799	3.7249	7.9649	4.1905	8.8499	4.6561	15
28 0	2977	8168	1806	2863	0636	7558	9465	2252	8295	6947	62 0
15	2853	8399	1662	3132	0471	7866	9280	2599	8089	7332	45
30	2729	8630	1517	3401	0305	8173	9094	2944	7882	7716	30
45	2604	8859	1371	3669	0138	8479	8905	3289	7673	8099	15
29 0	2477	9089	1223	3937	6.9970	8785	8716	3633	7462	8481	61 0
15	2350	9317	1075	4203	9800	9090	8525	3976	7250	8862	45
30	2221	9545	0925	4470	9628	9394	8332	4318	7036	9242	30
45	2092	9773	0774	4735	9456	9697	8138	4659	6820	9622	15
30 0	1962	3.0000	0622	5000	9282	4.0000	7942	5000	6603	5.0000	60 0
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.
	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		

TRAVERSE TABLE (*Continued*)

Course.	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
° ′											° ′
30 15	0.8638	0.5038	1.7277	1.0075	2.5915	1.5113	3.4553	2.0151	4.3192	2.5189	59 45
30	8616	5075	7233	0151	5849	5226	4465	0302	3081	5377	30
45	8594	5113	7188	0226	5782	5339	4376	0452	2970	5565	15
31 0	8572	5150	7142	0301	5715	5451	4287	0602	2858	5752	59 0
15	8549	5188	7098	0375	5647	5563	4196	0751	2746	5939	45
30	8526	5225	7053	0450	5579	5675	4106	0900	2632	6125	30
45	8504	5262	7007	0524	5511	5786	4014	1049	2518	6311	15
32 0	8480	5299	6961	0598	5441	5898	3922	1197	2402	6496	58 0
15	8457	5336	6915	0672	5372	6008	3829	1345	2286	6681	45
30	8434	5373	6868	0746	5302	6119	3736	1492	2170	6865	30
45	0.8410	0.5410	1.6821	1.0819	2.5231	1.6229	3.3642	2.1639	4.2052	2.7049	15
33 0	8387	5446	6773	0893	5160	6339	3547	1786	1934	7232	57 0
15	8363	5483	6726	0966	5089	6449	3451	1932	1814	7415	45
30	8339	5519	6678	1039	5017	6558	3355	2077	1694	7597	30
45	8315	5556	6629	1111	4944	6667	3259	2223	1573	7779	15
34 0	8290	5592	6581	1184	4871	6776	3162	2368	1452	7960	56 0
15	8266	5628	6532	1256	4798	6884	3064	2512	1329	8140	45
30	8241	5664	6483	1328	4724	6992	2965	2656	1206	8320	30
45	8216	5700	6433	1400	4649	7100	2866	2800	1082	8500	15
35 0	8192	5736	6383	1472	4575	7207	2766	2943	0958	8679	55 0
15	0.8166	0.5771	1.6333	1.1543	2.4499	1.7314	3.2666	2.3086	4.0832	2.8857	45
30	8141	5807	6282	1614	4423	7421	2565	3228	0706	9035	30
45	8116	5842	6231	1685	4347	7527	2463	3370	0579	9212	15
36 0	8090	5878	6180	1756	4271	7634	2361	3511	0451	9389	54 0
15	8064	5913	6129	1826	4193	7739	2258	3652	0322	9565	45
30	8039	5948	6077	1896	4116	7845	2154	3793	0193	9741	30
45	8013	5983	6025	1966	4038	7950	2050	3933	0063	9916	15
37 0	7986	6018	5973	2036	3959	8054	1945	4073	3.9932	3.0091	53 0
15	7960	6053	5920	2106	3880	8159	1840	4212	9800	0265	45
30	7934	6088	5867	2175	3801	8263	1734	4350	9668	0438	30
45	0.7907	0.6122	1.5814	1.2244	2.3721	1.8367	3.1628	2.4489	3.9534	3.0611	15
38 0	7880	6157	5760	2313	3640	8470	1520	4626	9400	0783	52 0
15	7853	6191	5706	2382	3560	8573	1413	4764	9266	0955	45
30	7826	6225	5652	2450	3478	8675	1304	4901	9130	1126	30
45	7799	6259	5598	2518	3397	8778	1195	5037	8994	1296	15
39 0	7771	6293	5543	2586	3314	8880	1086	5173	8857	1466	51 0
15	7744	6327	5488	2654	3232	8981	0976	5308	8720	1635	45
30	7716	6361	5432	2722	3149	9082	0865	5443	8581	1804	30
45	7688	6394	5377	2789	3065	9183	0754	5578	8442	1972	15
40 0	7660	6428	5321	2856	2981	9284	0642	5712	8302	2139	50 0
15	0.7632	0.6461	1.5265	1.2922	2.2897	1.9384	3.0529	2.5845	3.8162	3.2306	45
30	7604	6494	5208	2989	2812	9483	0416	5978	8020	2472	30
45	7576	6528	5151	3055	2727	9583	0303	6110	7878	2638	15
41 0	7547	6561	5094	3121	2641	9682	0188	6242	7735	2803	49 0
15	7518	6593	5037	3187	2555	9780	0074	6374	7592	2967	45
30	7490	6626	4979	3252	2469	9879	2.9958	6505	7448	3131	30
45	7461	6659	4921	3318	2382	9976	9842	6635	7303	3294	15
42 0	7431	6691	4863	3383	2294	2.0074	9726	6765	7157	3457	48 0
15	7402	6724	4804	3447	2207	0171	9609	6895	7011	3618	45
30	7373	6756	4746	3512	2118	0268	9491	7024	6864	3780	30
45	0.7343	0.6788	1.4686	1.3576	2.2030	2.0364	2.9373	2.7152	3.6716	3.3940	15
43 0	7314	6820	4627	3640	1941	0460	9254	7280	6568	4100	47 0
15	7284	6852	4567	3704	1851	0555	9135	7407	6419	4259	45
30	7254	6884	4507	3767	1761	0651	9015	7534	6269	4418	30
45	7224	6915	4447	3830	1671	0745	8895	7661	6118	4576	15
44 0	7193	6947	4387	3893	1580	0840	8774	7786	5967	4733	46 0
15	7163	6978	4326	3956	1489	0934	8652	7912	5815	4890	45
30	7133	7009	4265	4018	1398	1027	8530	8036	5663	5045	30
45	7102	7040	4204	4080	1306	1120	8407	8161	5509	5201	15
45 0	7071	7071	4142	4142	1213	1213	8284	8284	5355	5355	45 0
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.
	Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		

Traverse Table (*Continued*)

Course.	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	
° ′											° ′
30 15	5.1830	3.0226	6.0468	3.5264	6.9107	4.0302	7.7745	4.5340	8.6384	5.0377	59 45
30	1698	0452	0314	5528	8930	0603	7547	5678	6163	0754	30
45	1564	0678	0158	5791	8753	0903	7347	6016	5941	1129	15
31 0	1430	0902	0002	6053	8573	1203	7145	6353	5717	1504	59 0
15	1295	1126	5.9844	6314	8393	1502	6942	6690	5491	1877	45
30	1158	1350	9685	6575	8211	1800	6738	7025	5264	2250	30
45	1021	1573	9525	6835	8028	2097	6532	7359	5035	2621	15
32 0	0883	1795	9363	7094	7844	2394	6324	7693	4805	2992	58 0
15	0744	2017	9201	7353	7658	2689	6116	8025	4573	3361	45
30	0603	2238	9037	7611	7471	2984	5905	8357	4339	3730	30
45	5.0462	3.2458	5.8873	3.7868	6.7283	4.3278	7.5694	4.8688	8.4104	5.4097	15
33 0	0320	2678	8707	8125	7094	3571	5480	9018	3867	4464	57 0
15	0177	2898	8540	8381	6903	3863	5266	9346	3629	4829	45
30	0033	3116	8372	8636	6711	4155	5050	9674	3389	5194	30
45	4.9888	3334	8203	8890	6518	4446	4832	5.0001	3147	5557	15
34 0	9742	3552	8033	9144	6323	4735	4613	0327	2904	5919	56 0
15	9595	3768	7861	9396	6127	5024	4393	0652	2659	6280	45
30	9448	3984	7689	9648	5930	5312	4171	0977	2413	6641	30
45	9299	4200	7515	9900	5732	5600	3948	1300	2165	7000	15
35 0	9149	4415	7341	4.0150	5532	5886	3724	1622	1915	7358	55 0
15	4.8998	3.4629	5.7165	4.0400	6.5331	4.6172	7.3498	5.1943	8.1664	5.7715	45
30	8847	4842	6988	0649	5129	6456	3270	2263	1412	8070	30
45	8694	5055	6810	0897	4926	6740	3042	2582	1157	8425	15
36 0	8541	5267	6631	1145	4721	7023	2812	2901	0902	8779	54 0
15	8387	5479	6451	1392	4516	7305	2580	3218	0644	9131	45
30	8231	5689	6270	1638	4309	7586	2347	3534	0386	9482	30
45	8075	5899	6088	1883	4100	7866	2113	3849	0125	9832	15
37 0	7918	6109	5904	2127	3891	8145	1877	4163	7.9864	6.0182	53 0
15	7760	6318	5720	2371	3680	8424	1640	4476	9600	0529	45
30	7601	6526	5535	2613	3468	8701	1402	4789	9335	0876	30
45	4.7441	3.6733	5.5348	4.2855	6.3255	4.8977	7.1162	5.5100	7.9069	6.1222	15
38 0	7281	6940	5161	3096	3041	9253	0921	5410	8801	1566	52 0
15	7119	7146	4972	3337	2825	9528	0679	5718	8532	1909	45
30	6956	7351	4783	3576	2609	9801	0435	6026	8261	2251	30
45	6793	7555	4592	3815	2391	5.0074	0190	6333	7988	2592	15
39 0	6629	7759	4400	4052	2172	0346	6.9943	6639	7715	2932	51 0
15	6464	7962	4207	4289	1951	0616	9695	6943	7439	3271	45
30	6297	8165	4014	4525	1730	0886	9446	7247	7162	3608	30
45	6131	8366	3819	4761	1507	1155	9196	7550	6884	3944	15
40 0	5963	8567	3623	4995	1284	1423	8944	7851	6604	4279	50 0
15	4.5794	3.8767	5.3426	4.5229	6.1059	5.1690	6.8691	5.8151	7.6323	6.4612	45
30	5624	8967	3228	5461	0832	1956	8437	8450	6041	4945	30
45	5454	9166	3030	5693	0605	2221	8181	8748	5756	5276	15
41 0	5283	9364	2830	5924	0377	2485	7924	9045	5471	5606	49 0
15	5110	9561	2629	6154	0147	2748	7666	9341	5184	5935	45
30	4937	9757	2427	6383	5.9916	3010	7406	9636	4896	6262	30
45	4763	9953	2224	6612	9685	3271	7145	9929	4606	6588	15
42 0	4589	4.0148	2020	6839	9452	3530	6883	6.0222	4314	6913	48 0
15	4413	0342	1815	7066	9217	3789	6620	0513	4022	7237	45
30	4237	0535	1609	7291	8982	4047	6355	0803	3728	7559	30
45	4.4059	4.0728	5.1403	4.7516	5.8746	5.4304	6.6089	6.1092	7.3432	6.7880	15
43 0	3881	0920	1195	7740	8508	4560	5822	1380	3135	8200	47 0
15	3702	1111	0986	7963	8270	4815	5553	1666	2837	8518	45
30	3522	1301	0776	8185	8030	5068	5284	1952	2537	8835	30
45	3342	1491	0565	8406	7789	5321	5013	2236	2236	9151	15
44 0	3160	1680	0354	8626	7547	5573	4741	2519	1934	9466	46 0
15	2978	1867	0141	8845	7304	5823	4467	2801	1630	9779	45
30	2795	2055	4.9928	9064	7060	6073	4193	3082	1325	7.0091	30
45	2611	2241	9713	9281	6815	6321	3917	3361	1019	0401	15
45 0	2426	2426	9497	9497	6569	6569	3640	3640	0711	0711	45 0
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Course.
	Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		

CHAPTER V

TRIGONOMETRY

When the surveyor reads the bearing of a compass needle he reads an angle and during his working life he is dealing with angular measurements. It is therefore necessary that he be skilled in that special branch of mathematics termed trigonometry.

Trigonometry was formerly said to deal with the properties of triangles and was divided into Plane, Analytical and Spherical Trigonometry. It is now defined as a branch of mathematics dealing with the functions of angles and their application in the solution of triangles. Plane and Analytical Trigonometry have been merged, so we now have only Plane and Spherical Trigonometry, the latter merely a particular application to cases where the sides of triangles are arcs of circles. It is proposed in this chapter to give the *minimum* amount of trigonometry every surveyor requires.

Every triangle consists of six parts, three sides and three angles. When three parts are known, one of which must be a side, the other three can be found. To the foregoing statement the exception must be made that when two 90° angles are given with a side opposite one of them the solution is indeterminate. However the surveyor will not meet with this particular case.

In arithmetic the three sides of a right-angled triangle are known as the base, the altitude and the hypothenuse.

Carpenters and practical men usually call the three sides the run, the rise and the slope.

For convenience it is usual in mathematical work to letter the sides and angles. The acute angle at the base is known as "*Angle A*," the side opposite being marked with a small letter and called "*side a*." The word "altitude" begins with "*a*," so the upright line designating the

altitude may be easily remembered as "line *a*." Similarly the base is "line *b*" and the acute angle at the top is "*Angle B*." The right angle at the base is "*Angle C*," the hypothenuse opposite the angle being "line *c*."

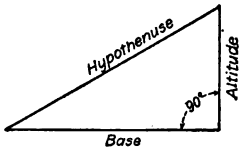


FIG. 150.

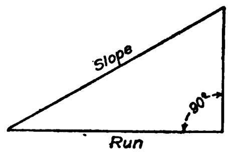


FIG. 151.

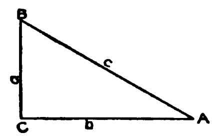


FIG. 152.

The right-angled triangle was used for illustration but the same conventions are used for oblique-angled triangles; i.e., capital letters indicate angles and small letters indicate sides.

THE PYTHAGOREAN THEOREM

Pythagoras, a noted mathematician of ancient times, is said to have been the first to demonstrate that

In a right-angled triangle the square on the hypothenuse is equal to the sum of the squares on the other two sides, so the statement is known as the Pythagorean Theorem and is the basis for a number of trigonometrical expressions.

Stated in a modern way (algebraically) it appears as follows:

$$c^2 = a^2 + b^2,$$

or

$$c = \sqrt{a^2 + b^2},$$

then

$$a = \sqrt{(c + b)(c - b)} = \sqrt{c^2 - b^2},$$

and

$$b = \sqrt{(c + a)(c - a)} = \sqrt{c^2 - a^2}.$$

GRAPHICAL SOLUTION OF A RIGHT TRIANGLE

Fig. 153 is reproduced, with some of the finer graduations omitted, from a diagram copyrighted in 1912 by Constantine K. Smoley.

The numbers may be taken to represent inches, feet, yards or meters. Given two sides of any triangle the third side may be found.

Given the base and altitude to find the hypothenuse. — Find the base on *A–B* and the altitude on *B–C*. Follow the

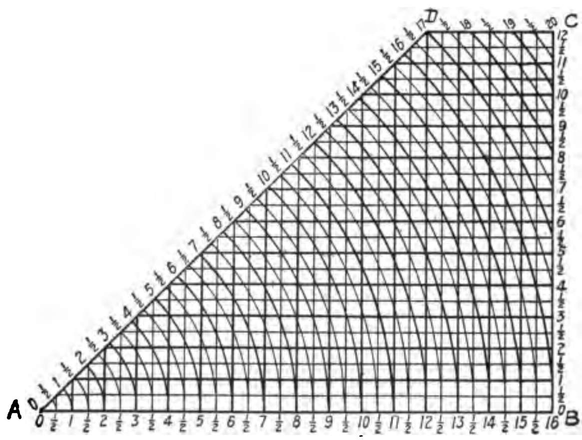


FIG. 153. Graphical solution of a right triangle.

lines to an intersection and then trace the arc found at this point to line *A–D*, on which read the length of the hypothenuse.

Given the hypothenuse and another side to find the third side. — Place a ruler on the line marking the second side. If this side is the altitude the ruler will be parallel with *A–B*, starting from *B–C*. If the second side is the base the ruler will be parallel with *B–C*, starting from *A–B*. Find the hypothenuse on *A–D* and follow the arc intersecting *A–D* at this point to an intersection with the ruler. Going from this intersection on a line perpendicular to the ruler read the required third side.

The diagram presented appears in the seventh edition of Smoley's Tables, the standard tables for engineering draftsmen, and is in inches. The author used similar diagrams for many years drawn on cross-section paper

divided decimally instead of in eighths. Such diagrams are in fairly common use and are of considerable value in certain kinds of work. For instructional purposes they are very good.

TRIGONOMETRIC FUNCTIONS

In trigonometry all angles are assumed to have angle A at the center of a circle, side b and side c being radial lines intercepting an arc on the circumference. Either b or c may be a radius.

If an arc is described and a perpendicular let fall from the end of the radius to the base, then

a = *sine*, written *sin*,
b = *cosine*, written *cos*,
c = *radius*, written *rad*,
$c - b$ = *versed sine*, written *versin*.

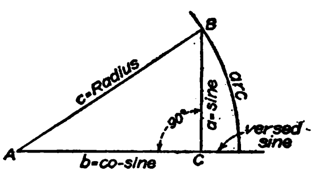


FIG. 154.

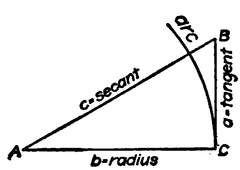


FIG. 155.

If an arc is described and a perpendicular erected from the end of the radius as the base, then

a = *tangent*, written *tan or tangt*,
b = *radius*, written *rad*,
c = *secant*, written *sec*.

The sine, tangent, etc., are called functions of an angle, a function in mathematics being any algebraic expression or quantity dependent for its value on another one.

The difference between any angle in a quadrant and 90 degrees, the full quadrantal angle, is said to be the complement of the angle. Thus in Fig. 156, angle B is the complement of angle A and *vice versa*.

The functions of the complement are said to be co-functions of the angle.

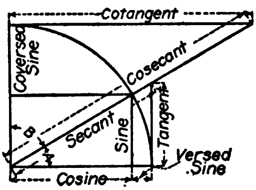


FIG. 156. Functions of angles.

The sine of A = cosine of B.
The sine of B = cosine of A.
The tangent of A = cotangent of B.
The cotangent of B = tangent of A, etc., as shown in Fig. 156.

The difference between any angle and 180 degrees is called the supplement of the angle. The functions of the supplement are the functions of the angle.

Side *a* may be the sine or tangent of angle *A* and side *b* may be the cosine, or it may be the radius of the circle in which a triangle is drawn having angle *A* at the center.

To avoid confusion it is considered best to regard the functions as ratios, that is, as pure numbers and not as lines. Side *a* and side *b* are lines but sin *A*, tan *A* and cos *A* are not always lines. Tangent *A* does not become a line until it is multiplied by the run of the triangle having angle *A* at the base, when it becomes side *a*. If sin *A* is multiplied by the slope it becomes side *a*.

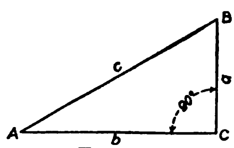


FIG. 157.

Using the "ratio concept" the functions are described as follows, referring to Fig. 157.

$$Sin\ A = \frac{a}{c}.$$

$$Cos\ A = \frac{b}{c}.$$

$$Tan\ A = \frac{a}{b}.$$

$$Cot\ A = \frac{b}{a}.$$

$$Sec\ A = \frac{c}{b}.$$

$$Cosec\ A = \frac{c}{a}.$$

The relations (ratios) here shown are true, no matter what may be the lengths of the lines. The ratios must be memorized or the student will find himself as helpless in trigonometrical work as he would be in arithmetic without knowing the multiplication tables.

The following trigonometrical equivalents must also be memorized, an easy task when the connection with the Pythagorean theorem is noted.

$$Sin = \frac{1}{cosec} = \frac{cos}{cot} = \sqrt{(1 - cos^2)} = \sqrt{(1 + cos)(1 - cos)}.$$

$$Cos = \frac{sin}{tan} = \frac{1}{sec} = sin \times cot = \sqrt{(1 - sin^2)}$$
$$= \sqrt{(1 + sin)(1 - sin)}.$$

$$Tan = \frac{sin}{cos} = \frac{1}{cot}.$$

$$Cot = \frac{cos}{sin} = \frac{1}{tan}.$$

$$Sec = \frac{tan}{sin} = \frac{1}{cos} = \sqrt{rad^2 + tan^2}.$$

$$Cosec = \frac{1}{sin}.$$

$$Rad = tan \times cot = \sqrt{sin^2 + cos^2}.$$

$$Versin = rad - cos = \frac{c - b}{c}.$$

$$Coversin = rad - sin = \frac{c - a}{c}.$$

NUMERICAL VALUES FOR THE TRIGONOMETRICAL RATIOS

Angle of 45°.

Draw a square A, B, C, D and in it draw the diagonal A, B. Angle $x = 45° = \frac{90°}{2}$ because side a = side b.

$$c^2 = a^2 + b^2,$$

and since $a = b = 1,$

$$c^2 = 2\,b^2 = 2\,a^2; \text{ or } c = \sqrt{2} \times b = \sqrt{2} \times a.$$

$$\therefore \quad \text{Sin } 45^\circ = \frac{a}{c} = \frac{1}{\sqrt{2}},$$

$$\text{Cos } 45^\circ = \frac{b}{c} = \frac{1}{\sqrt{2}},$$

$$\text{Tan } 45^\circ = \frac{a}{b} = 1.$$

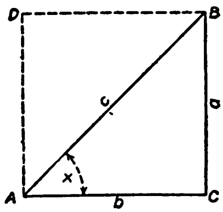


FIG. 158. Functions of 45°.

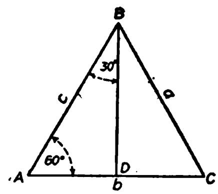


FIG. 159. Functions of 30° and 60°.

Angle of 60°.

In the equilateral triangle (Fig. 159) each of the three angles is equal to 60°. Drop the perpendicular BD to b. Then, assuming the length of each side = 1,

$$2\,AD = AB = AC = BC = a = b = c = 1,$$
$$C^2 = AD^2 + BD^2 = 1^2 + 1^2 = 2,$$
$$4\,AD^2 = BD^2 + AD^2 \quad \text{or} \quad BD^2 = 3\,AD^2;$$

that is,

$$4 \times \tfrac{1}{2}^2 = BD^2 + \tfrac{1}{2}^2 \quad \text{or} \quad BD^2 = 3 \times \tfrac{1}{2}^2.$$
$$\therefore \quad BD = \sqrt{3} \times \tfrac{1}{2} = \tfrac{1}{2}\sqrt{3}.$$

Since angle $A = 60^\circ$

$$\text{Sin } 60^\circ = \frac{BD}{c} = \frac{\sqrt{3} \times \frac{1}{2}}{1} = \frac{\sqrt{3}}{2},$$

$$\text{Cos } 60^\circ = \frac{AD}{C} = \frac{\frac{1}{2}}{1} = \frac{1}{2},$$

$$\text{Tan } 60^\circ = \frac{BD}{AD} = \frac{\sqrt{3} \times \frac{1}{2}}{\frac{1}{2}} = \sqrt{3}.$$

Angle of 30°.

$$\text{Sin } 30° = \cos 60° = \tfrac{1}{2}.$$

$$\text{Cos } 30° = \sin 60° = \frac{\sqrt{3}}{2}.$$

$$\text{Tan } 30° = \cot 60° = \frac{1}{\tan 60°} = \frac{1}{\sqrt{3}}.$$

$$\text{Sin } 15° = \frac{(\sqrt{3} - 1)}{2\sqrt{2}},$$

$$\text{Sin } 18° = \frac{(\sqrt{5} - 1)}{4}.$$

To obtain the remaining functions of the foregoing angles refer to the list of Trigonometrical Equivalents.

SIGNS OF THE TRIGONOMETRICAL RATIOS

It is convenient (hence the word convention) to assume that all directions up or down are measured from a horizontal line (or axis) $X \ldots X'$ and that all directions right or left are measured from a vertical line (or axis) $Y \ldots Y'$. The vertical axis is termed an ordinate and the horizontal axis an abscissa. The axes are known as co-ordinates and the point of intersection, O, the origin of co-ordinates.

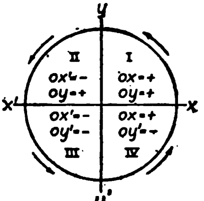


FIG. 160. Signs of functions of angles.

Describing a circle with O as a center and $O \ldots X$ as a radius the axes divide it into four quarters, or quadrants. Proceeding around the circle in a direction contrary to that followed by the hands of a clock — anti-clockwise — the quadrants are numbered as shown in Fig. 160.

All motion toward X (to the right) and towards Y (up) is considered positive (+). Motion to the left and down is considered negative (−).

The sine and cosine cannot extend beyond the circumference of a circle having a radius = 1 but the tangent, cotangent, secant and cosecant may extend to infinity (∞).

Representing the functions by lines, the signs of the functions, together with their limiting values, are shown in the following table, x being the angle at the origin.

If the angle is in quadrant.	Sin x.	Cos x.	Tan x.	Cot x.	Sec x.	Cosec x.
I. Sign.....	+	+	+	+	+	+
I. Value...	0 to 1	1 to 0	0 to ∞	∞ to 0	1 to ∞	∞ to 1
II. Sign.....	+	−	−	−	−	+
II. Value...	1 to 0	0 to 1	∞ to 0	0 to ∞	∞ to 1	1 to ∞
III. Sign.....	−	−	+	+	−	−
III. Value...	0 to 1	1 to 0	0 to ∞	∞ to 0	1 to ∞	∞ to 1
IV. Sign.....	−	+	−	−	+	−
IV. Value...	1 to 0	0 to 1	∞ to 0	0 to ∞	∞ to 1	1 to ∞

Summarizing the results of the investigation of the numerical values the following table has been prepared:

Functions. \ Angles,	0° or 360°	30°	45°	60°	90°	180°	270°
Sine...............	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	1
Cosine.............	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0
Tangent............	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞

$$\textit{Sin}\ 30^\circ = \tfrac{1}{2} = 0.5 = \textit{cos}\ 60^\circ.$$

$$\textit{Sin}\ 45^\circ = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = 0.7071 = \textit{cos}\ 45^\circ.$$

$$\textit{Sin}\ 60^\circ = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866 = \textit{cos}\ 30^\circ.$$

$$\textit{Tan}\ 30^\circ = \frac{1}{\sqrt{3}} = \frac{1}{1.732} = 0.5774 = \textit{cot}\ 60^\circ.$$

$$\textit{Tan}\ 60^\circ = \sqrt{3} = 1.732 = \textit{cot}\ 30^\circ.$$

TABLE OF NATURAL FUNCTIONS

The circumference of a circle is divided into 360 equal parts called degrees.

Each degree is divided into 60 equal parts called minutes.

Each minute is divided into 60 equal parts called seconds.

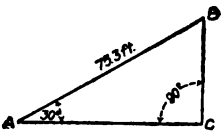


FIG. 161.

The size of an angle is indicated by the number of parts of a circle contained in the intercepted arc; thus, 25° 15′ 10″ which is read 25 degrees, 15 minutes, 10 seconds.

In Fig. 161 the angle at $A = 30°$, at $C = 90°$ and at $B = 60°$. The line $A–B$ is 75.3 ft. long. What are the lengths of the other sides?

Referring to the list of Trigonometrical Equivalents

$$Sec = \frac{tan}{sin},$$

therefore

$$sin \times sec = tan.$$

In the 30° triangle the *sin* = 0.5 and as the slope has a length of 75.3 ft., the tangent (line $B–C$) has a length of $0.5 \times 75.3 = 37.65$ ft.

Also the *secant* $= \dfrac{1}{cos}$,

therefore $\qquad cos \times sec = 1 = radius.$

The cos of $30° = 0.866$ and the length of $A–C = 75.3 \times 0.866 = 65.21$ ft.

Use of tangent.

(1) A triangle has a base of 65.21 ft. and an altitude of 37.65 ft. What is the angle A?

$$Tan = \frac{sin}{cos} = \frac{alt.}{base} = \frac{a}{b} = \frac{37.65}{65.21} = 0.5774.$$

The tangent for an angle of $30° = 0.5774$ so the angle $A = 30°$.

(2) A 30° right-angled triangle has a base of 65.21 ft. What is the altitude?

The tangent of 30° = 0.5774 so the altitude = 0.5774 × 65.21 = 37.65 ft.

By methods given in college textbooks, tables have been computed which contain values of all the functions. So many tables are in existence it is unnecessary for any one save professional mathematicians, as a part of their training, to compute such tables.

The table of Natural Functions of Angles here presented gives values of the ratios for each ten minutes of arc, this being sufficient for the examples in this book, which are intended to give practice in the solution of triangles and the use of tables. Tables in common use give values for each minute of arc, with numbers whereby interpolations may be made for smaller angles. Books of tables usually contain directions for use.

Example. — Find the sine of 33° 20′.

Turning to the table find 33 in the column headed (Deg.) and find 20 in the column headed (Min.). The sine is in the column headed Sine and = 0.564007. The radius has a value = 1.000000.

The values of other functions are found in the same way in the proper column.

Example. — Find the tangent of 54° 30′.

The tangent of 54° 30′ = 1.4019483.

Read the note at the bottom of each page of the table. The functions of angles are co-functions of the complements of the angles so it is therefore necessary to compute the values of functions for angles between 0° and 45° only. For angles between 45° and 90° read up from the bottom of the pages. Notice that the column with

> *Sine* at the top has *Cosine* at the bottom,
> *Tangent* at the top has *Cotangent* at the bottom,
> *Secant* at the top has *Cosecant* at the bottom

and *vice versa*.

Example. — Find the sine of 146° 40′.

Subtract from	179° 60′ (180°)
	146° 40′
and find sine of	33° 20′

Example. — Find the tangent of 125° 30′.

Subtract from	$179° 60'$ (180°)
	$\underline{125° 30'}$
and find tangent of	$54° 30'$

These two examples show that the functions of an angle are the functions of the supplement of the angle.

In adding and subtracting angles it is a good safe habit to write the work on paper instead of attempting to do it mentally until considerable experience is had. It is well to *see* such things.

In subtraction *lessen* the degrees by 1 and *add* 60 to the *minutes* of the larger angle. When seconds are given *lessen* the number of *minutes* by 1 and add 60 to the seconds.

The tables in this book give values to 10 minutes of arc but for all practical purposes the values for intermediate angles may be obtained by interpolation.

Example. — Find sine of 27° 13'.

$$\begin{aligned}
\text{Sine } 27° 20' &= 0.459166 \\
\text{Sine } 27° 10' &= \underline{0.456580} \\
&\ 0.002586 \text{ diff. for } 10'
\end{aligned}$$

3 minutes = 0.3 of 10 minutes, therefore

$$\begin{aligned}
&0.002586 \text{ diff. for } 10' \\
&\underline{\quad 0.3 \quad} \\
&0.0007758 \\
\text{Sine } 27° 10' \ &\underline{0.4565800} \\
&0.4573558 = \sin 27° 13'.
\end{aligned}$$

In using the tables in this book for arcs smaller than 10 minutes, the final results should be considered as being correct for the first five places (figures). If the sixth figure is 5 or more, increase the *fifth* figure by 1 and reject the figures following. Thus, the sine of 27° 13' correct to five places = 0.45736. If the sixth figure is less than 5 drop all figures following the *fifth*.

GRAPHICAL NATURAL FUNCTIONS

Paper protractors 8 and 14 ins. in diameter are printed from engine divided plates on rectangular sheets. On such a sheet draw a line through the center passing through the 270° and 90° graduations. Normal to this line draw

a line from the center through the 0° point. Make each line 5 or 10 ins. long and divide in ten equal parts. Draw lines through each division so the paper will be ruled in squares which are to be numbered from 0 to 10, starting at the center of the protractor. Each division should be divided into ten parts. A fine line drawn from the center through any angle will represent the hypothenuse of a triangle. This hypothenuse is the secant of the angle with a radius = 1.0. In making practical use of such a diagram a fine line is drawn on a narrow strip of tracing cloth or transparent celluloid and this line is graduated to correspond with the horizontal and vertical lines. A fine needle is put through the zero on this line and the center of the protractor. The transparent strip may then be swung so the line will intersect any angle and the lengths of the three sides of a triangle having this angle at the base may be read.

If the angle A alone is given the secant may be extended an infinite distance and an infinite number of triangles be formed in which

$$B = 90^\circ - A,$$
$$C = 90^\circ,$$

for the sum of the three interior angles of any triangle = $2 \times 90 = 180^\circ$.

An angle measures the amount of divergence between two lines starting from a common point. The base is one line and the secant another line defining an angle. When the secant is defined on the diagram as described and the angle A thus marked, the angle B may also be read off, for it is the complement of the angle at the base. To assist in obtaining the complementary angle a second set of graduations may be placed on the circumference of the protractor in an opposite direction to the regular marks. This second set should be in red ink to avoid mistakes.

The radius being 1.00 the functions of an angle are ratios expressed in per cent of the radius.

If the radius is ten inches, the heavy lines drawn at intervals of one inch and the lighter lines at intervals of one-tenth of an inch, values of the functions may be read to three decimal places.

Fig. 162 is an engine-divided diagram used in Lewis Institute, Chicago, Ill., and reproduced by permission of Prof. Phillip B. Woodworth. The only difference between this engine-divided protractor and the diagram just described is in the angular graduations, which are placed on vertical and horizontal lines outside of the quadrille ruling.

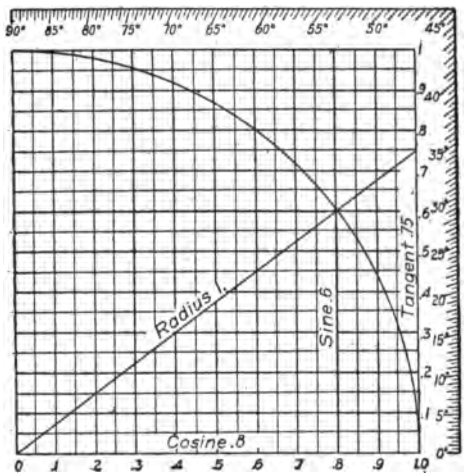


FIG. 162. Trigonometer.

Arranged as shown in the illustration this device is called a Trigonometer and is of considerable value in checking calculations for latitudes and departures. Metal trigonometers were formerly sold by instrument dealers but the ease with which a good draftsman may make one probably caused the manufacture to be discontinued, for they seemingly are no longer advertised. The writer used one for many years.

SOLUTIONS OF RIGHT TRIANGLE

Given A and c to find B, a and b.

$$B = 90° - A,$$
$$a = c \sin A,$$
$$b = c \cos A.$$

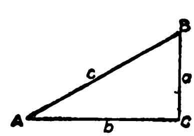


FIG. 163.

Given A and a, to find B, b and c.

$$B = 90° - A,$$
$$b = a \cot A,$$
$$c = \frac{a}{\sin A}.$$

Given A and b, to find B, a and c.

$$B = 90° - A,$$
$$a = b \tan A,$$
$$c = \frac{b}{\cos A}.$$

Given c and a, to find A, B and b.

$$\text{Sin } A = \frac{a}{c},$$
$$B = 90° - A,$$
$$b = a \cot A.$$

Given a and b, to find A, B and c.

$$\text{Tan } A = \frac{a}{b},$$
$$B = 90° - A,$$
$$c = \frac{a}{\sin A}.$$

$$\text{Area} = \frac{ab}{2}.$$

Either acute angle may be assumed to be the angle at the base in which case the adjacent side becomes the base.

PROBLEMS

1. $A = 48° 17'$, $c = 324$ ft. Find B, a, b, area.
2. $A = 51° 19'$, $b = 1254$ ft. Find B, a, c, area.
3. $A = 43° 38'$, $a = 186$ ft. Find B, b, c, area.
4. $a = 249$ ft., $c = 415$ ft. Find A, B, b, area.
5. $a = 67$, $b = 53$. Find A, B, c, area.
6. $c = 893$, $b = 586$. Find A, B, a, area.
7. $A = 64° 40'$, $b = 326$. Find B, a, c, area.
8. $A = 71° 24'$, $a = 286$. Find B, b, c, area.
9. $A = 41° 48'$, $c = 963$. Find B, b, a, area.

In solving problems the student is greatly assisted by drawing the triangle free hand and writing the given values in the proper places. Each part as found should be placed on the sketch. *Check.* — Draw the triangles to scale, using a protractor to measure the angles.

The expression abc means $a \times b \times c$, the multiplication sign being understood when letters are used. *Sin A* means the sine of the angle A, and $b \sin C$ means side $b \times \sin C$, the value of the sine being given in the tables.

Sine $\frac{1}{2}A$ means the sine of one-half the angle A and does not mean half the sine of A, which is something entirely different. Beginners often get into trouble over this matter.

ALGEBRAIC THEOREMS

1. *The square of the sum of two quantities is equal to the square of the first, plus twice the product of the first multiplied by the second, plus the square of the second.*

Example. — $(a + b)^2 = a^2 + 2ab + b^2$.

$$\begin{array}{l} a + b \\ \underline{a + b} \\ a^2 + ab \\ \underline{\quad + ab + b^2} \\ a^2 + 2ab + b^2 \end{array}$$

2. *The square of the difference of two quantities is equal to the square of the first minus twice the product of the first by the second, plus the square of the second.*

Example. — $(a - b)^2 = a^2 - 2ab + b^2$.

$$\begin{array}{l} a - b \\ \underline{a - b} \\ a^2 - ab \\ \underline{\quad - ab + b^2} \\ a^2 - 2ab + b^2 \end{array}$$

3. *The product of the sum and difference of two quantities is equal to the difference of their squares.*

Example. — $(a + b)(a - b) = a^2 - b^2$.

$$\begin{array}{l} a + b \\ \underline{a - b} \\ a^2 + ab \\ \underline{\quad - ab - b^2} \\ a^2 \qquad - b^2 \end{array}$$

(*Note.* — In the preceding chapter it was explained that in multiplication like signs produce + and unlike signs produce —.)

TRIGONOMETRIC LAWS

The following laws should be thoroughly understood and in their development appears the Pythagorean Theorem and the algebraic rules and theorems just given. The

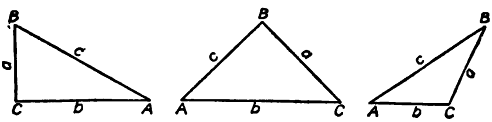


FIG. 164.

student should work the expressions by assuming the following values, $a = 9$, $b = 12$, $c = 15$.

Law of sines.

In any triangle the sides are to one another as the sines of their opposite angles.

$$\frac{a}{b} = \frac{\sin A}{\sin B};\ \frac{b}{c} = \frac{\sin B}{\sin C};\ \frac{a}{c} = \frac{\sin A}{\sin C}.$$

Law of cosines.

$$a^2 = b^2 + c^2 - 2\,bc\cos A.$$
$$b^2 = a^2 + c^2 - 2\,ac\cos B.$$
$$c^2 = a^2 + b^2 - 2\,ab\cos c.$$

Law of tangents.

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)}.$$
$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(A - C)}{\tan \frac{1}{2}(A + C)}.$$
$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(B - C)}{\tan \frac{1}{2}(B + C)}.$$

Half the difference of two unequal quantities AB and BC, added to half their sum, gives the greater, and half the difference taken from half the sum, gives the less.

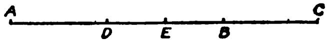


FIG. 165.

Proof. — Draw $AB + BC$.

Make $AD = BC$.

Then $AC =$ their sum and
$BD =$ their difference.

Bisect BD in E.

Then $BE = ED =$ half their difference and
$AE = EC =$ half their sum.

Consequently $AE + EB = AB$, the greater and
$EC - EB = BC$, the less.

Also, half the difference BE, added to the less BC, or taken from the greater AB, gives half the sum.

SOLUTION OF OBLIQUE TRIANGLES

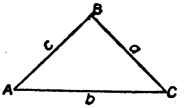


FIG. 166.

FIRST CASE.

Given A, B, a, to find C, b, c.

$$C = 180° - (A + B). \tag{1}$$

$$b = \sin B \frac{a}{\sin A}. \tag{2}$$

$$c = \sin C \frac{a}{\sin A}. \tag{3}$$

SECOND CASE.

Given A, a, b, to find B, C, c.

$$C = \tfrac{1}{2}(C + B) + \tfrac{1}{2}(C - B).$$
$$B = \tfrac{1}{2}(C + B) - \tfrac{1}{2}(C - B).$$

$$\text{Sin } B = \frac{\sin A}{a} b. \tag{4}$$

C use (1).
c use (3).

THIRD CASE.

Given A, b, c, to find a, B, C, area.

$$a = \sqrt{b^2 + c^2 - 2\,bc \cos A}. \qquad (5)$$

B use (4).

C use (1).

$$a = \sin A \frac{b}{\sin B}; \quad \text{or } \sin A \frac{c}{\sin C}. \qquad (6)$$

$$\text{Area} = \frac{bc \sin A}{2}. \qquad (7)$$

The second case has an angle at the base with the base and side opposite the angle given.

The third case has an angle and the two including sides given.

The second case often presents difficulties to the student. If the given angle is acute and the side opposite is the lesser side, then the angle found may be either obtuse (greater than 90°), or acute (less than 90°). The conditions of the problem should be known.

The sign $<$ means "*less than*" and the sign $>$ means "*greater than.*" The sign $\geqq$ means "*equal to or greater than.*"

Let b, c, B be the parts known. If $b < c \sin B$, no solution is possible; if $b = c \sin B$, then $\sin C = 90°$; if $b > c \sin B$ and $b < c$, and B is acute, two solutions are possible, but if $b \geqq c$ only one solution is possible and C is an acute angle.

Professor Stang in *Engineering News*, Dec. 25, 1913, presented the following formula for solving a triangle in which is given two sides and the included angle:

$$\cot B = \frac{a}{b \sin C} - \cot C.$$

FOURTH CASE.

Given a, b, c, the three sides, to find A, B, C, the three angles.

Let $$S = \frac{abc}{2},$$

$$\text{Sin } \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}}. \qquad (8)$$

B use 4.
C use 1.
Area $= \sqrt{S(s-a)(s-b)(s-c)}$. (9)

The following formulas may also be used for this case.

$$\text{Sin}\ \tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{ac}}. \qquad \text{Sin}\ \tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{ab}}.$$

$$\text{Cos}\ \tfrac{1}{2}A = \sqrt{\frac{s(s-a)}{bc}}. \qquad \text{Tan}\ \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

$$\text{Cos}\ \tfrac{1}{2}B = \sqrt{\frac{s(s-b)}{ac}}. \qquad \text{Tan}\ \tfrac{1}{2}B = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}.$$

$$\text{Cos}\ \tfrac{1}{2}C = \sqrt{\frac{s(s-c)}{ab}}. \qquad \text{Tan}\ \tfrac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}.$$

The following item appeared in *Engineering News*, July 25, 1912.

A triangle formula, to obtain the angles when the three sides are known, is given by Prof. C. Frank Allen (Mass. Inst. of Technology). While not new it may be unknown to many of our readers, as the formula

$$\sin \tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{bc}} \tag{1}$$

is usually quoted without reference to the other method of solution. In the above the sides are a, b and c, with the opposite angles A, B and C, and s is half the sum of the sides.

The derivation and formula, given by Professor Allen, are as follows: From angle C drop a perpendicular on side c, dividing c into the segments k and g, adjacent respectively to sides a and b. Calling the length of the perpendicular h, we have:

$$h^2 = b^2 - g^2 = b^2 - (c-k)^2,$$

and

$$h^2 = a^2 - k^2.$$

Then

$$a^2 - k^2 = b^2 - c^2 + 2ck - k^2.$$

Whence

$$2\,ck = c^2 - (b^2 - a^2),$$

and

$$k = \frac{1}{2}\left[c - \frac{(b + a)(b - a)}{c}\right]. \tag{2}$$

Since $g = c - k$, we get

$$g = \frac{1}{2}\left[c + \frac{(b + a)(b - a)}{c}\right]. \tag{3}$$

When g and k are computed, the angles are readily found, as

$$\cos A = \frac{g}{b}, \qquad \cos B = \frac{k}{a}.$$

For many uses these formulas are more convenient than Eq. (1), being probably less liable to errors of computation, and more quickly derived if forgotten.

PROBLEMS

In the following problems give both solutions when two are possible. Find in each problem all the parts not given and the area. Check the work by drawing to scale and read angles with a protractor.

1. $c = 532$, $b = 358$, $C = 107° \ 40'$.
2. $c = 232$, $b = 345$, $C = 37° \ 20'$.
3. $b = 560$, $a = 258$, $B = 63° \ 28'$.
4. $B = 63° \ 48'$, $A = 49° \ 25'$, $b = 275$.
5. $A = 49° \ 25'$, $C = 63° \ 48'$, $b = 275$.
6. A ship sailing due north observes a cape bearing $N \ 54° \ 12' \ W$ and after sailing 27 miles, the cape bore $S \ 70° \ 30' \ W$. Required her distances from it.
7. $c = 133$, $b = 176$, $A = 73° \ 16'$.
8. $c = 237$, $a = 482$, $B = 77° \ 48'$.
9. $b = 78$, $a = 168$, $C = 128° \ 26'$.
10. $a = 230$, $b = 365$, $c = 426$.
11. $a = 1248$, $b = 728$, $c = 956$.
12. $a = 375$, $b = 275$, $c = 196$.

FUNCTIONS OF HALF AN ANGLE

In Fig. 167 let b, o, c be the angle z, then

$$b, o, f = c, o, f = \tfrac{1}{2}z,$$
$$\sin c, o, f = \sin \tfrac{1}{2}z = c \ldots g,$$
$$\tan c, o, f = \tan \tfrac{1}{2}z = e \ldots f,$$
$$\sin z = a \ldots c,$$
$$\tan z = b \ldots d.$$

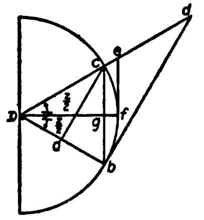


FIG. 167. Functions of half angles.

A study of this figure will show why the student must be careful to note the difference between $\frac{1}{2}\sin A$ and $\sin \frac{1}{2}A$.

$$\operatorname{Sin} \tfrac{1}{2}z = \pm\sqrt{\frac{1-\cos z}{2}}.$$

$$\operatorname{Tan} \tfrac{1}{2}z = \pm\sqrt{\frac{1-\cos z}{1+\cos z}}.$$

$$\operatorname{Cos} \tfrac{1}{2}z = \pm\sqrt{\frac{1+\cos z}{2}}.$$

$$\operatorname{Cot} \tfrac{1}{2}z = \pm\sqrt{\frac{1+\cos z}{1-\cos z}}.$$

Work the above expressions with $z = 45°$.
The sign $\pm$ means "positive or negative."

CIRCULAR MEASURE OF AN ANGLE

In certain kinds of calculations the *radian* is the unit of measure when the magnitude of an angle is to be measured.

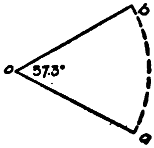


FIG. 168. The radian.

In Fig. 168 the arc ab is equal in length to the radius oa.

The ratio $\dfrac{arc}{radius}$ is called the circular or radian measure of an angle.

The circumference of a circle $= \pi d$, in which $\pi = 3.1416$ (pronounced pi) and $d =$ diameter $= 2 \times$ radius.

The circumference of a circle of radius r is $2\pi r$, or, if the radius is unity, 2π.

The angle 360° corresponds to an arc with the length 2π; the angle 180° to an arc $= \pi$; the angle of 90° to an arc $= \frac{1}{2}\pi$; etc.

The radian has a value when expressed in degrees as follows:

$$\frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi} = \frac{180^\circ}{3.1416} = 57^\circ\, 17'\, 44.8'' = 57.295^\circ.$$

The value usually given is 57.3°.

The radian, or 57.3 rule, is convenient when the size of angle between a random line and true line is wanted. If the angle is found to be less than 6° the rule is all right, but for larger angles may introduce a considerable error. This is because the offset is always measured on a straight line and in the 57.3 rule it is assumed to be an arc. In Fig. 169 this is illustrated.

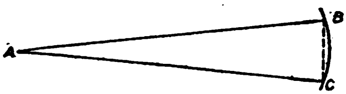


FIG. 169. Application of 57.3 rule.

The line AC is measured on the supposition that it is the line AB, which it however misses at B by the distance BC. For a small angle the difference between BC straight and curved is so small that

$$\text{Angle } BAC = \frac{57.3\, BC}{AC} = \frac{57.3 \times \text{offset}}{\text{distance}}.$$

In higher mathematics the radian measure of an angle is necessary but the instance just given is the only one in which the surveyor can advantageously use it.

HEIGHTS AND DISTANCES

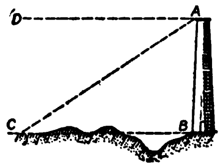


FIG. 170.

In Fig. 170, DA is parallel to BC and the angle $ABC = 90°$.

1. $BC = 236$ ft., angle $ACB = 35°\, 48'$. Find height AB.

2. $BC = 136$ ft., angle $ACB = 47°\, 25'$. Find height AB.

3. The angular elevation of a wall, taken from the edge of a ditch 18 ft. wide, was 62° 40′. Required the height of the wall and the length of a ladder to reach the top of it.

4. Let the sloping side of a hill AC be 268 ft., and the angle of depression at its top DAC be $33° 45'$. Required the horizontal distance BC and the vertical height AB.

To measure an inaccessible height AB on level ground.

Measure any distance CD in a straight line towards the object and at C and D read the angles of elevation; their difference is the angle CAD.

$$AB = \frac{\sin C \times \sin D \times CD}{\sin (C - D)}.$$

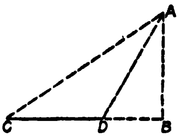


FIG. 171.

To measure an inaccessible height which has no level ground before it.

Take two stations C and D, in a vertical plane, and measure CD; at C read the vertical angle GCD and the two vertical angles ACF and BCF. At D take the angle ADE.

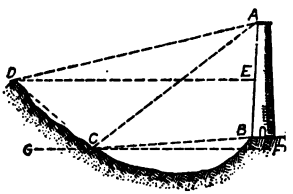


FIG. 172.

Since the angle $EDC = DCG$. $\therefore\ ADC = ADE + DCG$ and $DAC = ACF - ADE$. Then in the triangle ADE, the two angles ADC and DAC and the side CD are given to find the side AC. In the triangle ACB are given the angles $ACB = ACF \pm BCF$, and $ABC = 90° \pm BCF$, and the side AC to find the side AB.

If DE is above A, the angle DAC is the sum of ACF and ADE; otherwise it is their difference. Also in this case ADC is the difference of DCG and ADE; otherwise it is their sum. Also when F is below B, the angle ACB is the difference of ACF and BCF; otherwise it is their sum.

If the stations C and D cannot be conveniently taken in a vertical plane, they may be taken anywhere, and then the angles ADC and ACD must be measured. The triangle ACD will give the side AC.

To measure a line across a river or canyon.

With the instrument at B, Fig. 173, sight to the opposite bank and set the point C, so the points A, B and C will be

in the same straight line. An assistant standing on C sights back to B and marks out the line CD normal to line ABC. At D, just 10 ft. from C, set a point. The instrument man at B reads the angle CBD; then

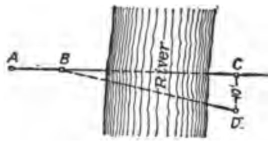


FIG. 173.

$$\text{Length } BC = CD \times \cot CBD.$$

Another method. — The line CD may be any length and it is not necessary that the triangle be right angled at C. The instrument is set at B, then at C and finally at D. Each angle is read. The surveyor then has a triangle with one side CD, and the three angles known, the distance from B to C being obtained according to the Law of Sines.

To obtain the depth and front of a lot on a diagonal street.

The streets shown in Fig. 174 have an angle x at the intersection of the lot lines, all the lines between the lots being normal to B Avenue.

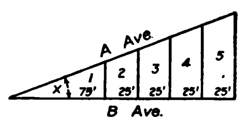


FIG. 174.

To obtain the lengths of the lines between the lots from A Street to B Avenue multiply the tangent of angle x by the distance of the line from the point; for example:

$$\text{Length of line between 1 and 2} = 75 \tan x,$$
$$\text{Length of line between 2 and 3} = 100 \tan x,$$
$$\text{Length of line between 3 and 4} = 125 \tan x.$$

To obtain the frontage on A Street multiply the secant of angle x by the width of the lot on B Avenue; for example:

$$\text{Front of lot 1 on A Street} = 75 \sec x,$$
$$\text{Front of lot 2 on A Street} = 25 \sec x,$$
$$\text{Front of lot 3 on A Street} = 25 \sec x, \text{ etc.}$$

TRAVERSES

In Fig. 175 it is desired to obtain the bearing and length of a straight line from A to B. Houses and shrubbery are in the way. The obvious thing to do is to run lines through

the cleared spaces and calculate the bearing and length of the line AB.

Starting from Sta. 0 (A) the line was run down the street

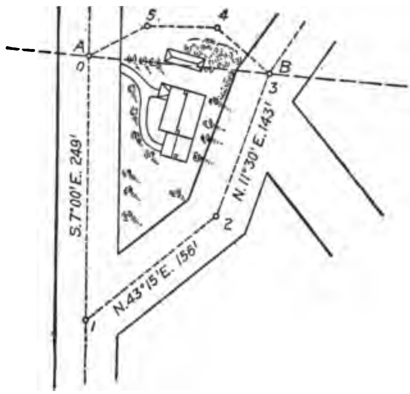


FIG. 175. Running a traverse line.

$S\ 7°\ 00'\ E$ 249 ft.; thence $N\ 43°\ 15'\ E$ 156 ft.; thence to B, $N\ 11°\ 30'$ 143 ft.

To compute the missing line set the work down as follows:

Station.	Bearing.	Distance.	N+	S−	E+	W−
0	S 7° 00′ E	249	....	247.16	30.35	....
1	N 43° 15′ E	156	113.63	....	106.89	....
2	N 11° 00′ E	143	140.37	....	27.28	....
			254.00	247.16	164.52	
			247.16			
			6.84			

The southings lack 6.84 ft. of balancing the northings and the westings lack 164.52 ft. of balancing the eastings. This gives a triangle of the following shape and the required bearing will be southwest.

This is to be solved as a right triangle with a and b given, to find A, B and c;

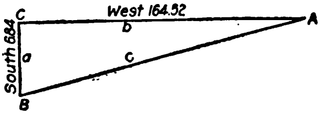


FIG. 176.

$$C = 90°.$$

$$\text{Tan } A = \frac{a}{b} = \frac{6.84}{164.52} = 0.0440.$$

$$\therefore \quad A = 2° \, 31'$$

$$B = 90° - A = 87° \, 29'.$$

The required bearing is $S\ 87° \, 29'\ W$.

$$\text{The length } c = \frac{a}{\sin A} = \frac{6.84}{0.04391} = 155.77 \text{ ft.}$$

If it is necessary to run the line from A to B without wanton cutting of shrubbery the bearing of the line is found in the manner just described and it is then run in from B towards A, or the bearing is made to read $N\ 87° \, 29'\ E$ and the line is run from A to B.

If the work is carefully done the bearing and distance should check with the computations.

A surveyor must never neglect to check his work. An experienced surveyor would make a "closed traverse" by endeavoring to run his line through open spaces from B to 4, for example; then to 5 and back to A. He would compute the latitudes and departures and get them to balance, as described in the chapter on Compass Surveying. Then the bearing and length of line AB should be calculated as already described, using courses 0, 1, 2, to Sta. B. An independent calculation would then be made using courses 3, 4, 5 to Sta. A. The differences will be small and can be averaged, after which the line can be staked out.

"Running a traverse" is a common operation in surveying. Any number of courses may be run and the bearing and length of a straight line joining the ends of the first and last obtained. In fact the bearing and length of a line joining any two points on a traverse may be found by tabulating only the courses involved.

The captain of a ship finds her position by traversing, the work of surveyors and navigators being in many respects similar.

OMISSIONS

The supplying of omitted bearings and distances in surveys is an extension of the principles of traversing.

Many reasons may be given to explain why a portion of the field notes are omitted but a principal reason is forgetfulness, which is one manifestation of carelessness. In the example under the head of traversing one reason for the omission to run one course was given. It sometimes happens that two courses may lie in swampy, or otherwise inaccessible, places, in which case the surveyors should run a closing line to make a closed survey of all accessible corners, the inaccessible courses then forming with the closing line a separate survey. All errors are thrown into the omitted parts.

There are four cases in omissions, each of which will now be illustrated.

CASE I. — *Bearing and length of one course omitted.*

Station.	Bearing.	Distance.	Latitudes.		Departures.	
			N+	S−	E+	W−
0	N 16° 30′ E	22.10	21.190	....	6.276	....
1	N 82° 05′ E	19.62	2.702	....	19.433	....
2	S 16° 54′ E	23.97	....	22.935	6.968	....
3	S 36° 58′ W	22.11	....	17.666	....	13.296
4	Omitted	Omitted	....	....	....	....
			23.892	40.601	32.677	13.296
				23.892	13.296	
				16.709	19.381	

$$\text{Nat. tan of bearing} = \frac{19.381}{16.709} = 1.15991.$$

$$\text{Bearing} = N\,49^\circ\,14'\,W.$$

$$\text{Length of course} = \frac{\text{dep.}}{\text{sin bearing}} = \frac{19.381}{0.75738} = 25.59 \text{ chains.}$$

The traverse table in the chapter on compass surveying gives values to quarter degrees only, whereas in work done with a transit angles are read to one minute of arc, except in extremely high-grade work when angles are read to one

second. Surveyors' transits usually read to minutes and engineers' transits read to one-half or one-third of a minute. When traversing is done by means of an instrument reading closer than a compass it is plain that a compass traverse table cannot be used.

It is common to use a table of sines and cosines, for

$$\begin{aligned} \textit{Cosine} \times \textit{distance} &= \textit{latitude}. \\ \textit{Sine} \times \textit{distance} &= \textit{departure}. \end{aligned}$$

The student must memorize this.

In all tables of latitudes and departures, for any angle, the value given in column 1 for the latitude is the cosine of that angle, and the value given in column 1 for the departure is the sine of that angle.

The actual labor of multiplying and dividing can be greatly lessened by using Crelle's Tables ($5.00), by using a slide rule, or by logarithms. Crelle's Tables contain the products of all numbers up to 1000 × 1000 and by following the instructions, products of much larger numbers may be obtained by inspection. The tables are as readily used for division of one number by another. The slide rule is not so good for this class of problems as for others and is not recommended for traversing calculations except for checking, in which work it is better than a trigonometer. Logarithms are great labor savers and the degree of accuracy of logarithmic work is fixed by the number of significant figures to which the tables are computed.

The greatest labor savers in calculating latitudes and departures are traverse tables, which reduce all multiplication to addition. The author has read in several textbooks written by teachers, that traverse tables do not save time as compared with tables of logarithms and logarithmic functions of angles. For a number of years the work of the author was principally surveying and he found that traverse tables are far superior to logarithms in saving time and lessening liability of errors.

The oldest known traverse tables with Latitude and Departure computed for each minute of angle are those of Gen. J. T. Boileau ($5.00). In 1900 H. Louis and G. W. Caunt brought out a book somewhat more convenient to

use, with larger and clearer type. The price of this book is two dollars, the page measuring six by nine inches.

The most complete Traverse Tables are those of R. L. Gurden. The latitudes and departures are computed to four places of decimals and for every minute of angle up to 100 of distance. The size is 9 ins. by 14 ins.; the binding is cloth and half morocco. When the author bought his copy in 1889, the price was $12.50, but is now $7.50. The Louis and Caunt tables are admirable for field use while the Gurden tables are ideal for the office. The difference in the tables lies in the fact that the Boileau and the Louis and Caunt tables are computed only for all distances up to 10. The following example will serve as an illustration.

Find the latitude and departure of S 16° 54′ E 23.97 chains.

1. Boileau.
2. Louis & Caunt. 16° 54′

	Lat.	Dep.
20.00	19.136	5.814
3.00	2.8704	0.8721
0.90	0.86113	0.26163
0.07	0.06698	0.02035
	22.93451	6.96808

3. Gurden. 16° 54′

	Lat.	Dep.
23.00	22.0064	6.6861
0.97	0.92811	0.28198
	22.93451	6.96808

To obtain the same results by using logarithms it will be necessary to take from one table the logarithm of 23.97 and from another table the logarithm of the sine and of the cosine of 16° 54′. These are added and the numbers corresponding to the resulting logarithms found.

By logarithms:

	Lat.		Dep.
log. 23.97	= 1.379668	log. 23.97	= 1.379668
log. cos 16° 54′	= 9.980827	log. sin 16° 54′	= 9.463448
	11.360495		10.843116
subtract	10.	subtract	10.
log. lat.	= 1.360495	log. dep.	= 0.843116
Latitude	= 22.9345	Departure	= 6.968

CASE II. — *Lengths of two courses omitted.*

This might be a case where bearings are taken to a corner, which is visible but inaccessible.

In the assumed problem the two courses are adjacent. When the courses are not adjacent the method is the same, or the method for Case IV may be used.

If the two sides are parallel the problem is indeterminate.

The whole process in Cases II, III and IV is to discard the omitted parts and calculate the bearing and length of a closing line. A triangle is then formed on this closing line as a base and the missing parts computed.

Station.	Bearing.	Distance.	Latitudes.		Departure.	
			N+	S−	E+	W−
0	N 16° 30′ E	22.10	21.190	....	6.276	....
1	N 82° 05′ E	19.62	2.702	....	19.433	....
2	S 16° 54′ E	23.97	....	22.935	6.968	....
3	S 36° 58′ W	Omitted	....	....	....	....
4	N 49° 14′ W	Omitted	....	....	....	....
			23.892	22.935	32.677	
			22.935			
			0.957			

$$\text{Nat. tan of bearing} = \frac{\text{dep.}}{\text{lat.}} = \frac{32.677}{0.957} = 34.14524.$$

$$\text{Bearing} = \text{S } 88° \ 19' \text{ W.}$$

$$\text{Length of course} = \frac{\text{dep.}}{\text{sin bearing}} = \frac{32.677}{\sin 88° \ 19'}$$

$$= \frac{32.677}{0.99957} = 32.691 \text{ chains.}$$

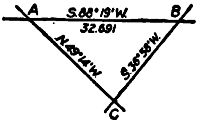


FIG. 177.

Draw a triangle (free hand) as shown in Fig. 177 and from the bearings compute angles A, B and C.

$$\begin{array}{rl} \text{S } 88° \ 19' \text{ W} = & \text{S } 87° \ 79' \text{ W} \\ & \underline{\text{S } 36° \ 58' \text{ W}} \\ & \quad 51° \ 21' \quad = \text{angle } B. \end{array}$$

$$\begin{array}{l} \text{S } 36° \ 58' \text{ W} \\ \underline{\text{N } 49° \ 14' \text{ W}} \\ \quad 85° \ 72' \quad = 86° \ 12' = \text{angle } C \end{array}$$

S 88° 19′ W		To Check
N 49° 14′ W		51° 21′
137° 33′		86° 12′
179° 60′		42° 27′
42° 27′	= angle A	179° 60′ = 180° 00′

$$\frac{\text{Sin } 86° \ 12'}{32.691} = \frac{\sin A}{a} = \frac{\sin B}{b}.$$

$$\therefore a = \sin 42° \ 27' \left(\frac{32.691}{\sin 86° \ 12'}\right) = \frac{0.67495 \times 32.691}{0.99780} = 22.11 \text{ chains}$$

and

$$b = \sin 51° \ 21' \left(\frac{32.691}{\sin 86° \ 12'}\right) = \frac{0.78098 \times 32.691}{0.99780} = 25.59 \text{ chains.}$$

CASE III. — *Bearings of two courses omitted.*

This problem may have two solutions but the ambiguity is practically unimportant as the surveyor is presumed to have a clear idea of the shape of the field.

Station.	Bearing.	Distance.	Latitudes.		Departures.	
			N+	S−	E+	W−
0	N 16° 30′ E	22.10	21.190	....	6.276	....
1	N 82° 05′ E	19.62	2.702	....	19.433	....
2	S 16° 54′ E	23.97	....	22.935	6.968	....
3	Omitted	22.11	....	....	....	....
4	Omitted	25.59	....	....	....	....
			23.892		32.677	
			22.935			
			0.957			

This gives the same bearing and length for the closing line as were found for the problem illustrating Case II. The triangle is shown in Fig. 178.

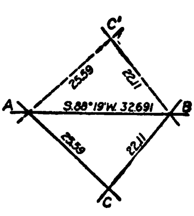


FIG. 178.

If the instrument man does not make a sketch in his notes to guide the computer in the office, the latter might assume the courses to run as indicated by the dotted lines. The interior angles

of the closing triangle will not be affected but the shape and area of the field will differ considerably from the truth.

$$Let\ S = \frac{a+b+c}{2} = \frac{22.11 + 25.59 + 32.69}{2} = 40.20.$$

$$Sin\ \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}} = \sqrt{\frac{14.61 \times 7.51}{25.59 \times 32.69}} = 0.36216.$$

$$A = 42^\circ\ 28'.$$

$$Sin\ \tfrac{1}{2} B = \sqrt{\frac{(s-a)(s-c)}{ac}} = \sqrt{\frac{18.09 \times 7.51}{22.11 \times 32.69}} = 0.43355.$$

$$B = 51^\circ\ 22'.$$

$$C = 180^\circ - (A + B) = 179^\circ 60' - 93^\circ 50' = 86^\circ 10'.$$

$$\begin{array}{llll} & \text{S } 88^\circ\ 19'\ \text{W} & & 86^\circ\ 10'\ \text{L} \\ & \underline{51^\circ\ 22'\ \text{L}} & \text{S } 36^\circ\ 47'\ \text{W} = & \text{N } \underline{36^\circ\ 57'}\ \text{E} \\ a = & \text{S } 36^\circ\ 57'\ \text{W} & b = & \text{N } 49^\circ\ 13'\ \text{W} \end{array}$$

CASE IV. — *Length of one course and bearing of another omitted.*

The field is to be revolved until the course of which only the bearing is known is in the meridian, thus throwing all the unknown departure into the course of which only the length is known. The omitted parts may be on adjacent or non-adjacent courses.

Station.	Bearing.	Revolved to left.	New bearing.	Distance.
0	N 16° 30′ E	36° 58′	N 20° 28′ W	22.10
1	N 82° 05′ E	36° 58′	N 45° 07′ E	19.62
2	S 16° 54′ E	36° 58′	S 53° 52′ E	23.97
3	S 36° 58′ W	36° 58′	South	Omitted
4	Omitted	....	Omitted	25.59

When the computations are finished a doubt sometimes arises as to whether the latitude of the course whose bearing is omitted is a northing or a southing. This produces two sets of values, either of which will satisfy the problem, though one will give a wrong area. The knowledge the surveyor has of the general directions of the courses and the shape of the field must settle all such ambiguities.

The habit of making free-hand sketches to supplement the field notes is one every instrument man should early acquire.

Sta.	Bearing.	Distance.	Latitudes.		Departures.	
			N+	S−	E+	W−
0	N 20° 28′ W	22.10	20.705	....	....	7.727
1	N 45° 07′ E	19.62	13.845	....	13.902	....
2	S 53° 52′ E	23.97	....	14.134	19.360	....
3	South	Omitted	....	....	....	....
4	Omitted	25.59	....	....	....	....
			34.550	14.134	33.262	7.727
			14.134		7.727	
			20.416		25.535	

All the difference in departure is thrown into course 4, the length of which is 25.59 chains. The following triangle is obtained.

The sine of $A = \frac{25.535}{25.59} = 0.99785$,

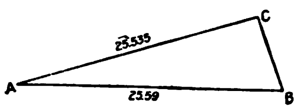


FIG. 179.

thus making the bearing of course 4 = 86° 14′ but some uncertainty exists as to whether it is a northing or a southing. From the differences in latitudes and departures it is apparently S 86° 14′ W.

To settle the matter apply the angle of revolution.

$$\begin{array}{lr} \text{S} & 86^\circ\ 14'\ \text{W} \\ & \underline{36^\circ\ 58'\ \text{R}} \\ & 123^\circ\ 12' \\ & \underline{179^\circ\ 60'} \\ \text{N} & 56^\circ\ 48'\ \text{W} \end{array}$$

The bearing was known (from the study of Cases I, II and III) to be N 49° 14′ W so another trial must be made.

$$\begin{array}{lr} \text{N} & 86^\circ\ 14'\ \text{W} \\ & \underline{36^\circ\ 58'\ \text{R}} \\ \text{N} & 49^\circ\ 16'\ \text{W} \end{array}$$

The bearing is evidently a northing. The difference of two minutes is due to the computations in some of the examples being carried to a greater degree of refinement than in others. It will make no appreciable difference in the length of course 4, but the difference in angle introduces a closing error of about $1\frac{1}{2}$ links. No survey is free from error, but all errors being thrown into the courses supplied by computation a false idea of accuracy is obtained by doing work in the office that belongs to the field. "Measure in haste and repent in the office," is a true proverb.

The latitude of the course = cosine 86° 16′ × 25.59 = 1.681, and, this amount is to be added to the latitude difference to obtain the southing for course 3, so the northings and southings will balance.

Latitude for course 3 = 20.416 + 1.681 = 22.097.

The latitudes and departures are now

Station.	Latitudes.		Departures.	
	+	−	+	−
0	20.705	......	......	7.727
1	13.845	......	13.902	......
2	......	14.134	19.360	......
3	......	22.097	......	......
4	1.681	......	......	25.535
...	36.231	36.231	33.262	33.262

and the bearings may be restored by revolving the field *to the right* through an angle of 36° 58′.

The field is not altered in size or shape by being revolved so the area may be computed by using the values already found for the latitudes and departures.

LOGARITHMS

A logarithm is an exponent, and in an earlier chapter it was shown that

$$a^3 \times a^2 = a^{3+2} = a^5,$$

and

$$\frac{a^3}{a^2} = a^{3-2} = a^1 = a.$$

A series of quantities which increase or decrease by a common difference is called an Arithmetical Progression; as 1, 2, 3, 4, 5, etc., or 75, 72, 69, 66, etc.

A series of quantities which increase by a constant multiplier, or decrease by a constant divisor, is called a Geometrical Progression; as 2, 8, 32, 128, etc., the multiplier being 4; or 567, 189, 63, etc., the divisor being 3.

We can write down a line of figures in arithmetical progression over a line in geometrical progression, and the upper line will contain the exponents of the second line so that multiplication of two quantities in the lower line may be accomplished by adding their exponents.

0.5	0.4	0.3	0.2	0.1	0	1	2	3	4	5	6
0.00001	0.0001	0.001	0.01	0.1	1	10	100	1000	10,000	100,000	1,000,000

Add 2 and 3, the sum is 5. Under 5 is found 100,000 which is plainly the product of 100 (under 2) and 1000 (under 3).

Subtract 4 from 5, the difference is 1. Under 5 is found 100,000 and under 4 is found 10,000. Under 1 is found $10 = \frac{100,000}{10,000}$.

Multiply 2 by 3, the product is 6. Under 6 is found 1,000,000 which is the *square* of 1000, found under 3.

Divide 6 by 2, the quotient is 3. Under 3 is found 1000 which is the *square root* of 1,000,000 found under 6.

The student for an exercise may carry these series, ascending and descending, to 10 or more places each way, and multiply and divide, raise to powers and extract roots, keeping always in an ascending or a descending series.

Logarithms are a series of artificial numbers used as exponents for numbers placed in *arithmetical progression.* By their use addition takes the place of multiplication and subtraction takes the place of division. Numbers are raised to any power by multiplying the logarithm of the number by the exponent representing that power, and roots are extracted by dividing the logarithm by the index of the required root.

Logarithms are useful for many calculations, especially those involving proportion, or a combination of multiplication and division.

An expression containing a positive or negative sign is not in shape for logarithmic computation and must be differently written to eliminate every such sign. For example

$$\sin (x + y) + \sin (x - y)$$

can be adapted for logarithmic computation by writing it as follows:

$$2 \sin x \times \cos y.$$

Adding logarithms is equivalent to multiplying their numbers, and subtracting logarithms is equivalent to performing the operation of division with their numbers.

Some surveyors never use logarithms. Other surveyors use them upon every occasion. Whether to use "logs" or "naturals" is something each man must settle for himself and is not worth discussing. For involved calculations logarithms save time and lessen the liability of error.

TO USE A TABLE OF LOGARITHMS

A complete logarithm consists of two parts. The *mantissa* is the decimal part printed in the table. The *characteristic* is a whole number and depends upon the number of figures in the number.

The log. of 2500 is 3.3979 of which 0.3979 is the *mantissa* taken from the table and 3 is the characteristic.

The log. of 8435 = 3.926085.	The log. of 8.435 = 0.926085.
The log. of 843.5 = 2.926085.	The log. of 0.8435 = − 1.926085.
The log. of 84.35 = 1.926085.	The log. of 0.08435 = − 2.926085.

The characteristic is seen by the above illustration to be a number indicating 1 less than the number of integral figures of which the number consists. In decimal numbers the characteristic is negative and indicates the distance of the first significant figure from the decimal point.

In some tables of logarithms there is a column headed "Tabular Difference." In the table in this book the tabular difference is not shown. The logarithm of 243 = 2.38561 and the logarithm of 244 = 2.38739. The difference is 0.00178 and would be printed as Tab. Diff. 178. The Tabular Difference for each horizontal line is therefore the *average* difference between the units figures on the line.

TO FIND THE LOGARITHM OF A NUMBER

First fix the characteristic. Then look in the table for the mantissa. The number may have only three figures, in which case the mantissa will be found at once. If it contains four or more figures the mantissa for the three first figures is found and the difference between this and the next higher figure used in the following way:

Find the log. of 2369.

The characteristic = 3. The mantissa for 2360 = 0.37291. To find the mantissa find 23 in the column headed "No.," and proceed horizontally to the right to the column headed "6."

The remainder of 9 cannot be ignored. Find the mantissa for 237 = 0.37475. The difference between 0.37475 and 0.37291 = 0.00184 and this must be multiplied by 0.9 = 0.001656, the product being added to the mantissa for 2360. The complete log. is as follows:

$$\begin{array}{ll} \text{Mantissa of } 236 & = 0.37291 \\ \text{Mantissa of } 0.9 & = \underline{0.00166} \\ & \ 0.37457 \\ \text{Log. of } 2369 & = 3.37457. \end{array}$$

The reason for designating the 9 remainder as 0.9 may be found by considering the mantissas as being given in the table for 2370 and for 2360, but not for fractional parts, therefore 9 is nine-tenths the difference between 60 and 70. Had the number been 236,924 the process would have been the same but the characteristic would be 5 and the difference 0.00184 would be multiplied by 0.924 = 0.00170.

$$\begin{array}{ll} \text{Mantissa of } 236 & = 0.37291 \\ \text{Mantissa of } 0.924 & = \underline{0.00179} \\ & \ 0.37470 \\ \text{Log. of } 236924 & = 5.37470. \end{array}$$

Tables of logarithms having a column of tabular differences show the difference of 0.00184 as a whole number, 184, to save space.

The student must remember that the mantissa is a decimal fraction and therefore the tabular difference is a decimal fraction containing the same number of figures, the differ-

ence being made up by placing ciphers in front of the significant figures.

To find the number corresponding to a given logarithm.

The log. is 3.37457.

In the table of logs. find the mantissa nearest to the given mantissa but *below*. This we find to be 0.37291, corresponding to number 2360, for as we have 3 for a characteristic there must be four figures in the number.

Given log.	= 3.37457
Log. of 2360	= 3.37291
Diff.	= 0.00166

Tabular difference between mantissas of 2370 and 2360 = 0.00184.

$$\frac{0.00166}{0.00180} = 0.913,$$

which gives a number = 2369.13. The 0.13 is an exceedingly small per cent of 2369 but proves that a table of logarithms of numbers from 0 to 1000 is not accurate for numbers containing more than four figures, when the mantissa is carried to only five places. Tables of logarithms from 0 to 10,000 are accurate for numbers containing as many figures as there are decimal places in the mantissa. Tables of logarithms from 0 to 108,000 are accurate enough for practical use for numbers containing one figure more than the decimal figures in the mantissa. For all the work of surveyors the five-place table here given is accurate enough for all *practical purposes* for numbers containing not more than five figures. A table for numbers from 0 to 10,000 is more convenient to use. In the examples following notice the inaccuracy of the work due to the table used.

To multiply two numbers by using logs.

1. 54.3 × 6.19 = ?

Log. 54.3	= 1.73480
Log. 6.19	= 0.79169
	2.52649
Log. of 336	= 2.52633
	0.00016

Tab. diff. 336 and 337 = 0.52763 − 0.52633 = 0.00130.

$$\frac{0.00016}{0.00130} = 0.123.$$

By logs. 54.3 × 6.19 = 336.123.
By arithmetic = 336.117.

2. 54.3 × 6.19 × 27 = ?

Log. 54.3	=	1.73480
Log. 6.19	=	0.79169
Log. 27	=	1.43136
		3.95785
Log. of 9070	=	3.95761
		0.00024

Tab. diff. 9070 and 9080 = 0.95809 − 0.95761 = 0.00048.

$$\frac{0.00024}{0.00048} = 0.5.$$

By logs. 54.3 × 6.19 × 27 = 9075.000.
By arithmetic = 9075.159.

The rule for multiplying numbers by using logs. is to add the logarithms of the numbers and find from the table the number corresponding to the new logarithm thus obtained.

To divide a number by another by using logs.

$$\frac{54.3}{6.19} = ?$$

Log. 54.3	=	1.73480
Log. 6.19	=	0.79169
		0.94311
Log. 8.77	=	0.94300
Diff.	=	0.00011

Tab. diff. 878 and 877 = 0.94349 − 0.94300 = 0.00049.

$$\frac{0.00011}{0.00049} = 0.225.$$

By logs. 54.3 ÷ 6.19 = 8.77225.
By arithmetic = 8.77205.

To divide one number by another find the difference between their logarithms. The resulting logarithm is the log. of the quotient of the numbers.

To raise a number to any power.

Find the square of 6.19.

$$
\begin{array}{lrl}
\text{Log. } 6.19 & = & 0.79169 \\
 & & \underline{2} \\
 & & 1.58338 \\
\text{Log. } 38.3 & = & \underline{1.58320} \\
 & & 0.00018
\end{array}
$$

$$\frac{\text{Diff.}}{\text{Tab. diff.}} = \frac{0.00018}{0.00133} = 0.135.$$

$$6.19^2 \begin{cases} \text{By logs.} & = 38.3135. \\ \text{By arithmetic} & = 38.3161. \end{cases}$$

The rule, which is general, is to multiply the log. by the exponent of the power. The number corresponding to the log. thus obtained is the power sought.

To extract the root of any number.

What is the square root of 54.3?

$$
\begin{array}{lcl}
\text{Log. } 54.3 = \dfrac{1.73480}{2} & = & 0.86740 \\
\text{Log. } 7.36 = & & \underline{0.86688} \\
 & & 0.00052
\end{array}
$$

$$\frac{\text{Diff.}}{\text{Tab. diff.}} = \frac{0.00052}{0.00059} = 0.881.$$

$$\sqrt{5.43} \begin{cases} \text{By logs.} & = 7.36881. \\ \text{By arithmetic} & = 7.368+. \end{cases}$$

The rule, which is general, is to divide the log. of the number by the index of the root. The number corresponding to the log. thus obtained is the root sought.

ARITHMETICAL COMPLEMENT

A logarithm may be mentally subtracted from 10, an integer, or the right-hand figure may be subtracted from 10, and all the rest from 9.

By using the arithmetical complement of a logarithm, division instead of being subtraction becomes addition. That is, adding an arithmetical complement is equivalent to subtracting its logarithm. In the product, 10 must be subtracted from the characteristic.

NEGATIVE CHARACTERISTIC

The characteristic is negative when the number is a decimal fraction.

A negative characteristic must be subtracted when the logarithm is added and must be added when the logarithm is subtracted.

After multiplying a negative index subtract from the resulting index the amount carried from the mantissa.

Example. — Raise 0.009 to the third power.

$$\begin{array}{rr} \text{Log. } 0.009 = & \bar{3}.95424 \\ & 3 \\ \hline & \bar{7}.86272 \end{array}$$

The -7 was obtained as follows: 2 was carried from the multiplication of the mantissa. The product of $3 \times (-3) = -9$ and $+2 - 9 = -7$. It is customary to place the negative sign above the characteristic instead of placing it in front.

Sometimes a positive characteristic is used, in which case 10 times the exponent of the power lessened by 1 must be taken from the final characteristic, and the result added to -10.

Example. — Raise 0.0437 to the fourth power.

$$\begin{array}{lrcr} & \text{Neg. char.} & & \text{Pos. char.} \\ \text{Log. } 0.0437 & = \bar{2}.64048 & \text{or} & 8.64048 \\ & 4 & & 4 \\ \hline \text{Log. } 0.000003649 & = \bar{6}.56192 & & +\ 4.56192 \\ & & & -10 \\ \hline & & & \bar{6}.56192 \end{array}$$

The $\bar{6}$ was obtained when the negative characteristic was used by subtracting the 2 carried from the mantissa from the product of $-2 \times 4 = -8$.

In the case of the positive characteristic $4 \times 8 = 32$ and adding the 2 carried from the mantissa the result was 34. From this was subtracted $10 \times (4 - 1) = 30$. Then $+4 - 10 = -6$.

When extracting roots if the given number be a decimal, and its characteristic positive, subtract 1 from the index of

the root and add the remainder to the characteristic before dividing.

If the characteristic be negative, add to it the least number that will make the sum divisible by the index of the root; the quotient is the characteristic of the log. of the root. In dividing the mantissa, only the number added is to be considered as the characteristic.

Example. —

$$\begin{array}{ll} \sqrt[4]{0.00130321} & = ? \\ \text{Log. } 0.00130321 & = \bar{3}.11501 \\ \text{Add} & \quad 1 \\ \hline \text{Divide by } 4 & = \bar{1}.27875 \end{array}$$

The method by which the characteristic $\bar{1}$ was obtained is clearly seen. In dividing the mantissa the characteristic $\bar{3}$ was ignored and the 1 substituted.

To Work Proportion by Logarithms

Add the logarithms of the second and third terms together. From their sum subtract the logarithm of the first term. The remainder is the logarithm of the fourth term.

Or; add together the arithmetical complement of the first term and the logarithms of the other two. The sum, with 10 subtracted from the characteristic, is the logarithm of the answer.

Example. — Illustrating the two methods.

$$36 : 144 :: 28 : x$$

$$\begin{array}{lll} \text{Log. } 144 & = 2.15836 & \qquad 2.15836 \\ \text{Log. } 28 & = \underline{1.44716} & \qquad 1.44716 \\ & 3.60552 & \\ \text{Log. } 36 & = \underline{1.55630} \text{ a.c.} & = \underline{8.44370} \\ & 2.04922 & \qquad 2.04922 \end{array}$$

$$x = 112.$$

(The 10 is subtracted mentally.)

LOGARITHMIC FUNCTIONS OF ANGLES

Logarithmic functions of angles are merely logarithms of the natural functions. Characteristics are placed in the tables but as the natural functions are decimal fractions

the actual characteristics are negative. The numbers used as characteristics are positive for convenience, being the difference between 10 and the true, negative characteristic. Tables of logarithmic functions are for use with tables of logarithms of numbers.

Example. — Refer to example in Case III of Omissions, and compare the work required when using "naturals" and using "logs."

$$\text{Sin}\ \tfrac{1}{2}A = \sqrt{\frac{14.61 \times 7.51}{25.59 \times 32.69}}.$$

$$\begin{array}{lll}
\text{Log. } 14.61 = 1.16465 & & \\
\text{Log. } 7.51 = \underline{0.87564} & & 2.04029 \\
\text{Log. } 25.59 = 1.40807 & & \\
\text{Log. } 32.69 = \underline{1.51442} & & \\
\qquad 2.92249 \text{ a.c.} & = & \underline{7.07751} \\
 & & 9.11780 = \bar{1}.11780
\end{array}$$

$$2 \,\underline{\big|\, \bar{1}.11780 \atop 1}$$

$$\bar{1}.55890 = 9.55890 = \sin 21^\circ\ 14'$$

$$A = 21^\circ\ 14' \times 2 = 42^\circ\ 28'.$$

The student, for exercise, should work all the examples in this chapter by using logarithms. In this way a comparison can be made of the advantages and disadvantages of logarithms and their limitations in some kinds of work, bearing in mind the degree of accuracy possible with the small table of logs. in this book.

PROPORTIONAL PARTS IN LOGARITHMIC TABLES

The Five Place Logarithmic-Trigonometric Tables by Constantine Smoley, C.E., are printed on a thin, tough bond paper and are bound in flexible cloth. The price is 50 cents. They consist of logarithms of numbers from 1 to 10,000; Logarithms of the sine and tangent varying by ten seconds from 0° to 3° and of the cosine and cotangent varying by ten seconds from 87° to 90°; logarithms of the sine, cosine, tangent and cotangent, secant and cosecant for each minute of arc; tables of squares, cubes, square and cube roots, etc.

In the table of logarithms of numbers a star is frequently used as follows:

N.	0	1	2	3	4	5	6	7	8	9
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63,043	053	063	073	083	094	104	114	124	134

The mantissa is carried to five figures but the first two figures are omitted wherever possible in order to save space and make the tables more easily read. The mantissa of 4260 is 62,941, of 4265 it is 62,992, etc. The star in front of the mantissa for 4266 indicates that the first two figures are increased, so this mantissa is 63,002; the mantissa for 4268 is 63,022, etc.

On the right hand of each page is a column headed P.P. (Proportional Parts) in which figures appear as follows:

	10
1	1.0
2	2.0
3	3.0
4	4.0
5	5.0
6	6.0
7	7.0
8	8.0
9	9.0

Example. — Find the log. of 426,758.

The tabular difference between logs. of 4267 and 4268 = 10, which number heads the P.P. column.

$$\begin{array}{ll} \text{Log. of } 426{,}700 & = 5.63012 \\ \text{P.P. } 50 & = 0.000050 \\ \text{P.P. } 8 & = \underline{0.000008} \\ & \ 5.630178 \end{array}$$

(*Note.* — These values of P.P. for differences of 50 and 8 apply only to tab. diff. of 10.

When tab. diff. = 14 then P.P. for 5 = 7.0 and for 8 = 11.2, etc.)

Example. — Find number corresponding to log. 5.63018.

$$\begin{array}{rl} & \text{Log. } 5.63018 \\ \text{No. } 426{,}700 = & \text{Log. } \underline{5.63012} \\ & \quad\ \ 0.00006 \end{array}$$

The difference between log. of 426,700 and 426,800 = 10, so in the P.P. column find 10. On the right of the vertical line in the column find 6 and on the left of the line is found 6.0 which is placed after the 7 in the number and we thus obtain

No. 426,760 log. = 5.63018.

In the tables of logarithms of functions of angles the P.P. column refers to seconds. Few surveyors or engineers read angles closer than the nearest minute and few problems are encountered in which seconds appear in the angles. When the angles do contain seconds the proportional parts are used in the way just described for logarithms of numbers. For angles smaller than 3° the differences are so great between consecutive minutes that a separate table is given in which the values of the functions differ by 10″ and the P.P. column gives the differences for single seconds. With the explanation given, the student can with a little practice use the columns of proportional parts readily.

Natural Functions of Angles

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
0	0	0.000000	Infinite.	0.000000	Infinite.	1.00000	1.000000	0	90
	10	0.002909	343.77516	0.002909	343.77371	1.00000	0.999996	50	
	20	0.005818	171.88831	0.005818	171.88540	1.00002	0.999983	40	
	30	0.008727	114.59301	0.008727	114.58865	1.00004	0.999962	30	
	40	0.011635	85.945609	0.011636	85.939791	1.00007	0.999932	20	
	50	0.014544	68.757360	0.014545	68.750087	1.00011	0.999894	10	
1	0	0.017452	57.298688	0.017455	57.289962	1.00015	0.999848	0	89
	10	0.020361	49.114062	0.020365	49.103881	1.00021	0.999793	50	
	20	0.023269	42.975713	0.023275	42.964077	1.00027	0.999729	40	
	30	0.026177	38.201550	0.026186	38.188459	1.00034	0.999657	30	
	40	0.029085	34.382316	0.029097	34.367771	1.00042	0.999577	20	
	50	0.031992	31.257577	0.032009	31.241577	1.00051	0.999488	10	
2	0	0.034899	28.653708	0.034921	28.636253	1.00061	0.999391	0	88
	10	0.037806	26.450510	0.037834	26.431600	1.00072	0.999285	50	
	20	0.040713	24.562123	0.040747	24.541758	1.00083	0.999171	40	
	30	0.043619	22.925586	0.043661	22.903766	1.00095	0.999048	30	
	40	0.046525	21.493676	0.046576	21.470401	1.00108	0.998917	20	
	50	0.049431	20.230284	0.049491	20.205553	1.00122	0.998778	10	
3	0	0.052336	19.107323	0.052408	19.081137	1.00137	0.998630	0	87
	10	0.055241	18.102619	0.055325	18.074977	1.00153	0.998473	50	
	20	0.058145	17.198434	0.058243	17.169337	1.00169	0.998308	40	
	30	0.061049	16.380408	0.061163	16.349855	1.00187	0.998135	30	
	40	0.063952	15.636793	0.064083	15.604784	1.00205	0.997357	20	
	50	0.066854	14.957882	0.067004	14.924417	1.00224	0.997763	10	
4	0	0.069756	14.335587	0.069927	14.300666	1.00244	0.997564	0	86
	10	0.072658	13.763115	0.072851	13.726738	1.00265	0.997357	50	
	20	0.075559	13.234717	0.075776	13.196888	1.00287	0.997141	40	
	30	0.078459	12.745495	0.078702	12.706205	1.00309	0.996917	30	
	40	0.081359	12.291252	0.081629	12.250505	1.00333	0.996685	20	
	50	0.084258	11.868370	0.084558	11.826167	1.00357	0.996444	10	
5	0	0.087156	11.473713	0.087489	11.430052	1.00382	0.996195	0	85
	10	0.090053	11.104549	0.090421	11.059431	1.00408	0.995937	50	
	20	0.092950	10.758488	0.093354	10.711913	1.00435	0.995671	40	
	30	0.095846	10.433431	0.096289	10.385397	1.00463	0.995396	30	
	40	0.098741	10.127522	0.099226	10.078031	1.00491	0.995113	20	
	50	0.101635	9.8391227	0.102164	9.7881732	1.00521	0.994822	10	
6	0	0.104528	9.5667722	0.105104	9.5143645	1.00551	0.994522	0	84
	10	0.107421	9.3091699	0.108046	9.2553035	1.00582	0.994214	50	
	20	0.110313	9.0651512	0.110990	9.0098261	1.00614	0.993897	40	
	30	0.113203	8.8336715	0.113936	8.7768874	1.00647	0.993572	30	
	40	0.116093	8.6137901	0.116883	8.5555468	1.00681	0.993238	20	
	50	0.118982	8.4045586	0.119833	8.3449558	1.00715	0.992896	10	83
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 83° 10′ to 90° read from bottom of table upward.

NATURAL FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
7	0	0.121869	8.2055090	0.122785	8.1443464	1.00751	0.992546	0	83
	10	0.124756	8.0156450	0.125738	7.9530224	1.00787	0.992187	50	
	20	0.127642	7.8344335	0.128694	7.7703506	1.00825	0.991820	40	
	30	0.130526	7.6612976	0.131653	7.5957541	1.00863	0.991445	30	
	40	0.133410	7.4957100	0.134613	7.4287064	1.00902	0.991061	20	
	50	0.136292	7.3371909	0.137576	7.2687255	1.00942	0.990669	10	
8	0	0.139173	7.1852965	0.140541	7.1153697	1.00983	0.990268	0	82
	10	0.142053	7.0396220	0.143508	6.9682335	1.01024	0.989859	50	
	20	0.144932	6.8997942	0.146478	6.8269437	1.01067	0.989442	40	
	30	0.147809	6.7654691	0.149451	6.6911562	1.01111	0.989016	30	
	40	0.150686	6.6363293	0.152426	6.5605538	1.01155	0.988582	20	
	50	0.153561	6.5120812	0.155404	6.4348428	1.01200	0.988139	10	
9	0	0.156434	6.3924532	0.158384	6.3137515	1.01247	0.987688	0	81
	10	0.159307	6.2771933	0.161368	6.1970279	1.01294	0.987229	50	
	20	0.162178	6.1660674	0.164354	6.0844381	1.01342	0.986762	40	
	30	0.165048	6.0588980	0.167343	5.9757644	1.01391	0.986286	30	
	40	0.167916	5.9553625	0.170334	5.8708042	1.01440	0.985801	20	
	50	0.170783	5.8553921	0.173329	5.7693688	1.01491	0.985309	10	
10	0	0.173648	5.7587705	0.176327	5.6712818	1.01543	0.984808	0	80
	10	0.176512	5.6653331	0.179328	5.5763786	1.01595	0.984298	50	
	20	0.179375	5.5749258	0.182332	5.4845052	1.01649	0.983781	40	
	30	0.182236	5.4874043	0.185339	5.3955172	1.01703	0.983255	30	
	40	0.185095	5.4026333	0.188359	5.3092793	1.01758	0.982721	20	
	50	0.187953	5.3204860	0.191363	5.2256647	1.01815	0.982178	10	
11	0	0.190809	5.2408431	0.194380	5.1445540	1.01872	0.981627	0	79
	10	0.193664	5.1635924	0.197401	5.0658352	1.01930	0.981068	50	
	20	0.196517	5.0886284	0.200425	4.9894027	1.01989	0.980500	40	
	30	0.199368	5.0158317	0.203452	4.9151570	1.02049	0.979925	30	
	40	0.202218	4.9451687	0.206483	4.8430045	1.02110	0.979341	20	
	50	0.205065	4.8764907	0.209518	4.7728568	1.02171	0.978748	10	
12	0	0.207912	4.8097343	0.212557	4.7046301	1.02234	0.978148	0	78
	10	0.210756	4.7448206	0.215599	4.6382457	1.02298	0.977539	50	
	20	0.213599	4.6816748	0.218645	4.5736287	1.02362	0.976921	40	
	30	0.216440	4.6202263	0.221695	4.5107085	1.02428	0.976296	30	
	40	0.219279	4.5604080	0.224748	4.4494181	1.02494	0.975662	20	
	50	0.222116	4.5021565	0.227806	4.3896940	1.02562	0.975020	10	
13	0	0.224951	4.4454115	0.230868	4.3314759	1.02630	0.974370	0	77
	10	0.227784	4.3901158	0.233934	4.2747066	1.02700	0.973712	50	
	20	0.230616	4.3362150	0.237004	4.2193318	1.02770	0.973045	40	
	30	0.233445	4.2836576	0.240079	4.1652998	1.02842	0.972370	30	
	40	0.236273	4.2323943	0.243158	4.1125614	1.02914	0.971687	20	
	50	0.239098	4.1823785	0.246241	4.0610700	1.02987	0.970995	10	76
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 76° 10′ to 83° 00′ read from bottom of table upward.

NATURAL FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
14	0	0.241922	4.1335655	0.249328	4.0107809	1.03061	0.970296	0	76
	10	0.244743	4.0859130	0.252420	3.9616518	1.03137	0.969588	50	
	20	0.247563	4.0393804	0.255517	3.9136420	1.03213	0.968872	40	
	30	0.250380	3.9939292	0.258618	3.8667131	1.03290	0.968148	30	
	40	0.253195	3.9495224	0.261723	3.8208281	1.03363	0.967415	20	
	50	0.256008	3.9061250	0.264834	3.7759519	1.03447	0.966675	10	
15	0	0.258819	3.8637033	0.267949	3.7320508	1.03528	0.965926	0	75
	10	0.261628	3.8222251	0.271069	3.6890927	1.03609	0.965169	50	
	20	0.264434	3.7816596	0.274195	3.6470467	1.03691	0.964404	40	
	30	0.267238	3.7419775	0.277325	3.6058835	1.03774	0.963630	30	
	40	0.270040	3.7031506	0.280460	3.5655749	1.03858	0.962849	20	
	50	0.272840	3.6651518	0.283600	3.5260938	1.03944	0.962059	10	
16	0	0.275637	3.6279553	0.286745	3.4874144	1.04030	0.961262	0	74
	10	0.278432	3.5915363	0.289896	3.4495120	1.04117	0.960456	50	
	20	0.281225	3.5558710	0.293052	3.4123626	1.04206	0.959642	40	
	30	0.284015	3.5209365	0.296214	3.3759434	1.04295	0.958820	30	
	40	0.286803	3.4867110	0.299380	3.3402326	1.04385	0.957990	20	
	50	0.289589	3.4531735	0.302553	3.3052091	1.04477	0.957151	10	
17	0	0.292372	3.4203036	0.305731	3.2708526	1.04569	0.956305	0	73
	10	0.295152	3.3880820	0.308914	3.2371438	1.04663	0.955450	50	
	20	0.297930	3.3564900	0.312104	3.2040638	1.04757	0.954588	40	
	30	0.300706	3.3255095	0.315299	3.1715948	1.04853	0.953717	30	
	40	0.303479	3.2951234	0.318500	3.1397194	1.04950	0.952838	20	
	50	0.306249	3.2653149	0.321707	3.1084210	1.05047	0.951951	10	
18	0	0.309017	3.2360680	0.324920	3.0776835	1.05146	0.951057	0	72
	10	0.311782	3.2073673	0.328139	3.0474915	1.05246	0.950154	50	
	20	0.314545	3.1791978	0.331364	3.0178301	1.05347	0.949243	40	
	30	0.317305	3.1515453	0.334595	2.9886850	1.05449	0.948324	30	
	40	0.320062	3.1243959	0.337833	2.9600422	1.05552	0.947397	20	
	50	0.322816	3.0977363	0.341077	2.9318885	1.05657	0.946462	10	
19	0	0.325568	3.0715535	0.344328	2.9042109	1.05762	0.945519	0	71
	10	0.328317	3.0458352	0.347585	2.8769970	1.05869	0.944568	50	
	20	0.331063	3.0205693	0.350848	2.8502349	1.05976	0.943609	40	
	30	0.333807	2.9957443	0.354119	2.8239129	1.06085	0.942641	30	
	40	0.336547	2.9713490	0.357396	2.7980198	1.06195	0.941666	20	
	50	0.339285	2.9473724	0.360680	2.7725448	1.06306	0.940684	10	
20	0	0.342020	2.9238044	0.363970	2.7474774	1.06418	0.939693	0	70
	10	0.344752	2.9006346	0.367268	2.7228076	1.06531	0.938694	50	
	20	0.347481	2.8778532	0.370573	2.6985254	1.06645	0.937687	40	
	30	0.350207	2.8554510	0.373885	2.6746215	1.06761	0.936672	30	
	40	0.352931	2.8334185	0.377204	2.6510867	1.06878	0.935650	20	
	50	0.355651	2.8117471	0.380530	2.6279121	1.06995	0.934619	10	69
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 69° 10′ to 76° 00′ read from bottom of table upward.

NATURAL FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
21	0	0.358368	2.7904281	0.383864	2.6050891	1.07115	0.933580	0	69
	10	0.361082	2.7694532	0.387205	2.5826094	1.07235	0.932534	50	
	20	0.363793	2.7488144	0.390554	2.5604649	1.07356	0.931480	40	
	30	0.366501	2.7285038	0.393911	2.5386479	1.07479	0.930418	30	
	40	0.369206	2.7085139	0.397275	2.5171507	1.07602	0.929348	20	
	50	0.371908	2.6888374	0.400647	2.4959661	1.07727	0.928270	10	
22	0	0.374607	2.6694672	0.404026	2.4750869	1.07853	0.927184	0	68
	10	0.377302	2.6503962	0.407414	2.4545061	1.07981	0.926090	50	
	20	0.379994	2.6316180	0.410810	2.4342172	1.08109	0.924989	40	
	30	0.382683	2.6131259	0.414214	2.4142136	1.08239	0.923880	30	
	40	0.385369	2.5949137	0.417626	2.3944889	1.08370	0.922762	20	
	50	0.388052	2.5769753	0.421046	2.3750372	1.08503	0.921638	10	
23	0	0.390731	2.5593047	0.424475	2.3558524	1.08636	0.920505	0	67
	10	0.393407	2.5418961	0.427912	2.3369287	1.08771	0.919364	50	
	20	0.396080	2.5247440	0.431358	2.3182606	1.08907	0.918216	40	
	30	0.398749	2.5078428	0.434812	2.2998425	1.09044	0.917060	30	
	40	0.401415	2.4911874	0.438276	2.2816693	1.09183	0.915896	20	
	50	0.404078	2.4747726	0.441748	2.2637357	1.09323	0.914725	10	
24	0	0.406737	2.4585933	0.445229	2.2460368	1.09464	0.913545	0	66
	10	0.409392	2.4426448	0.448719	2.2285676	1.09606	0.912358	50	
	20	0.412045	2.4269222	0.452218	2.2113234	1.09750	0.911164	40	
	30	0.414693	2.4114210	0.455726	2.1942997	1.09895	0.909961	30	
	40	0.417338	2.3961367	0.459244	2.1774920	1.10041	0.908751	20	
	50	0.419980	2.3810650	0.462771	2.1608958	1.10189	0.907533	10	
25	0	0.422618	2.3662016	0.466308	2.1445069	1.10338	0.906308	0	65
	10	0.425253	2.3515424	0.469854	2.1283213	1.10488	0.905075	50	
	20	0.427884	2.3370833	0.473410	2.1123348	1.10640	0.903834	40	
	30	0.430511	2.3228205	0.476976	2.0965436	1.10793	0.902585	30	
	40	0.433135	2.3087501	0.480551	2.0809438	1.10947	0.901329	20	
	50	0.435755	2.2948685	0.484137	2.0655318	1.11103	0.900065	10	
26	0	0.438371	2.2811720	0.487733	2.0503038	1.11260	0.898794	0	64
	10	0.440984	2.2676571	0.491339	2.0352565	1.11419	0.897515	50	
	20	0.443593	2.2543204	0.494955	2.0203862	1.11579	0.896229	40	
	30	0.446198	2.2411585	0.498582	2.0056897	1.11740	0.894934	30	
	40	0.448799	2.2281681	0.502219	1.9911637	1.11903	0.893633	20	
	50	0.451397	2.2153460	0.505867	1.9768050	1.12067	0.892323	10	
27	0	0.453990	2.2026893	0.509525	1.9626105	1.12233	0.891007	0	63
	10	0.456580	2.1901947	0.513195	1.9485772	1.12400	0.889682	50	
	20	0.459166	2.1778595	0.516876	1.9347020	1.12568	0.888350	40	
	30	0.461749	2.1656806	0.520567	1.9209821	1.12738	0.887011	30	
	40	0.464327	2.1536553	0.524270	1.9074147	1.12910	0.885664	20	
	50	0.466901	2.1417808	0.527984	1.8939971	1.13083	0.884309	10	62
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 62° 10′ to 69° 00′ read from bottom of table upward.

NATURAL FUNCTIONS OF ANGLES (*Continued*).

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
28	0	0.469472	2.1300545	0.531709	1.8807265	1.13257	0.882948	0	62
	10	0.472038	2.1184737	0.535547	1.8676003	1.13433	0.881578	50	
	20	0.474600	2.1070359	0.539195	1.8546159	1.13610	0.880201	40	
	30	0.477159	2.0957385	0.542956	1.8417409	1.13789	0.878817	30	
	40	0.479713	2.0845792	0.546728	1.8290628	1.13970	0.877425	20	
	50	0.482263	2.0735556	0.550515	1.8164892	1.14152	0.876026	10	
29	0	0.484810	2.0626653	0.554309	1.8040478	1.14335	0.874620	0	61
	10	0.487352	2.0519061	0.558118	1.7917362	1.14521	0.873206	50	
	20	0.489890	2.0412757	0.561939	1.7795524	1.14707	0.871784	40	
	30	0.492424	2.0307720	0.565773	1.7674940	1.14896	0.870356	30	
	40	0.494953	2.0203929	0.569619	1.7555590	1.15085	0.868920	20	
	50	0.497479	2.0101362	0.573478	1.7437453	1.15277	0.867476	10	
30	0	0.500000	2.0000000	0.577350	1.7320508	1.15470	0.866025	0	60
	10	0.502517	1.9899822	0.581235	1.7204736	1.15665	0.864567	50	
	20	0.505030	1.9800810	0.585134	1.7090116	1.15861	0.863102	40	
	30	0.507538	1.9702944	0.589045	1.6976631	1.16059	0.861629	30	
	40	0.510043	1.9606206	0.592970	1.6864261	1.16259	0.860149	20	
	50	0.512543	1.9510577	0.596908	1.6752988	1.16460	0.858662	10	
31	0	0.515038	1.9416040	0.600861	1.6642795	1.16663	0.857167	0	59
	10	0.517529	1.9322578	0.604827	1.6533663	1.16868	0.855665	50	
	20	0.520016	1.9230173	0.608807	1.6425576	1.17075	0.854156	40	
	30	0.522499	1.9138809	0.612801	1.6318517	1.17283	0.852640	30	
	40	0.524977	1.9048469	0.616809	1.6212469	1.17493	0.851117	20	
	50	0.527450	1.8959138	0.620832	1.6107417	1.17704	0.849586	10	
32	0	0.529919	1.8870799	0.624869	1.6003345	1.17918	0.848048	0	58
	10	0.532384	1.8783438	0.628921	1.5900238	1.18133	0.846503	50	
	20	0.534844	1.8697040	0.632988	1.5798079	1.18350	0.844951	40	
	30	0.537300	1.8611590	0.637079	1.5696856	1.18569	0.843391	30	
	40	0.539751	1.8527073	0.641167	1.5596552	1.18790	0.841825	20	
	50	0.542197	1.8443476	0.645280	1.5497155	1.19012	0.840251	10	
33	0	0.544639	1.8360785	0.649408	1.5398650	1.19236	0.838671	0	57
	10	0.547076	1.8278985	0.653531	1.5301025	1.19463	0.837083	50	
	20	0.549509	1.8198065	0.657710	1.5204261	1.19691	0.835488	40	
	30	0.551937	1.8118010	0.661886	1.5108352	1.19920	0.833886	30	
	40	0.554360	1.8038809	0.666077	1.5013282	1.20152	0.832277	20	
	50	0.556779	1.7960449	0.670285	1.4919039	1.20386	0.830661	10	
34	0	0.559193	1.7882916	0.674509	1.4825610	1.20622	0.829038	0	56
	10	0.561602	1.7806201	0.678749	1.4732983	1.20859	0.827407	50	
	20	0.564007	1.7730290	0.683007	1.4641147	1.21099	0.825770	40	
	30	0.566406	1.7655173	0.687281	1.4550090	1.21341	0.824126	30	
	40	0.568801	1.7580837	0.691573	1.4459801	1.21584	0.822475	20	
	50	0.571191	1.7507273	0.695881	1.4370268	1.21830	0.820817	10	55
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 55° 10′ to 62° 00′ read from bottom of table upward.

NATURAL FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
35	0	0.573576	1.7434468	0.700208	1.4281480	1.22077	0.819152	0	55
	10	0.575957	1.7362413	0.704552	1.4193427	1.22327	0.817480	50	
	20	0.578332	1.7291096	0.708913	1.4106098	1.22579	0.815801	40	
	30	0.580703	1.7220508	0.713293	1.4019483	1.22833	0.814116	30	
	40	0.583069	1.7150639	0.717691	1.3933571	1.23089	0.812423	20	
	50	0.585429	1.7081478	0.722108	1.3848355	1.23347	0.810723	10	
36	0	0.587785	1.7013016	0.726543	1.3763810	1.23607	0.809017	0	54
	10	0.590136	1.6945244	0.730996	1.3679959	1.23869	0.807304	50	
	20	0.592482	1.6878151	0.735469	1.3596764	1.24134	0.805584	40	
	30	0.594823	1.6811730	0.739961	1.3514224	1.24400	0.803857	30	
	40	0.597159	1.6745970	0.744472	1.3432331	1.24669	0.802123	20	
	50	0.599489	1.6680864	0.749003	1.3351075	1.24940	0.800383	10	
37	0	0.601815	1.6616401	0.753554	1.3270448	1.25214	0.798636	0	53
	10	0.604136	1.6552575	0.758125	1.3190441	1.25489	0.796882	50	
	20	0.606451	1.6489376	0.762716	1.3111046	1.25767	0.795121	40	
	30	0.608761	1.6426796	0.767627	1.3032254	1.26047	0.793353	30	
	40	0.611067	1.6364828	0.771959	1.2954057	1.26330	0.791579	20	
	50	0.613367	1.6303462	0.776612	1.2876447	1.26615	0.789798	10	
38	0	0.615661	1.6242692	0.781286	1.2799416	1.26902	0.788011	0	52
	10	0.617951	1.6182510	0.785981	1.2722957	1.27191	0.786217	50	
	20	0.620235	1.6122908	0.790698	1.2647062	1.27483	0.784416	40	
	30	0.622515	1.6063879	0.795436	1.2571723	1.27778	0.782608	30	
	40	0.624789	1.6005416	0.800196	1.2496933	1.28075	0.780794	20	
	50	0.627057	1.5947511	0.804080	1.2422685	1.28374	0.778973	10	
39	0	0.629320	1.5890157	0.809784	1.2348972	1.28676	0.777146	0	51
	10	0.631578	1.5833318	0.814612	1.2275786	1.28980	0.775312	50	
	20	0.633831	1.5777077	0.819463	1.2203121	1.29287	0.773472	40	
	30	0.636078	1.5721337	0.824336	1.2130970	1.29597	0.771625	30	
	40	0.638320	1.5666121	0.829234	1.2059327	1.29909	0.769771	20	
	50	0.640557	1.5611424	0.834155	1.1988184	1.30223	0.767911	10	
40	0	0.642788	1.5557238	0.839100	1.1917536	1.30541	0.766044	0	50
	10	0.645013	1.5503558	0.844069	1.1847376	1.30861	0.764171	50	
	20	0.647233	1.5450378	0.849062	1.1777698	1.31183	0.762292	40	
	30	0.649448	1.5397690	0.854081	1.1708496	1.31509	0.760406	30	
	40	0.651657	1.5345491	0.859124	1.1639763	1.31837	0.758514	20	
	50	0.653861	1.5293773	0.864193	1.1571495	1.32168	0.756615	10	
41	0	0.656059	1.5242531	0.869287	1.1503684	1.32501	0.754710	0	49
	10	0.658252	1.5191759	0.874407	1.1436326	1.32838	0.752798	50	
	20	0.660439	1.5141452	0.879553	1.1369414	1.33177	0.750880	40	
	30	0.662620	1.5091605	0.884725	1.1302944	1.33519	0.748956	30	
	40	0.664796	1.5042211	0.889924	1.1236909	1.33864	0.747025	20	
	50	0.666966	1.4993267	0.895151	1.1171305	1.34212	0.745088	10	48
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 48° 10′ to 55° 00′ read from bottom of table upward.

NATURAL FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
42	0	0.669131	1.4944765	0.900404	1.1106125	1.34563	0.743145	0	48
	10	0.671289	1.4896703	0.905685	1.1041365	1.34917	0.741195	50	
	20	0.673443	1.4849073	0.910994	1.0977020	1.35274	0.739239	40	
	30	0.675590	1.4801872	0.916331	1.0913085	1.35634	0.737277	30	
	40	0.677732	1.4755095	0.921697	1.0849554	1.35997	0.735309	20	
	50	0.679868	1.4708736	0.927091	1.0786423	1.36363	0.733335	10	
43	0	0.681998	1.4662792	0.932515	1.0723687	1.36733	0.731354	0	47
	10	0.684123	1.4617257	0.937968	1.0661341	1.37105	0.729367	50	
	20	0.686242	1.4572127	0.943451	1.0599381	1.37481	0.727374	40	
	30	0.688355	1.4527397	0.948965	1.0537801	1.37860	0.725374	30	
	40	0.690462	1.4483063	0.954508	1.0476598	1.38242	0.723369	20	
	50	0.692563	1.4439120	0.960083	1.0415767	1.38628	0.721357	10	
44	0	0.694658	1.4395565	0.965689	1.0355303	1.39016	0.719340	0	46
	10	0.696748	1.4352393	0.971326	1.0295203	1.39409	0.717316	50	
	20	0.698832	1.4309602	0.976996	1.0235461	1.39804	0.715286	40	
	30	0.700909	1.4267182	0.982697	1.0176074	1.40203	0.713251	30	
	40	0.702981	1.4225134	0.988432	1.0117088	1.40606	0.711209	20	
	50	0.705047	1.4183454	0.994199	1.0058348	1.41012	0.709161	10	
45	0	0.707107	1.4142136	1.000000	1.0000000	1.41421	0.707107	0	45
Deg.	Min.	Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.	Min.	Deg.

For functions from 45° 0′ to 48° 00′ read from bottom of table upward.

Logarithms of Numbers from 0 to 1000

No.	0	1	2	3	4	5	6	7	8	9
0	0	00000	30103	47712	60206	69897	77815	84510	90309	95424
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720
33	51851	51983	52114	52244	52375	52504	52633	52763	52892	53020
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864
38	57978	58093	58206	58320	58433	58546	58659	58771	58883	58995
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957

LOGARITHMS OF NUMBERS FROM 0 TO 1000 (*Continued*)

No.	0	1	2	3	4	5	6	7	8	9
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224
65	81291	81358	81425	81491	81558	81624	81690	81756	81823	81889
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448
75	87506	87564	87622	87680	87737	87795	87852	87910	87967	88024
76	88081	88138	88196	88252	88309	88366	88423	88480	88536	88593
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957

Logarithmic Functions of Angles

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
0	0	— ∞	∞	— ∞	∞	10.00000	10.00000	0	**90**
	10	7.46373	12.53627	7.46373	12.53627	10.00000	9.99999	50	
	20	7.76475	12.23525	7.76476	12.23524	10.00000	9.99999	40	
	30	7.94084	12.05916	7.94086	12.05914	10.00002	9.99998	30	
	40	8.06578	11.93422	8.06581	11.93419	10.00003	9.99997	20	
	50	8.16268	11.83732	8.16272	11.83727	10.00005	9.99995	10	
1	0	8.24186	11.75814	8.24192	11.75808	10.00007	9.99993	0	**89**
	10	8.30879	11.69121	8.30888	11.69111	10.00009	9.99991	50	
	20	8.36678	11.63322	8.36689	11.63311	10.00012	9.99988	40	
	30	8.41792	11.58208	8.41807	11.58193	10.00015	9.99985	30	
	40	8.46366	11.53634	8.46385	11.53615	10.00018	9.99982	20	
	50	8.50504	11.49496	8.50527	11.49473	10.00022	9.99978	10	
2	0	8.54282	11.45718	8.54308	11.45692	10.00026	9.99974	0	**88**
	10	8.57757	11.42243	8.57788	11.42212	10.00031	9.99969	50	
	20	8.60973	11.39027	8.61009	11.38990	10.00036	9.99964	40	
	30	8.63968	11.36032	8.64009	11.35991	10.00041	9.99959	30	
	40	8.66769	11.33231	8.66816	11.33184	10.00047	9.99953	20	
	50	8.69400	11.30600	8.69453	11.30547	10.00053	9.99947	10	
3	0	8.71880	11.28120	8.71940	11.28060	10.00060	9.99940	0	**87**
	10	8.74226	11.25774	8.74292	11.25708	10.00066	9.99934	50	
	20	8.76451	11.23549	8.76525	11.23475	10.00074	9.99926	40	
	30	8.78567	11.21432	8.78648	11.21351	10.00081	9.99919	30	
	40	8.80585	11.19415	8.80674	11.19326	10.00089	9.99911	20	
	50	8.82513	11.17487	8.82610	11.17390	10.00097	9.99903	10	
4	0	8.84358	11.15642	8.84464	11.15536	10.00106	9.99894	0	**86**
	10	8.86128	11.13872	8.86243	11.13757	10.00115	9.99885	50	
	20	8.87829	11.12171	8.87953	11.12047	10.00124	9.99876	40	
	30	8.89464	11.10536	8.89598	11.10402	10.00134	9.99866	30	
	40	8.91040	11.08960	8.91185	11.08815	10.00144	9.99856	20	
	50	8.92561	11.07439	8.92716	11.07284	10.00155	9.99845	10	
5	0	8.94030	11.05970	8.94195	11.05805	10.00166	9.99834	0	**85**
	10	8.95450	11.04550	8.95627	11.04373	10.00177	9.99823	50	
	20	8.96825	11.03175	8.97013	11.02987	10.00188	9.99812	40	
	30	8.98157	11.01843	8.98358	11.01642	10.00200	9.99800	30	
	40	8.99450	11.00550	8.99662	11.00337	10.00213	9.99787	20	
	50	9.00704	10.99296	9.00930	10.99070	10.00225	9.99775	10	
6	0	9.01923	10.98076	9.02162	10.97838	10.00239	9.99761	0	**84**
	10	9.03109	10.96891	9.03361	10.96639	10.00252	9.99748	50	
	20	9.04262	10.95738	9.04528	10.95472	10.00266	9.99734	40	
	30	9.05386	10.94614	9.05666	10.94334	10.00280	9.99720	30	
	40	9.06480	10.93519	9.06775	10.93225	10.00295	9.99705	20	
	50	9.07548	10.92452	9.07858	10.92142	10.00310	9.99690	10	**83**
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 83° 10′ to 90° 00′ read from bottom of table upward.

Logarithmic Functions of Angles (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
7	0	9.08589	10.91411	9.08914	10.91086	10.00325	9.99675	0	83
	10	9.09606	10.90394	9.09947	10.90053	10.00341	9.99659	50	
	20	9.10599	10.89401	9.10956	10.89044	10.00357	9.99643	40	
	30	9.11570	10.88430	9.11943	10.88057	10.00373	9.99627	30	
	40	9.12519	10.87481	9.12909	10.87091	10.00390	9.99610	20	
	50	9.13447	10.86553	9.13854	10.86146	10.00407	9.99593	10	
8	0	9.14356	10.85644	9.14780	10.85220	10.00425	9.99575	0	82
	10	9.15245	10.84755	9.15688	10.84312	10.00443	9.99557	50	
	20	9.16116	10.83884	9.16577	10.83423	10.00461	9.99539	40	
	30	9.16970	10.83030	9.17450	10.82550	10.00480	9.99520	30	
	40	9.17807	10.82193	9.18306	10.81694	10.00499	9.99501	20	
	50	9.18628	10.81372	9.19146	10.80854	10.00518	9.99482	10	
9	0	9.19433	10.80567	9.19971	10.80029	10.00538	9.99462	0	81
	10	9.20223	10.79777	9.20782	10.79218	10.00558	9.99442	50	
	20	9.20999	10.79001	9.21578	10.78422	10.00579	9.99421	40	
	30	9.21761	10.78239	9.22361	10.77639	10.00600	9.99400	30	
	40	9.22509	10.77491	9.23130	10.76870	10.00621	9.99379	20	
	50	9.23244	10.76756	9.23887	10.76113	10.00643	9.99357	10	
10	0	9.23967	10.76033	9.24632	10.75368	10.00665	9.99335	0	80
	10	9.24677	10.75323	9.25365	10.74635	10.00687	9.99313	50	
	20	9.25376	10.74624	9.26086	10.73914	10.00710	9.99290	40	
	30	9.26063	10.73937	9.26797	10.73203	10.00733	9.99267	30	
	40	9.26739	10.73261	9.27496	10.72503	10.00757	9.99243	20	
	50	9.27405	10.72595	9.28186	10.71814	10.00781	9.99219	10	
11	0	9.28060	10.71940	9.28865	10.71135	10.00805	9.99195	0	79
	10	9.28705	10.71295	9.29535	10.70465	10.00830	9.99170	50	
	20	9.29340	10.70660	9.30195	10.69805	10.00855	9.99145	40	
	30	9.29966	10.70034	9.30846	10.69154	10.00881	9.99119	30	
	40	9.30582	10.69418	9.31489	10.68511	10.00901	9.99093	20	
	50	9.31189	10.68811	9.32122	10.67878	10.00933	9.99067	10	
12	0	9.31788	10.68212	9.32747	10.67253	10.00960	9.99040	0	78
	10	9.32378	10.67622	9.33365	10.66635	10.00987	9.99013	50	
	20	9.32960	10.67040	9.33974	10.66026	10.01014	9.98986	40	
	30	9.33534	10.66466	9.34576	10.65424	10.01042	9.98958	30	
	40	9.34100	10.65900	9.35170	10.64830	10.01070	9.98930	20	
	50	9.34658	10.65342	9.35757	10.64243	10.01099	9.98901	10	
13	0	9.35209	10.64791	9.36336	10.63664	10.01128	9.98872	0	77
	10	9.35752	10.64248	9.36909	10.63091	10.01157	9.98843	50	
	20	9.36289	10.63711	9.37476	10.62524	10.01187	9.98813	40	
	30	9.36819	10.63181	9.38035	10.61965	10.01217	9.98783	30	
	40	9.37341	10.62659	9.38589	10.61411	10.01247	9.98752	20	
	50	9.37858	10.62142	9.39136	10.60864	10.01278	9.98722	10	76
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 76° 10′ to 83° 00′ read from bottom of table upward.

LOGARITHMIC FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
14	0	9.38368	10.61632	9.39677	10.60323	10.01310	9.98690	0	76
	10	9.38871	10.61129	9.40212	10.59788	10.01341	9.98659	50	
	20	9.39369	10.60631	9.40742	10.59258	10.01373	9.98627	40	
	30	9.39860	10.60140	9.41266	10.58734	10.01406	9.98594	30	
	40	9.40346	10.59654	9.41784	10.58216	10.01439	9.98561	20	
	50	9.40825	10.59175	9.42297	10.57703	10.01472	9.98528	10	
15	0	9.41300	10.58700	9.42805	10.57195	10.01506	9.98494	0	75
	10	9.41768	10.58232	9.43308	10.56692	10.01540	9.98460	50	
	20	9.42232	10.57768	9.43806	10.56194	10.01574	9.98426	40	
	30	9.42690	10.57310	9.44299	10.55701	10.01609	9.98391	30	
	40	9.43143	10.56857	9.44787	10.55213	10.01644	9.98356	20	
	50	9.43590	10.56409	9.45271	10.54729	10.01680	9.98320	10	
16	0	9.44034	10.55966	9.45750	10.54250	10.01716	9.98284	0	74
	10	9.44472	10.55528	9.46224	10.53776	10.01752	9.98248	50	
	20	9.44905	10.55095	9.46694	10.53305	10.01789	9.98211	40	
	30	9.45334	10.54666	9.47160	10.52840	10.01826	9.98174	30	
	40	9.45758	10.54242	9.47622	10.52378	10.01864	9.98136	20	
	50	9.46178	10.53822	9.48080	10.51920	10.01902	9.98098	10	
17	0	9.46594	10.53406	9.48534	10.51466	10.01940	9.98060	0	73
	10	9.47005	10.52995	9.48984	10.51016	10.01979	9.98021	50	
	20	9.47411	10.52589	9.49430	10.50570	10.02018	9.97982	40	
	30	9.47814	10.52186	9.49872	10.50128	10.02058	9.97942	30	
	40	9.48213	10.51787	9.50311	10.49689	10.02098	9.97902	20	
	50	9.48607	10.51393	9.50746	10.49254	10.02138	9.97861	10	
18	0	9.48998	10.51002	9.51178	10.48822	10.02179	9.97821	0	72
	10	9.49385	10.50615	9.51606	10.48394	10.02221	9.97779	50	
	20	9.49768	10.50232	9.52031	10.47969	10.02262	9.97738	40	
	30	9.50148	10.49852	9.52452	10.47548	10.02304	9.97696	30	
	40	9.50523	10.49477	9.52870	10.47130	10.02347	9.97653	20	
	50	9.50896	10.49104	9.53285	10.46715	10.02390	9.97610	10	
19	0	9.51264	10.48736	9.53697	10.46303	10.02433	9.97567	0	71
	10	9.51629	10.48371	9.54106	10.45894	10.02477	9.97523	50	
	20	9.51991	10.48009	9.54512	10.45488	10.02521	9.97479	40	
	30	9.52350	10.47650	9.54915	10.45085	10.02565	9.97435	30	
	40	9.52705	10.47295	9.55315	10.44685	10.02610	9.97390	20	
	50	9.53057	10.46944	9.55712	10.44288	10.02656	9.97344	10	
20	0	9.53405	10.46595	9.56107	10.43893	10.02701	9.97299	0	70
	10	9.53751	10.46249	9.56498	10.43502	10.02748	9.97252	50	
	20	9.54093	10.45907	9.56887	10.43113	10.02794	9.97206	40	
	30	9.54433	10.45567	9.57274	10.42726	10.02841	9.97159	30	
	40	9.54769	10.45231	9.57658	10.42342	10.02889	9.97111	20	
	50	9.55102	10.44898	9.58039	10.41961	10.02936	9.97063	10	69
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 69° 10′ to 76° 00′ read from bottom of table upward.

LOGARITHMIC FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
21	0	9.55433	10.44567	9.58418	10.41582	10.02985	9.97015	0	69
	10	9.55761	10.44239	9.58794	10.41206	10.03034	9.96966	50	
	20	9.56085	10.43915	9.59168	10.40832	10.03083	9.96917	40	
	30	9.56408	10.43592	9.59540	10.40460	10.03132	9.96868	30	
	40	9.56727	10.43273	9.59909	10.40091	10.03182	9.96818	20	
	50	9.57044	10.42956	9.60276	10.39724	10.03233	9.96767	10	
22	0	9.57358	10.42642	9.60641	10.39359	10.03283	9.96717	0	68
	10	9.57669	10.42331	9.61004	10.38996	10.03335	9.96665	50	
	20	9.57978	10.42022	9.61364	10.38636	10.03386	9.96614	40	
	30	9.58284	10.41716	9.61722	10.38278	10.03438	9.96562	30	
	40	9.58588	10.41412	9.62079	10.37921	10.03491	9.96509	20	
	50	9.58889	10.41111	9.62433	10.37567	10.03544	9.96456	10	
23	0	9.59188	10.40812	9.62785	10.37215	10.03597	9.96403	0	67
	10	9.59484	10.40516	9.63135	10.36865	10.03651	9.96349	50	
	20	9.59778	10.40222	9.63484	10.36516	10.03706	9.96294	40	
	30	9.60070	10.39930	9.63830	10.36170	10.03760	9.96240	30	
	40	9.60359	10.39641	9.64175	10.35825	10.03815	9.96185	20	
	50	9.60646	10.39354	9.64517	10.35483	10.03871	9.96129	10	
24	0	9.60931	10.39069	9.64858	10.35142	10.03927	9.96073	0	66
	10	9.61214	10.38786	9.65197	10.34803	10.03983	9.96017	50	
	20	9.61494	10.38506	9.65535	10.34465	10.04040	9.95960	40	
	30	9.61773	10.38227	9.65870	10.34130	10.04098	9.95902	30	
	40	9.62049	10.37951	9.66204	10.33795	10.04156	9.95845	20	
	50	9.62323	10.37677	9.66537	10.33463	10.04214	9.95786	10	
25	0	9.62595	10.37405	9.66867	10.33133	10.04272	9.95728	0	65
	10	9.62865	10.37135	9.67196	10.32804	10.04332	9.95668	50	
	20	9.63133	10.36867	9.67524	10.32476	10.04391	9.95609	40	
	30	9.63398	10.36602	9.67850	10.32150	10.04451	9.95549	30	
	40	9.63662	10.36338	9.68174	10.31826	10.04512	9.95488	20	
	50	9.63924	10.36076	9.68497	10.31503	10.04573	9.95427	10	
26	0	9.64184	10.35816	9.68818	10.31182	10.04634	9.95366	0	64
	10	9.64442	10.35558	9.69138	10.30862	10.04696	9.95304	50	
	20	9.64698	10.35302	9.69457	10.30543	10.04758	9.95242	40	
	30	9.64953	10.35047	9.69774	10.30226	10.04821	9.95179	30	
	40	9.65205	10.34795	9.70089	10.29911	10.04884	9.95116	20	
	50	9.65456	10.34544	9.70404	10.29596	10.04948	9.95052	10	
27	0	9.65705	10.34295	9.70717	10.29283	10.05012	9.94988	0	63
	10	9.65952	10.34048	9.71028	10.28972	10.05077	9.94923	50	
	20	9.66197	10.33803	9.71338	10.28661	10.05142	9.94858	40	
	30	9.66441	10.33559	9.71648	10.28352	10.05207	9.94793	30	
	40	9.66682	10.33318	9.71955	10.28045	10.05273	9.94727	20	
	50	9.66923	10.33078	9.72262	10.27738	10.05340	9.94660	10	62
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 62° 10′ to 69° 00′ read from bottom of table upward.

Logarithmic Functions of Angles (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
28	0	9.67161	10.32839	9.72567	10.27433	10.05406	9.94593	0	62
	10	9.67398	10.32602	9.72872	10.27128	10.05474	9.94526	50	
	20	9.67633	10.32367	9.73175	10.26825	10.05542	9.94458	40	
	30	9.67866	10.32134	9.73476	10.26524	10.05610	9.94390	30	
	40	9.68098	10.31902	9.73777	10.26223	10.05679	9.94321	20	
	50	9.68328	10.31672	9.74077	10.25923	10.05748	9.94252	10	
29	0	9.68557	10.31443	9.74375	10.25625	10.05818	9.94182	0	61
	10	9.68784	10.31216	9.74673	10.25327	10.05888	9.94112	50	
	20	9.69010	10.30990	9.74969	10.25031	10.05959	9.94041	40	
	30	9.69234	10.30766	9.75264	10.24736	10.06030	9.93970	30	
	40	9.69456	10.30544	9.75558	10.24442	10.06102	9.93898	20	
	50	9.69677	10.30323	9.75852	10.24148	10.06174	9.93826	10	
30	0	9.69897	10.30103	9.76144	10.23856	10.06247	9.93753	0	60
	10	9.70115	10.29885	9.76435	10.23565	10.06320	9.93680	50	
	20	9.70332	10.29668	9.76726	10.23275	10.06394	9.93606	40	
	30	9.70547	10.29453	9.77015	10.22985	10.06468	9.93532	30	
	40	9.70761	10.29239	9.77303	10.22697	10.06543	9.93457	20	
	50	9.70973	10.29027	9.77591	10.22409	10.06618	9.93382	10	
31	0	9.71184	10.28816	9.77877	10.22123	10.06693	9.93307	0	59
	10	9.71393	10.28607	9.78163	10.21837	10.06770	9.93230	50	
	20	9.71602	10.28398	9.78448	10.21552	10.06846	9.93154	40	
	30	9.71809	10.28191	9.78732	10.21268	10.06923	9.93077	30	
	40	9.72014	10.27986	9.79015	10.20985	10.07001	9.92999	20	
	50	9.72218	10.27782	9.79297	10.20703	10.07079	9.92921	10	
32	0	9.72421	10.27579	9.79579	10.20421	10.07158	9.92842	0	58
	10	9.72622	10.27378	9.79860	10.20140	10.07237	9.92763	50	
	20	9.72823	10.27177	9.80140	10.19860	10.07317	9.92683	40	
	30	9.73022	10.26978	9.80419	10.19581	10.07397	9.92603	30	
	40	9.73219	10.26781	9.80697	10.19303	10.07478	9.92522	20	
	50	9.73416	10.26584	9.80975	10.19025	10.07559	9.92441	10	
33	0	9.73611	10.26389	9.81252	10.18748	10.07641	9.92359	0	57
	10	9.73805	10.26195	9.81528	10.18472	10.07723	9.92277	50	
	20	9.73997	10.26003	9.81803	10.18197	10.07806	9.921194	40	
	30	9.74189	10.25811	9.82078	10.17922	10.07889	9.92111	30	
	40	9.74379	10.25621	9.82352	10.17648	10.07973	9.92027	20	
	50	9.74568	10.25432	9.82626	10.17374	10.08058	9.91942	10	
34	0	9.74756	10.25244	9.82899	10.17101	10.08143	9.91857	0	56
	10	9.74943	10.25057	9.83171	10.16829	10.08228	9.91772	50	
	20	9.75128	10.24872	9.83442	10.16558	10.08314	9.91686	40	
	30	9.75313	10.24687	9.83713	10.16287	10.08401	9.91599	30	
	40	9.75496	10.24504	9.83984	10.16016	10.08488	9.91512	20	
	50	9.75678	10.24322	9.84254	10.15746	10.08575	9.91425	10	55
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 55° 10′ to 62° 00′ read from bottom of table upward.

LOGARITHMIC FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
35	0	9.75859	10.24141	9.84523	10.15477	10.08664	9.91336	0	**55**
	10	9.76039	10.23961	9.84791	10.15209	10.08752	9.91248	50	
	20	9.76218	10.23782	9.85059	10.14941	10.08842	9.91158	40	
	30	9.76395	10.23604	9.85327	10.14673	10.08931	9.91069	30	
	40	9.76572	10.23428	9.85594	10.14406	10.09022	9.90978	20	
	50	9.76747	10.23253	9.85860	10.14140	10.09113	9.90887	10	
36	0	9.76922	10.23078	9.86126	10.13874	10.09204	9.90796	0	**54**
	10	9.77095	10.22905	9.86392	10.13608	10.09296	9.90704	50	
	20	9.77268	10.22732	9.86656	10.13344	10.09389	9.90611	40	
	30	9.77439	10.22561	9.86921	10.13079	10.09482	9.90518	30	
	40	9.77609	10.22391	9.87185	10.12815	10.09576	9.90424	20	
	50	9.77778	10.22222	9.87448	10.12552	10.09670	9.90330	10	
37	0	9.77946	10.22054	9.87711	10.12289	10.09765	9.90235	0	**53**
	10	9.78113	10.21887	9.87974	10.12026	10.09861	9.90139	50	
	20	9.78280	10.21720	9.88236	10.11764	10.09957	9.90043	40	
	30	9.78445	10.21555	9.88498	10.11502	10.10053	9.89947	30	
	40	9.78609	10.21391	9.88759	10.11241	10.10151	9.89849	20	
	50	9.78772	10.21228	9.89020	10.10980	10.10248	9.89752	10	
38	0	9.78934	10.21066	9.89281	10.10719	10.10347	9.89653	0	**52**
	10	9.79095	10.20905	9.89541	10.10459	10.10446	9.89554	50	
	20	9.79256	10.20744	9.89801	10.10199	10.10545	9.89455	40	
	30	9.79415	10.20585	9.90061	10.09939	10.10646	9.89354	30	
	40	9.79573	10.20427	9.90320	10.09680	10.10746	9.89254	20	
	50	9.79731	10.20269	9.90578	10.09422	10.10848	9.89152	10	
39	0	9.79887	10.20113	9.90837	10.09163	10.10950	9.89050	0	**51**
	10	9.80043	10.19957	9.91095	10.08905	10.11052	9.88948	50	
	20	9.80197	10.19803	9.91353	10.08647	10.11156	9.88844	40	
	30	9.80351	10.19649	9.91610	10.08390	10.11259	9.88741	30	
	40	9.80504	10.19496	9.91868	10.08132	10.11364	9.88636	20	
	50	9.80656	10.19344	9.92125	10.07875	10.11469	9.88531	10	
40	0	9.80807	10.19193	9.92381	10.07619	10.11575	9.88425	0	**50**
	10	9.80957	10.19043	9.92638	10.07362	10.11681	9.88319	50	
	20	9.81106	10.18894	9.92894	10.07106	10.11788	9.88212	40	
	30	9.81254	10.18746	9.93150	10.06850	10.11895	9.88105	30	
	40	9.81402	10.18598	9.93406	10.06594	10.12004	9.87996	20	
	50	9.81548	10.18451	9.93661	10.06339	10.12113	9.87887	10	
41	0	9.81694	10.18306	9.93916	10.06084	10.12222	9.87778	0	**49**
	10	9.81839	10.18161	9.94171	10.05829	10.12332	9.87668	50	
	20	9.81983	10.18017	9.94426	10.05574	10.12443	9.87557	40	
	30	9.82126	10.17874	9.94681	10.05319	10.12554	9.87446	30	
	40	9.82269	10.17731	9.94935	10.05065	10.12666	9.87334	20	
	50	9.82410	10.17590	9.95190	10.04810	10.12779	9.87221	10	**48**
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 48° 10′ to 55° 00′ read from bottom of table upward

LOGARITHMIC FUNCTIONS OF ANGLES (*Continued*)

Deg.	Min.	Sine.	Cosec.	Tang.	Cotang.	Sec.	Cosine.	Min.	Deg.
42	0	9.82551	10.17449	9.95444	10.04556	10.12893	9.87107	0	48
	10	9.82691	10.17309	9.95698	10.04302	10.13007	9.86993	50	
	20	9.82830	10.17170	9.95952	10.04048	10.13121	9.86879	40	
	30	9.82968	10.17032	9.96205	10.03795	10.13237	9.86763	30	
	40	9.83106	10.16894	9.96459	10.03541	10.13353	9.86647	20	
	50	9.83242	10.16758	9.96712	10.03288	10.13470	9.86530	10	
43	0	9.83378	10.16622	9.96966	10.03034	10.13587	9.86413	0	47
	10	9.83513	10.16487	9.97219	10.02781	10.13705	9.86295	50	
	20	9.83648	10.16352	9.97472	10.02528	10.13824	9.86176	40	
	30	9.83781	10.16219	9.97725	10.02275	10.13944	9.86056	30	
	40	9.83914	10.16086	9.97978	10.02022	10.14064	9.85936	20	
	50	9.84046	10.15954	9.98231	10.01769	10.14185	9.85815	10	
44	0	9.84177	10.15823	9.98484	10.01516	10.14307	9.85693	0	46
	10	9.84308	10.15692	9.98737	10.01263	10.14429	9.85571	50	
	20	9.84437	10.15563	9.98989	10.01011	10.14552	9.85448	40	
	30	9.84566	10.15434	9.99242	10.00758	10.14676	9.85324	30	
	40	9.84694	10.15306	9.99495	10.00505	10.14800	9.85200	20	
	50	9.84822	10.15178	9.99747	10.00253	10.14926	9.85074	10	
45		9.84949	10.15052	10.00000	10.00000	10.15052	9.84949		45
		Cosine.	Sec.	Cotang.	Tang.	Cosec.	Sine.		

For functions from 45° 00′ to 48° 00′ read from bottom of table upward.

CHAPTER VI

TRANSIT SURVEYING

Surveying was an established calling many centuries before the compass was known, there being a well-developed system of mensuration in Egypt in the time of Joseph. Nothing however is known of the instruments used in the most ancient times.

In the second century B.C. there lived in Alexandria Heron the Elder, a mathematician and practical surveyor. He has been styled "The first engineer," because of a number of inventions, one being the æolipile, the first steam engine. A book entitled "Dioptra," the first known treatise on surveying, is supposed to have been written by him although some writers believe it to be the work of a later writer of the same name. The "Dioptra" is a treatise on the use of the diopter, a surveying instrument in common use up to the end of the Middle Ages. With this instrument the Romans laid out their cities, roads, aqueducts and all public works.

Venturi wrote: "Dioptra were instruments resembling the modern theodolites. The instrument consisted of a rod, four yards long, with little plates at the end for aiming. This rested upon a circular disk. The rod could be moved horizontally and also vertically. By turning the rod around until stopped by two suitably located pins on the circular disk, the surveyor could work off a line perpendicular to a given direction. The level and plumb line were also used." From an illustration given by Heron of a simple diopter and from an illustration given by Venturi of a later type it is evident that the instrument was merely a large surveyors' cross for setting out perpendiculars. At what date the plate was graduated so other angles could be set off no one seems to know. The compass was not long in use before a needle was placed on the plate of the

diopter, later followed by two concentric plates so that angles could be read independently of the needle. Transversals were used to subdivide the graduations to enable fine readings to be taken.

About the year 1608 a Dutch spectacle maker named Lippershey discovered the principle of the telescope and Galileo, in 1609, made the first telescope. In 1631 the vernier was invented and in 1640 cross-hairs were used to define the optical axis, or line of sight, of a telescope. When the vernier was used to read the graduated circles and the improved telescope took the place of the sighting disks, the diopter became a theodolite. This instrument seems to have been first mentioned in print about the year 1674. The first English telescopic theodolite is believed to have been made in 1723. Fig. 180, copied from an old edition of Davies' Surveying, shows the type in use in 1835. The etymology of the word is doubtful, some writers believing it to come from three Greek words meaning "to see a way plainly," while others believe it to have been named as a compliment to M. Theodolus, a French mathematician who wrote a treatise concerning its use, and who may have been the man who was responsible for the final form the diopter assumed before the name was changed. The first may have been called "Dioptra Theodolus."

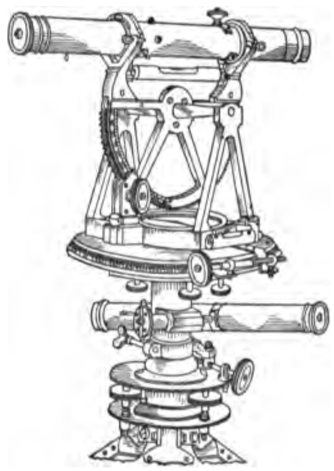


FIG. 180. Cradle theodolite.

The theodolite was not well adapted to the work required of a surveying instrument in America and the compass held sway until the commencement of steam railway construction. The graduated plates with verniers made the theodolite a good instrument for laying out railway curves, a work for which the compass was plainly not fit. The telescope with its cross-hairs enabled points to

be set with an accuracy equal to that of the graduations but rapid work could not be done because the telescope was mounted in wyes (cradles) and could only be reversed for backsights by changing it in the wyes end for end. In 1831 Mr. Young, an instrument maker in Philadelphia, applied the principle of the astronomical transit instrument to a mounting for a theodolite telescope and the "Portable American Transit for Engineers" appeared. It is the standard surveying instrument in America today for everything but work of the highest character. On such work theodolites are used, the word in the United States being limited to high-grade surveying instruments without a compass, the omission of the needle permitting of great rigidity in the mounting of the telescope. Few, if any, "cradle" theodolites are made. In Europe the word theodolite is used to describe all surveying instruments having graduated horizontal and vertical circles, with and without needles, and regardless of how the telescopes are mounted.

The transit is mounted on a vertical compound center. That is, the vertical shaft has a central solid spindle fitting in an outer one which works in a deep socket attached to the leveling device. The outer portion of the vertical center carries a horizontal disk with a circumferential band of silver on top, the inner edge of the band being graduated. The solid spindle carries a horizontal disk extending over the graduated ring. Upon this covering plate is placed the standards carrying the telescope, verniers for reading the graduations closely and a compass.

The compass needle is as long as possible, for much work is done with the needle in sections of the country where land is cheap. The compass is always fitted with a screw for lifting the needle and is usually supplied with a variation plate. The compass box occupies much space and it is necessary to place the telescope standards near the edge of the plate. This lessens the stability somewhat, in the opinion of hypercritical persons, but not enough to affect much of the work done by the majority of engineers and surveyors. For high-grade city surveying the standards are U-shape, thus bringing them closer to the center at the base, and a trough compass is used. The trough com-

pass consists of a needle mounted in a narrow box so it can swing only a few degrees to the right or left of the meridian. A needle thus mounted serves to set the line of sight in the magnetic meridian, from which base line other courses are run wholly by angles read on the horizontal plate.

In Fig. 181 is shown a typical American transit used for most of the work done by engineers and surveyors. The transit is attached to a tripod, the upper end of which is shown.

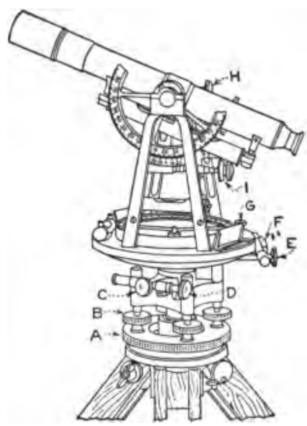


FIG. 181. Modern American transit.

A. The ring containing the threads for attaching the transit to the tripod. It is often called the "lower screw plate."

B. The leveling screws. These screws work in dustproof screw caps attached to a plate, made in the shape of a cross for lightness. This cross, "upper screw plate," contains the socket in which the instrument center spindle works. On the lower end of this socket is attached a ball joint working in a plate underneath the lower screw plate. The hole in this plate is much larger than the socket stem so by loosening the screws the entire instrument may be moved from one side to the other, the device being termed a "shifting center."

Four screws are commonly used, but for many years only three screws have been customary on the highest grade instruments for geodetic work. Within recent years three-screw bases have been made which are almost as compact as four-screw bases, and many engineers now have three instead of four leveling screws on their transits and levels. No man who is accustomed to using three screws goes back willingly to four. The latter are much the slower, are not as stable and require the use of two hands to keep the bubbles centered in the plate levels. Leveling screws are used to keep the horizontal plate

level. If the plate is not level all horizontal angles read will be too small.

C. The lower clamp screw. This screw works in a clamp attached to the spindle socket, the inner end of the screw resting against a block. When turned it presses the block against the outer spindle thus clamping the spindle to the socket.

D. Lower tangent screw. This is fastened to the upper screw plate and works against the lower clamp screw for the purpose of making a final adjustment. Formerly two opposing screws were used but now there is but one screw as shown, the cylindrical case on the other side of the clamp screw containing a strong German silver spring.

E. The upper clamp screw used to clamp the inner and outer spindles together.

F. The upper tangent screw with opposing spring for making a final adjustment of the line of sight.

G. The needle lifter. A similar screw not shown is used to shift the variation plate.

H. Clamp screw for the vertical arc. This screw is on an arm which it clamps to the axis of the telescope. The lower end of the arm is between the vertical arc tangent screw and opposing spring.

I. Vertical arc tangent screw. After the clamp screw clamps the telescope axis the tangent screw is used to make the final small vertical movement required for accurate pointing.

The graduated disk shown ahead of the vertical arc tangent screw is a gradienter. The author had two transits equipped with gradienters and used these transits for more than sixteen years. In all that time he found no occasion when he could use a gradienter to advantage. Furthermore he has yet to meet an engineer who uses a gradienter in preference to stadia wires or the vertical arc. In purchasing a transit five dollars are saved when the gradienter is omitted. If the gradienter is to be of service the threads of the tangent screw must be accurately cut and be in perfect condition. After a few years of use all screw threads are worn and while slight wear does not affect them for use on tangent screws it does make them unfit for micrometer work, the gradienter being a form of micrometer.

Two small levels are used to level the horizontal plate. One is set parallel with the line of sight, usually on the left standards as shown in the cut. The other is set across the line of sight on the edge of the plate under the object end of the telescope, and is the more important of the two. The bubbles are centered by means of the leveling screws *B*.

Through a rectangular opening in the upper plate the graduations and vernier are viewed. This opening is covered with glass and a reflector of celluloid, or ground glass, is used to enable the graduations to be seen readily.

On the best transits double verniers are used, 180 degrees apart, so readings may be checked and to assist in repeating readings. Some makers place the vernier on the sides between the standards; others directly under the telescope, but the best place is 30 degrees to the left of the line of sight and is most common. In this position the vernier may be read without disturbing the telescope and the surveyor does not have to step around the instrument.

The telescope is mounted on an axis revolving in bearings on top of the standards. It is generally of such a length as to permit of a complete revolution, but with a sunshade on the object end the eye end only will clear the compass glass. The small capstan screws on the telescope tube indicate the location of the cross-wires, which are brought into the field of view by moving the eyepiece in or out. This focusing is done in one of several ways: by a straight pull, by a screwing motion or by means of a screw near the eyepiece which moves a rack and pinion within the tube. The object glass is focused by means of a screw on the right-hand side of the telescope and is therefore not shown in the cut. Some makers place this screw on top.

For taking vertical angles a vernier is attached to the left-hand standards and an arc or a full circle is attached to the telescope axis. For practically ninety-five per cent of work done by surveyors an arc like that shown in the cut is sufficient and the author prefers it to a full circle which is more apt to be injured because of its projection above the top of the standards. A full circle is particularly exposed to damage in brushy land. When long lines are run and elevations are taken by means of vertical angles,

instead of a regular level, as happens frequently in underground surveying, a full circle with double and opposite verniers is advisable. It should be enclosed in a protective shield with glazed verniers.

Under the telescope is shown a long level. When in adjustment this may be used for leveling, the transit being therefore a leveling instrument as well as an angle measurer. First level the instrument so the bubbles in the plate levels remain stationary during a complete revolution horizontally. Then place the telescope in as nearly a horizontal position as possible and clamp the axis. By means of the vertical tangent screw bring the bubble of the long level under the telescope to the center. If in adjustment it will remain stationary during a revolution of the instrument on the vertical axis. If left in this position it will not be necessary on succeeding "set-ups" to first level the plate, the telescope being alternately leveled over each pair of screws, precisely like an engineer's level. Accurate leveling may be done with a transit but it is slow compared with a level.

When the transit is not being used as a level the telescope should be pointed vertically, or in line with the vertical axis when the instrument is carried between stations. It should be clamped as lightly as possible. In this way it offers the least obstruction in brush and a blow will cause it to revolve on the axis. The lower tangent screw should be loose, leaving the instrument free to revolve if accidently struck.

A plain transit has no level under the telescope and no vertical arc or circle. Instrument makers quote on plain transits, and give prices for extras. Plain transits are used on railway surveys. If the surveyor expects to do little compass work a large transit is not necessary if of a good make. When the author was young he bought a transit with a 5-in. needle. The weight was 18 lbs. and the tripod weighed 10 lbs. Although similar transits are now made the majority of engineers prefer transits weighing not to exceed 13 lbs. with 7-lb. tripods. For general use the surveyor will obtain very satisfactory results with a transit having a $3\frac{1}{2}$-in. needle, the combined weight of transit and tripod being under 16 lbs. The writer for a number

of years held a commission as a U. S. Deputy Mineral Surveyor in the far west, and for mountain work had a transit made to order. The needle was about $2\frac{3}{4}$ ins. long and the weight of the transit was $4\frac{1}{4}$ lbs. It was carried in a knapsack held by straps over the shoulders. The folding tripod weighed $3\frac{1}{4}$ lbs., and was carried in a small case slung to the bottom of the knapsack. Intended for mountain work only the instrument was gradually introduced on other work until finally it was used on all the work done by the author for nearly ten years. Because of its lightness it suffered less damage from falls than heavier transits. For the same reason it exhibited great steadiness in a wind. This instrument of course represented one extreme as the first one purchased by the author represented another extreme.

A man with thick thumbs and large fingers cannot be made to believe the smallest-sized transit is a reliable instrument. Lightness and portability are greatly appreciated in rough country as also in high altitudes where breathing is a conscious effort. In lower country more weight is not objectionable when it implies more easily read graduations. For the highest grade of engineering surveying, such as re-tracing boundaries of expensive city property, setting out lines for great bridges, important tunnels, subways, etc., the need for absolute accuracy makes the question of weight of relatively small importance. With such instruments, however, a compass is not required and this fact effects a saving in weight by a gain in compactness.

Graduations were formerly made on silver-coated brass plates. Now they are cut in the surface of a silver ring attached to the plate. All of the best makers today cut the graduations in solid silver, for by this means only can proper results be obtained.

For the horizontal circle five methods are used for numbering the graduations.

I. Figured clockwise (in the direction followed by the hands of a clock) from 0° to 360° with single opposite verniers.

II. Figured in quadrants with double opposite verniers. These quadrants correspond to the compass quadrants so angles may be readily checked by the needle.

III. A combination of I and II with double opposite verniers.

IV. Figured in one row, clockwise and anti-clockwise, from 0° to 180° each way with double opposite verniers.

V. Figured in two rows, one clockwise and one anti-clockwise, from 0° to 360°, with double opposite verniers. Some makers incline the figures in the direction in which they are to be read and some make further provision against error by coloring one set red and the other black.

Method V is the best for all purposes and the one used by instrument makers when the purchaser expresses no choice. Vertical circles and arcs are best graduated quadrantally.

VERNIERS

Horizontal and vertical circles are divided into degrees by short marks. Each fifth degree mark projects a little and the tenth degree marks project still more, the graduations being figured at each tenth degree only. On some circles the degree is further divided into quarter-degree, third-degree or half-degree graduations. For closer readings a vernier must be employed.

The vernier is a uniformly divided scale on a circle adjacent to the graduated circle, or limb. The end marks on the vernier coincide with marks on the limb but in the included space there is one more interval on the vernier than on the limb. If we assume a limb graduated to half degrees (30 minutes) and 29 divisions on the limb equal 30 divisions on the vernier then each division on the vernier is $\frac{1}{30}$ smaller than a division on the limb and the vernier reads to minutes.

Rule to find reading of a vernier. Divide the least reading of the limb by number of spaces on the vernier.

Rule for reading an angle by means of a vernier. Read the angle on the limb to the graduation nearest the zero of the vernier. Then read along the vernier in the same direction until a line is found that coincides with some line on the limb. Add the vernier reading to the limb reading.

The following illustrations and descriptions will serve to make the matter clear. The student is advised to study the readings obtained.

Fig.	Reading of limb.	Divisions of limb. =	Divisions of vernier	Reading of vernier.	Kind of vernier.
182	Degrees	11	12	5 minutes	Double direct
183	30 minutes	29	30	1 minute	Double direct
184	20 minutes	39	40	30 seconds	Double direct
185	20 minutes	59	60	20 seconds	Folded
186	30 minutes	29	30	1 minute	Folded
187	15 minutes	44	45	20 seconds	Double direct
188	15 minutes	49	50	$\frac{1}{200}$ degree	Double direct

Fig. 182 shows a double direct vernier, that is, one which reads from the center to either extreme division (60), that part being used in which the direction of the numbering corresponds to the direction in which the limb is numbered and read. The limb is graduated to degrees and the vernier (from 0 to 60) comprises 12 divisions, therefore the reading of the vernier is 60 minutes ÷ 12 = 5 minutes.

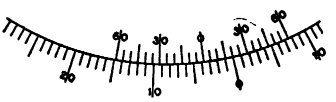


FIG. 182.

The figure reads 3° + 50′ = 3° 50′ from right to left.

Fig. 183 represents the usual graduations of an engineer's transit with its vernier. This is an ordinary

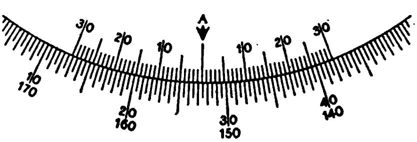


FIG. 183.

double-direct vernier reading from the center to either extreme division (30). The limb is graduated to half degrees and the vernier (from 0 to 30) comprises 30 divisions, therefore the reading of the vernier is 30 minutes ÷ 30 = 1 minute.

The figure reads 27° + 25′ = 27° 25′ from left to right, and 152° 30′ + 05′ = 152° 35′ from right to left.

Fig. 184 represents the graduations and vernier of a transit having finer graduations than that shown in Fig. 183, with a double-direct vernier reading from the center to either extreme division (20). The limb is graduated to

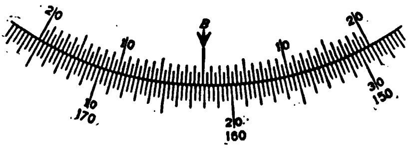


FIG. 184.

20 minutes and there are 40 divisions in the vernier, consequently the reading of the vernier is 20 minutes ÷ 40 = $\frac{1}{2}$ minute = 30 seconds.

The figure reads 17° 40′ + 12′ 30″ = 17° 52′ 30″ from left to right, and 162° + 7′ 30″ = 162° 7′ 30″ from right to left.

In Fig. 185 the transit has still finer divisions than those already considered. The vernier is a folded one reading from the center, indicated by the arrow, to either of the

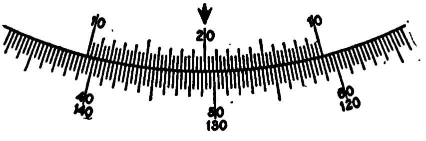


FIG. 185.

extreme divisions (10), and then forward in the same direction from the other extreme division (10) to the center division (20), the direction being determined by the numbering and reading of the limb. The limb is graduated to 20 minutes, while the vernier is composed of 60 equal parts, consequently the reading of the vernier is 20 minutes ÷ 60 = $\frac{1}{3}$ minute = 20 seconds.

The figure reads 49° + 14′ 20″ = 49° 14′ 20″ from left to right, and 130° 40′ + 5′ 40″ = 130° 45′ 40″ from right to left.

A transit still more finely graduated is shown in Fig. 186. This has a double-direct vernier reading from the center to either extreme division (45). The limb is graduated to 15 minutes and there are 45 divisions in the vernier, consequently the reading of the vernier is 15 minutes $\div$ 45 = $\frac{1}{3}$ minute = 20 seconds.

The figure reads 30° + 4′ 20″ = 30° 4′ 20″ from left to right and 149° 45′ + 10′ 40″ = 149° 55′ 40″ from right to left.

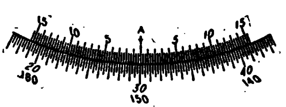


FIG. 186.

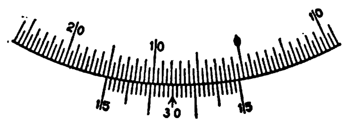


FIG. 187.

Fig. 187 represents a portion of a vertical circle or arc with folded vernier. The graduations are to half degrees, and the vernier is divided into 30 equal parts, consequently the reading of the vernier is 30 minutes $\div$ 30 = 1 minute.

The figure reads 7° 30′ + 21′ = 7° 51′ from right to left, an angle of depression.

In many kinds of work a decimal vernier has some minor advantages to recommend it and it is extremely useful in laying out railway curves. Its use is not common and many surveyors and engineers have never seen one. Fig. 188 shows the method of graduating the horizontal limb and vernier to read to decimals of a degree. This vernier is a double-direct vernier reading from the center to either extreme division (25), that part being used on which the direction of the numbering corresponds to the direction in which the limb is numbered and read. The limb is graduated to $\frac{1}{4}$ degree (0.25°) and the vernier divided into 50 parts, consequently the reading of the vernier is 0.25 $\div$ 50 = 0.005° which equals $\frac{1}{200}$ of a degree.

FIG. 188.

The figure reads 45° + 0.055 = 45.055° from left to right and 314.75° + 0.195 = 3.14945° from right to left.

In reading a vernier when the limb has two rows of figures care must be used to avoid reading the wrong row.

Never read the vernier by looking only at one line. The adjacent lines on each side of the coinciding line should be observed to see that they fail to coincide by equal amounts with lines on the limb.

Sometimes no coinciding lines can be found but two adjacent lines will be found which fail by equal amounts to coincide with lines on the limb. The true reading is between these lines; thus a closer reading may be obtained than is indicated by the least count of the vernier.

Every transit supplied with a vertical circle or arc should be equipped with stadia wires, the use of which will be described in the next chapter.

TO USE THE TRANSIT

The instrument should be set up firmly, the tripod legs being pressed into the ground, so as to bring the plates as nearly level as convenient. The plates should then be carefully leveled and properly clamped.

For precise work, in addition to leveling by the plate levels, it is always advisable, if the transit has such attachment, to level the plates by the telescope level, as this is much more sensitive than the levels on the plate. In this operation the position of the level on telescope must be observed over each pair of leveling screws in turn, and one-half the correction made by the axis tangent screw, the other half by the leveling screws.

Before an observation is made with the telescope, the eyepiece should be focused until the object is seen clear and well-defined, and the wires appear as if fastened to its surface. The intersection of the wires should be brought precisely upon the object to which the telescope is directed.

The zeros of the verniers and limb should be brought into line by the tangent screw of the leveling head. The angles taken are then read off upon the limb, without subtracting from those given by the verniers in any other position.

TO ADJUST THE TRANSIT

Every instrument should leave the hands of the maker in complete adjustment, but all adjustments are liable to derangement by accident or careless use so it is necessary to describe particularly those which are most likely to need attention.

The principal adjustments of the transit are: The Levels, the Line of Collimation, the Standards.

To adjust the levels. — Set the instrument upon its tripod as nearly level as may be, and having unclamped the plates, bring the two levels above, and on a line with, the two pairs of leveling screws. Clasp the heads of two opposite screws, and, turning both in or out, as may be needed, bring the bubble of the level directly over the screws exactly in the middle of the opening. Without moving the instrument, proceed in the same manner to bring the other bubble to the middle. The level first corrected may now be thrown a little out; if so, bring it in again, and when both are in place turn the instrument halfway around. If the bubbles are both in the middle they need no correction; but if not, turn the nuts at the end of the levels with the adjusting pin, until the bubbles are moved over half the error, bring the bubbles again into the middle by the leveling screws, and repeat the operation until the bubbles will remain in the middle during a complete revolution of the instrument.

To adjust the line of collimation. — This adjustment is to bring the cross-wires into such a position that the instrument, when placed at the middle of a straight line, will, by the transit of the telescope, cut the extremities of the line. Having leveled the instrument, determine if the vertical wire is plumb, by focusing on a defined point and observing if the wire remains on that point when the telescope is elevated or depressed. If not, loosen the cross-wire screws and by their heads turn the ring until correct, the openings in the telescope tube being slightly larger than the screws so that when the latter are loosened the ring can be rotated a short distance in either direction. Direct the intersection of the cross-wires on an object two or three hundred feet distant. Set the clamps and transit

to an object about the same distance in the opposite direction. Unclamp, turn the plates halfway around, and direct again to the first object; then transit to the second object. (*Note.* — To transit is to revolve the telescope on its axis. The telescope is thus reversed to get a sight on an object on the line in rear of the instrument.) If it strikes the same place the adjustment is correct. If not, the space which intervenes between the points bisected in the two observations will be double the deviation from a true straight line, since the error is the result of two observations.

In Fig. 189 let *A* represent the center of the instrument, and *BC* the imaginary straight line, upon the extremities of which the line of collimation is to be adjusted. *B* represents the object first selected, and *D* the point which the wires bisected when the telescope was reversed.

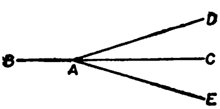


FIG. 189.

When the instrument is turned half around, and the telescope again directed to *B*, and once more reversed, the wires will bisect an object *E*, situated as far to one side of the true line as the point *D* is on the other side. The space *DE* is therefore the sum of two deviations of the wires from a true straight line, and the error is made very apparent.

In order to correct it, use the two capstan-head screws on the sides of the telescope, these being the ones which affect the position of the vertical wire. It must be kept in mind that the eyepiece apparently inverts the position of the wires, and therefore, in loosening one of the screws and tightening the other on the opposite side, the operator must proceed as if to increase the error observed.

The wires being adjusted, their intersection may now be brought into the center of the field of view by moving the screws holding the ring, which are slackened and tightened in pairs, the movement being now direct, until the wires are seen in their proper position.

The position of the line of collimation depends upon that of the objective solely, so that the eyepiece may, as in the case just described, be moved in any direction, or even removed and a new one substituted, without at all deranging the adjustment of the wires.

In case it becomes necessary to remove the cross-wire ring, the operator should proceed as follows: Take out the eyepiece, together with the ring by which it is centered, remove two opposite cross-wire screws, and with the others turn the ring until one of the screw holes is brought into view from the open end of the telescope tube. In this screw hole thrust a splinter of wood or a wire to hold the ring when the remaining screws are withdrawn. The ring can then be removed. It may be replaced by returning it to its position in the tube, and after either pair of screws is inserted the splinter or wire is removed, and the ring is turned until the other screws can be replaced, care being taken that the face of the diaphragm is turned toward the eyepiece. The eyepiece is next inserted, and its centering-ring brought into such a position that the screws in it can be replaced, and the ring into which the eyepiece is fixed is then screwed to the end of the telescope.

To adjust the standards. — In order that the point of intersection of the wires may trace a vertical line as the telescope is elevated or depressed, it is necessary that the standards of the telescope should be of precisely the same height. To ascertain this, and make the correction, if needed, proceed as follows:

Having the line of collimation properly adjusted, set up the instrument in a position where points of observation, such as the apex and base of a lofty spire, can be selected, giving a long range in a vertical direction.

Level the instrument, direct the telescope to the top of the object, and clamp to the spindle; then bring the telescope down until the wires bisect some well-defined point at the base. Turn the instrument half around, direct the telescope to the lower point, clamp to the spindle, and raise the telescope to the highest point. If the wires bisect it, the vertical adjustment is effected; if they are thrown to either side, this proves that the standard opposite to that side is the highest, the apparent error being double that actually due to this cause. When a transit does not have any means for adjusting the height of the standard it should be sent to the maker for adjustment.

Cross-wires are usually made of platinum. Occasionally spider webs are used although few modern instruments for

ordinary work are so equipped, as rough usage and long periods of damp weather injure spider hairs. When it is necessary to replace a broken wire or hair, hunt for the small black spider generally found in trees and underbrush, for all other spider web is too coarse. Allow the spider to run out on a pencil or small stick. When the end is reached it attaches a line and drops, thus stretching the line taut. The diaphragm having been removed from the telescope and the broken wires removed, grooves made by the instrument maker will be found. A drop of shellac is placed in each groove, and the stretched web fitted. The spider will not spin when cold and when sulky must be tossed and rolled gently until anxiety to escape is exhibited.

CARE OF TRANSITS AND LEVELS

Remarks on care of the compass apply equally to the compass attached to a transit.

All instruments should be protected during a rain by covering with a bag. For a transit use only oiled silk, for oiled linen and the sulphur in rubber will blacken the silver on the horizontal and vertical limbs.

In bright sunshine, on a hot day, it is a sensible idea to shade the instrument, to avoid errors due to expansion. Expansion sometimes injures the finer parts of an instrument. The use of a bag during wet weather is common but very few surveyors use a sunshade on hot days, a habit all should acquire.

Covering an instrument during a fog is wise and covering it in cold weather is to be done cautiously, for it may "sweat" with the frost. A soft linen cloth should be carried and the instrument wiped often to remove dust and moisture. When dry go over it with a large, soft camels-hair brush.

Never place an instrument in a box when damp. Be sure it is dry and then wipe and brush it before locking in the box. To dry an instrument place it in a dry, warm room.

Never leave an instrument standing in an open space without a person to guard it. In a room place it in a corner with the points of the tripod legs set in floor cracks. An instrument left on a tripod out in a room is easily knocked

over. After an instrument is dry and has been cleaned, place it in a box.

If, after exposure of an instrument in extremely hot or cold weather, it is found that the centers do not revolve as freely as usual, clean them as soon as possible.

In cleaning object and eyepiece glasses, use a soft rag or chamois leather. If the glasses should become greasy or very dirty, wash them with alcohol. As the fine polish on the object glass will be destroyed by wiping too often, the instrument man should be careful in this respect. Should telescope, compass or vernier glasses become moist, place the instrument in a room which is dry and moderately warm. Should it be impossible to follow this method, the glasses may be wiped dry, this latter process, however, affording an opportunity for dirt and dust to get into the instrument while the glasses are removed. The inner surfaces of protected glasses need seldom be wiped.

Due attention should be given the screws confining telescope bearings. They should be tightened sufficiently to make the bearing firm and still permit the telescope to revolve freely, yet be kept in position by the friction thus obtained. When there is too much friction in the telescope slide, take it out immediately and first scrape the rough place with the blade of a pocket knife, having its edge inclined a trifle, then use the back of blade for burnishing the spot. If possible, treat inside of tube in a similar manner, or at least wipe it out. The slide should then be slightly greased with watch oil, but all the grease must be wiped off before slide is replaced. No emery or emery paper should be used on any part of an instrument. When fretting begins it should be sent to an instrument maker for repairs. If this cannot be done promptly, no man should be afraid to take an instrument apart for cleaning. It is of course foolish to take an instrument apart when not necessary, but many surveyors, afraid to "examine the insides," keep on using a dirty instrument until it gets into such bad shape that an instrument maker loses money putting it into shape again.

A skilful man can take apart and put together an instrument without damage, while an unskilful man will certainly do harm by attempting such work.

Vaseline will clean the surface of the silvered ring on the limbs and should be thoroughly wiped off. *Use only good watch oil* for lubricating centers and cleaning screw threads. Use as little as possible and wipe it off so it will not gather dust and grit. Fretting centers should be treated in the same manner as fretting telescope tubes.

Overstraining of screws should be guarded against as it either stretches the threads, causing them to wear out in a short time, or gradually loosens some part of the instrument. When screws are too tight the instrument is sensitive to temperature changes; when too loose it is unsteady and reliable work cannot be done.

Tripod legs must be firm. If the screws are loose the instrument will be shaky and if too tight warping may cause the telescope to shift off the line of sight. The proper degree of restraint may be determined by raising the legs one at a time to a horizontal position. If the screw holds a leg up, it is too tight. If the leg falls rapidly, the screw is too loose. The downward turning should be slow and uniform.

TO READ ANGLES WITH A TRANSIT

The instrument is set at B (Fig. 190) and leveled by means of the leveling screws. All clamp screws being loose, bring the zeros on the horizontal limb and vernier together and clamp with the upper clamp screw. Then use the reading glass — which hangs by a cord around the neck — and with the upper tangent screw bring the zeros exactly together.

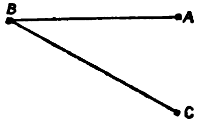


FIG. 190.

Revolve the instrument horizontally on its vertical axis, without touching the upper clamp or tangent screw, until the vertical cross-wire bisects the stake (or tack) at A. Clamp the lower clamp screw and with the lower tangent screw bring the cross-hair to the exact point desired.

The instrument is now clamped with zeros together and cross-hairs bisecting the object. It cannot be revolved on the vertical axis. Loosen the upper clamp screw and direct the telescope to C. Clamp the screw and by means

of the upper tangent screw bring the cross-hair to exactly bisect the point at *C*. Now with the reading glass the angle may be read. If the instrument is in perfect adjustment, well made, with no errors in graduation, and it is not disturbed during the operation, the angle is obtained correctly.

To check. — Unclamp the lower clamp screw without touching the upper screws. Reverse the telescope. Revolve the instrument horizontally and bisect *A* with the vertical wire. Clamp the lower clamp and bring the wire to an exact bisection by means of the lower tangent screw, the plate meanwhile remaining set at the angle read. Unclamp the upper screw and direct the telescope to *C*. Clamp, and with the upper tangent screw bisect *C* with the vertical wire. Now read the angle, which should be double the exact angle. The operation just described is called "Double-centering." The mean of the two readings is correct because every error in one direction on the first reading is balanced by an equal error in the opposite direction on the second reading. This, however, does not apply to errors in graduation. When running lines every angle "on line" should be "double-centered," but only one reading is necessary for side shots and lines where the closest possible accuracy is not required.

Repeating angles. — In triangulation work the method of repetition is followed. Read the angle by sighting to *A* and then to *C*. Keep the telescope erect and repeat the readings from *A* to *C* until a total of six are taken. The final reading divided by 6 gives an approximate value for the angle. Both verniers are read at the start and after the sixth repetition. The total reading of each is divided by 6. If there is a difference the mean of the final values is taken.

Reverse the telescope and repeat the process, but this time reading from right to left. Read both verniers, divide the totals by 6, and take the mean value of the six readings of each vernier.

We now have a mean value for the direct readings and one for the reversed readings. The mean of the means should be the true angle, plus or minus a probable error.

Let Σ = "the sum of all the quantities in the parenthesis,"

v = variation (or deviation) of each angle from the mean, provided each angle is read,

n = number of readings,

r = probable error,

then

$$r = \pm\, 0.8453 \frac{\Sigma\,(+\,v)}{\sqrt{n\,(n-1)}} \text{ for single observation,}$$

and

$$r = \pm\, 0.8453 \frac{\Sigma\,(+\,v)}{n\sqrt{n-1}} \text{ for all observations.}$$

When the deviation gives a larger angle than the mean it is positive and when smaller it is negative, but in obtaining the sum in the formulas above given the signs are disregarded. The formulas are known as Peter's Approximations of Bessel's Formula, which is based on the principle of least squares and is laborious to work.

"The probable error is not 'the most probable error,' nor, 'the most probable value of the actual error.' It determines the degree of confidence we may have in using the mean as the best representative value of a series of observations." (Mellor.) If the mean value of the angle is 17° 31′ 42″ and the probable error is 5.33″ then the odds are even that the true value lies somewhere between 17° 31′ 47.33″ and 17° 36′ 36.67.″ When the three angles of a triangle have been measured and the sum does not equal 180°, a computation of the most probable error for each angle will be a guide to balance the angles to obtain 180°. If the triangle has sides more than a mile in length it may be the error in closure is due to "spherical excess" by reason of the curvature of the earth, the triangle being spherical. Geodetic methods must then be used.

The computation for probable error is sometimes used in the case of a line which has been measured several times with a tape forward and back by different sets of chainmen. A small probable error is often said to be a measure of the accuracy, but as it refers only to the proportion in which errors of different magnitude occur it cannot be a measure of accuracy.

Formulas determining the probable error are worthless for a small number of observations. We may select ten as the least number, and the first step is to eliminate all *constant* errors. If a tape is too long or too short the constant error in each tape length is the difference between the length of the tape and a standard. If an error in graduation exists in some part of the circle of a transit, the part in which the error exists must not be used. In chaining, three sets of chainmen should measure the line forward and back at least twice, making a total of twelve measurements. By using one set of chainmen the *personal error* would not be eliminated, and as it is a *constant error* difficult to find and evaluate, other men must be used to give this error something of the character of a series of *accidental* errors. The careful measuring above described is done only on the highest grade work.

Constant errors are cumulative and accidental errors are compensating. The probable error is practically a mean square value of the positive and negative deviations from the arithmetical mean of all the accidental errors. The probable error is diminished by increasing the number of observations.

In chaining, the accuracy of the work is increased by taking plenty of time, and not working too fast. In repeating angles with a transit the accuracy of the work is increased when speed is increased. Clamping and unclamping disturbs an instrument and may throw it out of level. Standing a long time in one place and the handling tend to make one of the metal shod tripod legs sink into the earth. The temperature effects may be equalized in rapid work, for one side may be unduly heated or cooled if allowed to remain too long in one position.

A transit should always be tested for "index error." This may be due to faulty graduation, in which case the instrument should be returned to the maker, or it may be caused by a fall bending the plate. Even straightening the plate may not restore it, and the surveyor for some good reason may have to use it without re-graduation.

Set three stakes with tacks in the top so the small acute angle will lie between 10° and 20°. Start from 0° and read the angle successively around the circle without double

centering. This must be done rapidly. The angles should be equal. When a place is found where the reading differs from readings preceding or following it, test each degree in the space until the error is exactly located. The amount does not matter but the place must be marked and not used thereafter. Sometimes errors of very small value are found which show up only in a series of readings, and are distributed around the circle. Double centering will take care of them.

Unnecessary stepping around an instrument must be avoided. A good instrument-man never "straddles" a tripod leg. Transit work being closer than compass work every instrument station must be marked with a tack. The plumb-bob hanging from the center of the vertical axis must be centered over the tack, this being accomplished with a shifting center. The lower end of the axis is enlarged in a spherical shape and fits in a spherical socket on a small plate, thus enabling the leveling screws to bring the axis to a vertical position when the tripod head may not be level. The opening in the lower plate, which is screwed to the tripod head, is larger than the ball but smaller than the socket plate. By loosening the screws the center may be shifted to carry the plumb-bob to any side. When finally centered the screws are tightened and the plate leveled.

Fig. 191.

Plumb-bob lines must be lengthened and shortened and a number of devices are used to accomplish this without the line slipping in the hook at the upper end. Nearly all catch the wind and are objectionable for this reason. A knot in the string is best, but when loose enough to allow the cord to be slipped through it is a nuisance. By looping the cord twice around the hook the knot may be loose and the cord will not slip, the friction of the loop holding the plumb-bob in place.

In sighting lines, steel line rods are used and it takes considerable experience to enable a helper to hold the rod vertical. Sometimes a plumb line is held over the point but no man has so steady a hand that confidence can be

had in such a sight. Sometimes a line rod is stuck in the ground at a slant so it is above the tack. To this the plumb line is tied and moved along the rod until the point of the bob is centered on the tack. For quicker sights the helper places one knee on the ground and rests one hand, holding the plumb line, on the other knee. The plumb-bob is then centered over the tack and the white line is viewed with the leg as a background.

A target of white celluloid made in such size and manner that it may be carried in the pocket and attached to a plumb line when needed has been developed by Kolesch & Company of New York. It is circular in form with a diamond-shaped cut-out, which offers a strong contrast to the body of the target. It can be moved up and down the plumb line at will. The application of this device will be recognized by all surveyors who have taken sights against a background of shubbery or buildings, and it will enable the taking of longer "shots." It has the advantage also of being a more accurate means of setting a true point than can be attained with a flag or pole, that may be slightly out of plumb when the sight is taken.

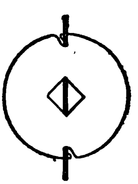


FIG. 192.

A piece of wood about the width of a lead pencil, sharpened on the lower end and stuck in a crack behind the tack is frequently used for a backsight.

In Fig. 190 let the transit be set at *A* with point *B* set. An angle is to be laid off and point *C* located. Turn off the angle and set a stake at *C*. The helper holds the rod on the stake and the transitman motions right or left until the rod is in line, when it is pressed slightly to make a mark. The telescope is reversed, the lower clamp loosened, the vertical hair set on *B* and the lower clamp tightened. The upper clamp is loosened and the telescope directed to *C*, clamped, and the upper tangent screw used to obtain exactly double the angle. The rod is now directed again to line and another mark made. The true point lies midway between the marks and a tack is driven to fix it.

In Fig. 193 is shown the general method of conducting a transit survey. Assume *A* and *F* to be set. The transit is set up at *A* with horizontal plate and vernier clamped at

zero. The lower clamp is loose. Reverse (transit) the telescope and sight F. Clamp the lower screw and with the lower tangent screw bisect the tack at F. Transit the telescope so it sights forward on the line FA.

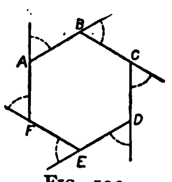


FIG. 193.

Unclamp the plate, turn the angle to the right and set point B. Move to B and set C from a backsight on A. Proceed around the field in this manner, finally closing on A, which may be occupied a second time to check the traverse.

In the example given all the angles were to the right. In Fig. 194 the angles are turned to the right and left, but the work is the same, the entries in the field book having the letter R, or L, as the case may be, following the amount of the angle.

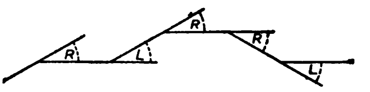


FIG. 194.

The needle should be allowed to swing to check the angles. This is a rule to be followed in all transit work. When carrying the transit lift the needle off the pivot and hold against the glass cover of the compass. The needle reading is not so accurate as angles read on the limb, but it affords a check so a mistake in setting down R for L, or vice versa, is quickly discovered. Such a mistake is common.

A mistake never gets past the person making it. An error is a mistake discovered by others and reflects upon the ability of the person responsible. All the field and office work of surveyors must be checked and re-checked so no errors can be found. Checking is a most important duty and cannot be neglected.

Angles may be read on a survey by setting the limb and vernier at zero on each station and double centering each time on important work. The bearing of the first line is set down in the field book and all calculations of bearings start from it as a base. The following rules are used to reduce angles to bearings:

Reduce bearings to azimuth angles, as follows:

All N E bearings are less than 90°.

S E bearings lie between 90° and 180° and are reduced to azimuth by subtracting from 180°.

S W bearings lie between 180° and 270° and are reduced to azimuth by adding to 180°.

N W bearings lie between 270° and 360° and are reduced to azimuth by subtracting from 360°.

The meridian being selected as an azimuth line and the meridian bearing of the starting course having been reduced to an azimuth bearing by the above method, consider all angles to the right as positive and all angles to the left as negative.

Sum less than 90°. Bearing N E.

Sum between 90° and 180°. Subtract from 180° and call bearing S E.

Sum between 180° and 270°. Subtract 180° and call bearing S W.

Sum between 270° and 360°. Subtract from 360° and call bearing N W.

Example. — Starting from the end of a line bearing S 18° 21′ E a line was run as follows:

		Feet
0.	R 14° 13′	280
1.	R 74° 21′	500
2.	L 47° 11′	320
3.	L 10° 05′	211
4.	R 27° 09′	419

Find the bearings of the lines, referred to the true meridian.

	179° 60′	
S	18° 21′	E
	161° 39′	= Azimuth of reference course.
0.	14° 13′	R
	175° 52′	
1.	74° 21′	R
	249° 73′	
2.	47° 11′	L
	202° 62′	
3.	10° 05′	L
	192° 57′	
4.	27° 09′	R
	219° 66′	

Course 0.	179° 60′	
	175° 52′	
S	40° 08′	E
Course 1.	250° 13′	
	180° 00′	
S	70° 13′	W
Course 2.	203° 02′	
	180° 00′	
S	23° 02′	W
Course 3.	192° 57′	
	180° 00′	
S	12° 57′	W
Course 4.	220° 06′	
	180° 00	
S	40° 06′	W

From a study of the foregoing example, the following shorter method is found:

$$\begin{array}{lrrl}
S & 18^\circ & 21' & E \\
 & 14^\circ & 13' & R \\
\hline
S & 4^\circ & 08' & E \\
 & 74^\circ & 21' & R \\
\hline
S & 70^\circ & 13' & W \\
 & 47^\circ & 11' & L \\
\hline
S & 23^\circ & 02' & W \\
 & 10^\circ & 05' & L \\
\hline
S & 12^\circ & 57' & W \\
 & 27^\circ & 09' & R \\
\hline
S & 40^\circ & 06' & W
\end{array}$$

The student may reduce angles to bearings by either method as the example given illustrates the principle.

The reason for recording the needle reading each time is now plain. If the *R* and *L* are always recorded properly the needle readings will check the computed bearings within a few minutes. With bearings nearly due north and south or east and west the check may not be valuable if local attraction is present, but it should never be omitted for it is the only check to be had when all angles start from zero and are read right and left.

Azimuth, according to the dictionary, is "the angle comprised between two vertical planes, one passing through the elevated pole, the other through the object." Herschel counted azimuth from 0° to 360°, and in this sense it is used by surveyors; that is, it indicates all angles referred to some meridian and independent of bearings. Bearings are never greater than 90° and are reckoned to the east and west from the north or south pole. When azimuth is reckoned from the north, the true meridian being the azimuth meridian, it is easy to convert azimuth into compass bearings, as in the example just given.

There is good reason for following this custom as compass bearings have been used many centuries because of the needle pointing north. When azimuth is reckoned clockwise from the north the needle readily checks all computed bearings. For about a generation a number of writers of surveying textbooks have reckoned azimuth from the south clockwise because the astronomical pole is

south. This may be scientifically correct for a very few astrophysicists, but it is confusing to the surveyor to assume one pole for angle work and another pole distant 180° for work involving compass bearings. It really makes no difference to the educated man which pole he uses but the surveyor is dealing with the public and the public always "starts from the north." The surveyor who reckons azimuth from the south must use two standard meridians and some day will make mistakes. The writer one summer employed a young graduate who reckoned azimuth from the south and was continually mixing his notes until he abandoned the practice. When the writer went to school, in the 80's, azimuth was any angle read clockwise from a chosen meridian to an object, so that an object 20° *L* would have an azimuth of 340°, thus preventing errors due to misplacing the letters *R* and *L*; no hint was given that azimuth should be referred to the south pole or the north pole.

With an instrument in perfect adjustment and handled by a competent man surveys are often made by azimuth bearings and no double centering is done. The instrument is set up at the starting point, the variation plate indicating the correct declination. The needle is let down and allowed to settle so it points to the true north. The limb and vernier are set to zero and the instrument clamped. The needle is then lifted and not again used.

Unclamping the plates the telescope is directed to the next station and the angle read and recorded after clamping the plates. Carrying the instrument to the next station, the telescope is reversed and sighted to the station just left; and the instrument clamped below. The telescope is transitted and then read forward on the line, the limb being still set at the azimuth. Unclamping the plate the telescope is directed to the next station and the azimuth read and recorded. These operations are repeated at each station and when the first station is again occupied and a sight taken to the second station the first azimuth should be obtained. Actually there may be a difference of one or two minutes, which may be distributed. On a closed traverse errors in reading azimuth tend to compensate, for the right and left readings balance; but

on other surveys errors are apt to be cumulative. Azimuth notes for the last example will read as follows:

Reference line	161° 39′	Needle S 18° 21′ E
0.	175° 52′	280 ft.
1.	250° 13′	500 ft.
2.	203° 02′	320 ft.
3.	192° 57′	211 ft.
4.	220° 06′	419 ft.

All angles are azimuth angles on stadia surveys. Not all surveyors double center their angles, a surprisingly large number using the azimuth method exclusively.

When surveying cheap land or doing work requiring no greater accuracy than may be obtained with a compass, the transit may be used as a compass and all readings taken with the needle.

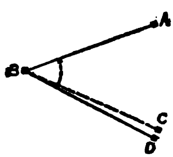


FIG. 195.

A very accurate method for setting off an angle is illustrated in Fig. 195.

On a given line are set two hubs *A* and *B*. A line is to run from *B* through a point *D* and this line must be laid off correctly. Setting on *B* the telescope is directed to *A*, plates clamped to zero and an angle *ABC* read, one minute smaller than the angle *ABD*, and stake *C* set. A tack is placed in *C* and the angle *ABC* is read by repetition to obtain the exact value. The length *BC* is measured with the greatest possible accuracy. The exact value of angle *ABC* is subtracted from the angle *ABD* and the tangent of the difference multiplied by the length *BC* gives the small distance *CD*, which may be measured off and a tack set. Or the angle *ABD* may be read and a broad stake driven at *D* with a tack in the top. The angle is then read by the repetitive method and if a difference is found from the true angle a correction may be measured and the tack set in the proper place.

A transit may be set on a line between two points, by setting as nearly as possible on line and leveling the instrument. Then sight on one point, clamp the plates and reverse the telescope for a sight on the other point. If it does not strike it, shift the transit and try again. A skilful instrument-man can get on a line in three trials. After

getting on line reverse the telescope for a sight each way, to eliminate errors in adjustment and disturbance due to handling.

Fig. 196 illustrates the operation, *A* and *B* being the points and *C* the transit station, the small circles near *C* showing trial set-up points.

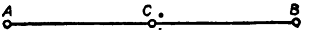


FIG. 196.

To prolong a line, the approved method is to set on a stake and take a backsight with telescope reversed on a stake on line in the rear.

Transit the telescope and set a stake ahead. Then double center on the stake and set a tack between the two marks. Move to the new stake and repeat for the next one.

To set a line of stakes between two stakes set on one, sight to the other and clamp the plates. The line of sight being fixed the telescope is plunged * on the horizontal axis and the cross-hairs set on the line rod used to locate the positions of the intermediate stakes and tacks are driven in them.

A good compass is graduated to ½ degree and by using care one-half this angle may be estimated. If this is done the maximum error in angle will be 4 minutes, of which the natural tangent is 0.00116. Few surveyors read a bearing closer than ¼ degree (15′) in which case the maximum error will be 8 minutes, of which the natural tangent is 0.00233.

Ordinary transits read by means of verniers to single minutes so the maximum error in angle cannot exceed 31 seconds, of which the natural tangent is practically 0.000146. It is not uncommon for transits to be graduated to read angles as small as 30, 20 or 10 seconds, while if angles are read by the repetitive method the exact size of an angle may be ascertained to the nearest 5 seconds with a vernier reading to one minute, or to the nearest second with a vernier reading to 20 seconds.

With a transit a skilful instrument-man can do all the angular work with as high a degree of accuracy as skilled chainmen can do the measuring with first-class tapes.

Methods for transit surveying are the same as methods

* To plunge a telescope is to turn it on its axis to set a line ahead. The telescope is clamped and is not inverted or reversed for a rear sight.

for compass work plus the increased accuracy. The angles right and left, or the azimuth bearings, are reduced to compass bearings and the computations for area are the same as those described in the chapter dealing with the compass. Computations for traverse work and for supplying omissions are described in the chapter on trigonometry.

Surveys are made for one of three purposes, sometimes for all:

1. To describe boundaries.
2. To obtain areas.
3. To make a map.

Boundary surveys are usually made by running out the boundary lines, offsetting a few feet when the boundary is obstructed with hedges, fences, etc.

Area surveys not calling for a description may be made by methods already described for chain surveys. The oldest book on surveying of which we have knowledge, Hero's Dioptra, shows that the custom fully twenty centuries ago was to lay off in the field a rectangular figure and erect on the sides of it, at regular intervals, perpendiculars to strike the boundary lines. The areas of all the divisions were added.

When the diopter was improved so angles other than right angles could be turned off, surveyors turned angles from points on the boundary of the interior rectangle to corners of the field and measured to the corners. The proceeding was duplicated in making the map and the areas of all the interior figures were computed by mensuration and added. With the introduction of plane trigonometry and the use of the compass the "radiation" method became standard and areas were computed by trigonometric methods. Sometimes the surveyor merely laid off a carefully measured base line and from the ends and some intermediate points took bearings to corners and measured the distances. This divided the field into triangles, the areas of which were computed trigonometrically. When the "double meridian" method for computing areas was developed in Ireland by Thomas Burgh, the custom came in of running out the exterior lines. The reason this was considered advisable was to save labor.

Prior to Burgh's discovery lines were run around the boundary to obtain data for writing a legal description, but surveys to compute areas were still made by mensuration and trigonometric methods. When the double meridian distance method for computing areas made it possible to use the boundary survey data, only the one survey was necessary.

With the passing years the "radiation" method of surveying land received less attention in textbooks, although all practical surveyors used the method where it proved to be time saving. The writer had to make several surveys by this method when at school and used it frequently in practical work.

The three approved methods of surveying a piece of land are:

1. The traverse method.
2. The radiation method.
3. The intersection method.

In the 1912 Proceedings of the Illinois Society of Engineers and Surveyors, a paper by Mr. G. W. Pickels, then an instructor in Civil Engineering at the University of Illinois, compared the three methods.

The results were as follows:

Speed in chaining. — Intersection method, first; radiation method, second; traverse method, third. In most instances the radiation method called for 20 per cent less chaining than the traverse method.

Speed in instrument work. — Intersection method, first; radiation method, second; traverse method, third.

Speed in computing results. — Traverse method, first; radiation method, second; intersection method, third.

The above comparisons prove the traverse method to be the more expensive, for slow work in the field means the employment of a party of men for a longer time. A little extra time in the office usually affects one man only. Accurate work involves careful checking and half the work of engineers and surveyors seems to consist in checking lengths, angles and computations.

Comparison of accuracy. — Assuming that no blunders are made, any one of the three methods is accurate enough for all practical work. With respect to actual accuracy

the intersection method is first; radiation method, second; traverse method, third.

The traverse method consists in running out the boundaries; or running on offset lines as closely as possible to the boundaries. To guard against error in reading the needle, or setting down the wrong angle, it is a common practice to read an angle or take a bearing to some one object from each station.

In Fig. 197 bearings were taken to one corner of the chimney of the house in the field. When making the map these bearings are all drawn and should intersect at a common point. This method has always been used by careful surveyors on compass surveys. Modern transits are equipped with stadia wires, or with gradienters, and distances should be checked with one or the other. In this one instance the gradienter, if new with sharp clean threads, has an advantage over the stadia. The gradienter can be used if one foot of rod can be seen, while the stadia requires a graduated rod. If a plain rod, however, is used the surveyor can direct the horizontal cross-hair to one mark and read the vertical angle. Then drop the wire two, three, or more feet on the rod and read a second angle. One-half the sum of the two angles gives the vertical angle to the mid-point, the rod being held vertically.

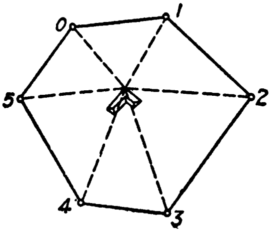


FIG. 197.

Let A = half the sum of the two angles,

D = distance on rod intercepted between the wire on the two sights in feet,

L = length of line in feet,

then $L = D \times Cos^2 A + 1.$

The formula is only closely approximate, but errors in measuring within reasonable limits are readily caught by such a check.

In the radiation method the transit is set up at one or more points, from which angles and lines are measured to the corners of the field.

Fig. 198 illustrates a field of less than 160 acres in fairly compact shape. Here all the corners could be seen from one set-up. To survey the field by the traverse method would call for the measurement of the five sides and for the occupation of each corner with the instrument. The sum of the lengths of the lines $OA + OB + OC + OD + OE$ is usually less than the perimeter of the field, but all lengths should be checked. Surveyors skilled in stadia work can dispense with the chain or tape.

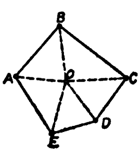


FIG. 198.

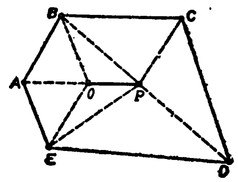


FIG. 199.

Fig. 199 illustrates a larger field, the corners of which cannot be seen from one point. The instrument is set at O and P, the angles at each point being turned from a backsight on the other. All the radiating lines are measured except PB and PE.

Fig. 200 illustrates a long narrow field and neither point O nor P is visible from the other. Flags are set on the boundaries at E and F and angles read to them, the lengths OE, OF, PE and PF being measured.

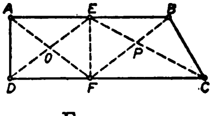


FIG. 200.

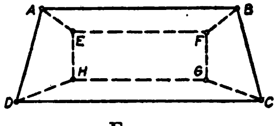


FIG. 201.

Fig. 201 illustrates the method to use with fields of considerable size, say exceeding 320 acres, in which the method shown in Fig. 200 may not be applicable. A small traverse is run within the field and angles and distances measured to the corners of the field.

The radiation method divides a field into triangles, two sides and the included angle of each being measured.

The area of each triangle $= \frac{1}{2}\, bc\, Sin\, A$ and the sum of the areas of the triangles equals the area of the field, Fig. 201 being an exception. If a description of the field is wanted each triangle must be solved for the third side and the base.

"The intersection method consists in establishing and carefully measuring a base line, from each end of which all the corners of the field are visible, and in measuring all the angles around each end formed by lines radiating to the corners. If the field is a small one and topography permits, one base line is sufficient, but in the case of a large field, two or more base lines will be necessary. The base line is taken inside the field when it is possible, as the lengths of the sights is thereby reduced to a minimum; but it is not essential that it be so taken, and it may be all outside of the field or partly inside and partly outside. The only imperative condition — using one base line — is that all corners must be visible from each end of the base line."

"In locating the base line care must be taken to avoid small angles, as the sines of small angles change rapidly. (Angles under 15° and over 165° must be obtained to the nearest 10 secs., which can be done by trebling the angle on the limb of the transit.) In the case of a rectangular field the best location for the base line is parallel to the short axis. This arrangement should be followed as closely as the topography will permit. The more irregular the field, the harder it is to avoid small angles, and more time and thought are required in the selection of the base line than in the case of the rectangular field. The length of the base line depends upon local conditions. In general, the longer it is, the more accurate the results; but it should never be less than one-third the length of the average sight. This is based on the assumption that the instrument used reads to 30 secs., and that each angle is doubled on the limb."

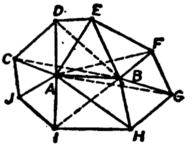


FIG. 202.

Fig. 202 illustrates a field surveyed by the intersection method, AB being the base line, the only line measured.

"The traverse method is applicable to any field regardless of the crops or topographical conditions. There is always a strip of land along the fence line that is not culti-

vated, and an open sight may be had along this strip, so the distance between corners may be chained without interfering with the growing crops. The radiation method may be used equally as well, provided that the growing crops do not interfere with the chaining or obscure the lines of sight. Farms are generally surveyed upon changing hands, and the principal thing to be ascertained is the acreage. Possession is taken in the spring before the crops are put in, and hence the surveyor is not limited in his choice of methods. The intersection method can be used only in level, open country, as all the corners of the field can seldom be seen from both ends of a base line in hilly wooded country. Hence with respect to applicability, the traverse method has a slight advantage over the radiation method, and both of these are far ahead of the intersection method."

"Which method of surveying should be used in any given case depends entirely upon the conditions peculiar to that particular case. Each of the methods is best in some instances. The use of the intersection method is limited, it is true, by topographical features, but it is the most accurate of all, and should be used when the nature of the country will permit. The radiation method should be given equal standing with the traverse method, for its saving in time more than compensates for its slightly limited use. Frequently the sides of a field have to be measured in order to re-locate the corners, and in such cases the traverse method is the quickest and should be used. As any one of the methods is accurate enough for all practical purposes, the one that is most applicable to the case in hand should be used."

Figs. 198 to 202 inclusive, and the quoted paragraphs are from the paper by Mr. Pickels.

ANGULAR LEVELING

The long level under the telescope of a transit may be used for leveling, as already described, or vertical angles may be read and the rise or fall computed by the formula

$$H = \pm\, l \tan A,$$

in which H = height or difference in elevation in feet,

l = length of sight in feet (horizontal),

A = angle of elevation or depression.

The intersection of the cross-hairs should be directed to a point at their height above the ground. A rod is generally used to sight on. The horizontal distance must be carefully measured. The vertical angle should be read at both stations, being an angle of elevation (+) at one and an angle of depression (−) at the other. If there is a small difference use the mean value. A large difference calls for a check and may be due to lack of adjustment. The long bubble should be adjusted so it will remain centered as the instrument revolves on its vertical axis. The cross-hairs should then be adjusted as described for the dumpy level, so the line of sight will be parallel to a horizontal line as indicated by the bubble. The zero of the vernier should be adjusted to coincide with the zero of the vertical arc.

Leveling in mining surveys is generally angular work underground. On steep ground and in rolling country leveling by vertical angles is rapid and when care is used is very accurate. It is not well adapted for work requiring elevations at intervals of about 100 ft. or less.

STADIA WIRES

When equipped with stadia wires so distances may be read on a rod, the transit becomes, as it has been advertised, "A universal surveying instrument."

Stadia wires are two horizontal wires, one above and one below the horizontal wire found in all transit telescopes. These two wires are fixed in a ring or diaphragm in such a way that the interval usually intercepts a space of one foot on a rod held at a point distant d ft. from the center of the transit, plus a small constant C.

In Fig. 203 the relations are shown:

c = distance from telescope axis to center of objective,
f = focal length, the distance from the stadia wires to the objective,
$D = d + C$ and $C = c + f$.

The rays cross each other so that the vertex of the visual angle is not at the center of the instrument, but at a distance in front of the objective equal to its focal length, measured when the telescope is focused on a distant object.

The relation between the size and distance of an object and the size of its image in a telescope is given by the formula

$$\frac{R^1}{R} = \frac{f}{d}. \quad \therefore \quad d = \frac{fR}{R^1},$$

in which R = space on rod intercepted between the two wires and called the "rod reading,"
R^1 = space between the wires,
d = distance from principal focal point to rod.

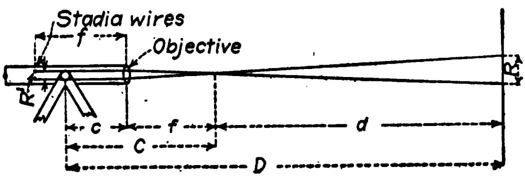


FIG. 203.

When the wires are set with a space = $\frac{1}{100}$ of the focal length between them we have the relation $\frac{R^1}{f} = \frac{1}{100}$, so the wires will read one foot on a rod held at a distance of 100 ft., one yard at a distance of 100 yds., one meter at a distance of 100 m., etc. Substituting this value in the above expression $d = 100\,R$, to which must be added the constant C.

This constant never varies for any given instrument and is independent of the distance. Focus the telescope on an object several miles away and measure from the axis to the objective for c; and from the screws holding the stadia diaphragm to the objective for f. The constant $C = c + f$ is noted by all instrument makers on a card or label affixed to the box of every instrument provided with stadia wires.

When using the stadia the rod is held vertically. The middle wire is directed to a point on the rod at the same height above the ground as the axis of the telescope. The difference between readings of the upper and lower wires gives the rod reading, if a level rod is used, or a rod having

numbers on the face. Regular stadia rods are seldom numbered, the intercepted space being read directly.

The rod is held vertically and as the line of sight is seldom horizontal this introduces relations between the resulting angles which are finally resolved into the following expressions:

$$D = R\cos^2 A + C\cos A,$$

$$H = R\frac{\sin 2A}{2} + C\sin A,$$

in which D = horizontal distance from center of transit to rod,
R = rod reading,
A = vertical angle,
$C = c + f$,
H = height or difference in elevation.

For ordinary work add C instead of $C\cos A$, to obtain horizontal distance; and add $0.015\,A$ instead of $C\sin A$, to obtain difference in elevation. The tables here given save a great deal of time in reducing inclined stadia readings to horizontal and vertical equivalents.

Example. — An angle $+18°\ 12'$ was read to a rod with a rod reading of 5.72 ft. Find horizontal distance and difference in elevation.

Turning to column headed 18° on the line opposite 12′ find: Hor. Dist. = 90.24, Diff. Elev. = 29.67. At the bottom find the values for $C = -0.95$ and 0.32, for we here assume C = 1.00 ft.

Hor. Dist. = $(5.72 \times 90.24) + 0.95 = 517.12$ ft.
Diff. Elev. = $(5.72 \times 29.67) + 0.32 = 170.03$ ft.

A number of excellent labor-saving slide rules and diagrams are sold by instrument dealers for reducing stadia rod readings.

When running lines with the stadia, readings should be taken forward and back for a check both for angle and rod reading. Angles should be kept below 20° when possible and readings distorted by heat are unreliable. When conditions are favorable and the surveyor is careful the limit of error can be kept within $\frac{1}{1500}$, a degree of accuracy superior to ordinary farm survey chaining.

The ratio 1 : 100 is convenient for the reason that ordinary level rods may be used and also the decimally divided rods sold by all dealers. If some other ratio is used the rods must be specially divided or all reduced readings be multiplied by a constant.

Instrument dealers advise the use of fixed wires and so do men in the government service who are constantly employed on stadia work. The author however worked for a number of years in regions where instrument repair shops were many days distant and not all instrument repairers were first class. Sudden changes in temperature and long-heated terms disturbed the wire interval so many times that finally adjustable wires were placed in his transits and all trouble ceased. He believes that all surveyors similarly circumstanced should use adjustable stadia wires and this after an extended experience in stadia work.

To adjust the wires, select a piece of ground practically level and lay off on it with a steel tape an accurately measured line several hundred feet long plus *C*. Set the transit at one end and have the rod held vertically at the other end. Level the telescope and sight on the rod. Deduct *C* from the exact distance and with the adjusting screws set the upper and lower wires to intercept this interval on the rod, taking care to keep them equidistant from the middle wire. Assume that the transit is 701 ft. from the rod and $C = 1.0$ ft.; the rod reading will be 7 ft. To catch changes in the wire interval the writer checked with a steel tape the first and last rod reading in the forenoon and the first and last rod reading in the afternoon of each day. Frequently the wires held the proper interval for 6 months and once no change was observed for a year. When the interval did change it was sudden and the ratio became as much as 1 : 104, generally however being 1 : 102.5 or 1 : 103.3, or a similarly annoying ratio. Instrument repairers generally blamed the cement used to hold the wires in place. Sometimes temperature effects on the diaphragm or too tight screwing up of adjusting screws may cause these alterations in the wire interval, which have been noticed by other writers. Alterations due to such causes disappear within a short time and the interval is apparently correct. By checking rod readings occasionally with a steel tape

and correcting readings by applying a constant thus found the stadia becomes one of the most useful of the tools of the surveyor. (See pp. 1041–2, Vol. LXXVII, *Trans. Am. Soc. C. E.*)

Stadia surveys of farms are made with a degree of accuracy suitable for the work. Either the radiation or the traverse method may be used, or a combination of these methods. Because the vertical angle must be read to reduce the inclined readings to horizontal distances information as to slope of surface is obtained while the measuring is being done. Distances are readily obtained with the stadia but cannot be set off, as with a tape or chain.

Stadia is a word with the same root as "stadium" the Latin form of "Stadion," the principal Greek measure of length, a little less than an eighth of a mile. Stadia rods are often called "telemeter" ("afar-off measuring") rods. In Europe a theodolite, designed to be used principally for topographical work and equipped with gradienter, stadia wires, etc., is called a "tachymeter," that is, "a rapid measurer."

Stadia Reduction Table

M.	0° Hor. dist.	0° Diff. Elev.	1° Hor. dist.	1° Diff. elev.	2° Hor. dist.	2° Diff. elev.
0'.........	100.00	0.00	99.97	1.74	99.88	3.49
2..........	100.00	0.06	99.97	1.80	99.87	3.55
4..........	100.00	0.12	99.97	1.86	99.87	3.60
6..........	100.00	0.17	99.96	1.92	99.87	3.66
8..........	100.00	0.23	99.96	1.98	99.86	3.72
10..........	100.00	0.29	99.96	2.04	99.86	3.78
12..........	100.00	0.35	99.96	2.09	99.85	3.84
14..........	100.00	0.41	99.95	2.15	99.85	3.90
16..........	100.00	0.47	99.95	2.21	99.84	3.95
18..........	100.00	0.52	99.95	2.27	99.84	4.01
20..........	100.00	0.58	99.95	2.33	99.83	4.07
22..........	100.00	0.64	99.94	2.38	99.83	4.13
24..........	100.00	0.70	99.94	2.44	99.82	4.18
26..........	99.99	0.76	99.94	2.50	99.82	4.24
28..........	99.99	0.81	99.93	2.56	99.81	4.30
30..........	99.99	0.87	99.93	2.62	99.81	4.36
32..........	99.99	0.93	99.93	2.67	99.80	4.42
34..........	99.99	0.99	99.93	2.73	99.80	4.48
36..........	99.99	1.05	99.92	2.79	99.79	4.53
38..........	99.99	1.11	99.92	2.85	99.79	4.59
40..........	99.99	1.16	99.92	2.91	99.78	4.65
42..........	99.99	1.22	99.91	2.97	99.78	4.71
44..........	99.98	1.28	99.91	3.02	99.77	4.76
46..........	99.98	1.34	99.90	3.08	99.77	4.82
48..........	99.98	1.40	99.90	3.14	99.76	4.88
50..........	99.98	1.45	99.90	3.20	99.76	4.94
52..........	99.98	1.51	99.89	3.26	99.75	4.99
54..........	99.98	1.57	99.89	3.31	99.74	5.05
56..........	99.97	1.63	99.89	3.37	99.74	5.11
58..........	99.97	1.69	99.88	3.43	99.73	5.17
60..........	99.97	1.74	99.88	3.49	99.73	5.23
$C=0.75$	0.75	0.01	0.75	0.02	0.75	0.03
$C=1.00$	1.00	0.01	1.00	0.03	1.00	0.04
$C=1.25$	1.25	0.02	1.25	0.03	1.25	0.05

STADIA REDUCTION TABLE (*Continued*)

M.	3°		4°		5°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	99.73	5.23	99.51	6.96	99.24	8.68
2	99.72	5.28	99.51	7.02	99.23	8.74
4	99.71	5.34	99.50	7.07	99.22	8.80
6	99.71	5.40	99.49	7.13	99.21	8.85
8	99.70	5.46	99.48	7.19	99.20	8.91
10	99.69	5.52	99.47	7.25	99.19	8.97
12	99.69	5.57	99.46	7.30	99.18	9.03
14	99.68	5.63	99.46	7.36	99.17	9.08
16	99.68	5.69	99.45	7.42	99.16	9.14
18	99.67	5.75	99.44	7.48	99.15	9.20
20	99.66	5.80	99.43	7.53	99.14	9.25
22	99.66	5.86	99.42	7.59	99.13	9.31
24	99.65	5.92	99.41	7.65	99.11	9.37
26	99.64	5.98	99.40	7.71	99.10	9.43
28	99.63	6.04	99.39	7.76	99.09	9.48
30	99.63	6.09	99.38	7.82	99.08	9.54
32	99.62	6.15	99.38	7.88	99.07	9.60
34	99.62	6.21	99.37	7.94	99.06	9.65
36	99.61	6.27	99.36	7.99	99.05	9.71
38	99.60	6.33	99.35	8.05	99.04	9.77
40	99.59	6.38	99.34	8.11	99.03	9.83
42	99.59	6.44	99.33	8.17	99.01	9.88
44	99.58	6.50	99.32	8.22	99.00	9.94
46	99.57	6.56	99.31	8.28	98.99	10.00
48	99.56	6.61	99.30	8.34	98.98	10.05
50	99.56	6.67	99.29	8.40	98.97	10.11
52	99.55	6.73	99.28	8.45	98.96	10.17
54	99.54	6.78	99.27	8.51	98.94	10.22
56	99.53	6.84	99.26	8.57	98.93	10.28
58	99.52	6.90	99.25	8.63	98.92	10.34
60	99.51	6.96	99.24	8.68	98.91	10.40
$C=0.75$	0.75	0.05	0.75	0.06	0.75	0.07
$C=1.00$	1.00	0.06	1.00	0.08	0.99	0.09
$C=1.25$	1.25	0.08	1.25	0.10	1.24	0.11

STADIA REDUCTION TABLE (*Continued*)

M.	6°		7°		8°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	98.91	10.40	98.51	12.10	98.06	13.78
2	98.90	10.45	98.50	12.15	98.05	13.84
4	98.88	10.51	98.48	12.21	98.03	13.89
6	98.87	10.57	98.47	12.26	98.01	13.95
8	98.86	10.62	98.46	12.32	98.00	14.01
10	98.85	10.68	98.44	12.38	97.98	14.06
12	98.83	10.74	98.43	12.43	97.97	14.12
14	98.82	10.79	98.41	12.49	97.95	14.17
16	98.81	10.85	98.40	12.55	97.93	14.23
18	98.80	10.91	98.39	12.60	97.92	14.28
20	98.78	10.96	98.37	12.66	97.90	14.34
22	98.77	11.02	98.36	12.72	97.88	14.40
24	98.76	11.08	98.34	12.77	97.87	14.45
26	98.74	11.13	98.33	12.83	97.85	14.51
28	98.73	11.19	98.31	12.88	97.83	14.56
30	98.72	11.25	98.29	12.94	97.82	14.62
32	98.71	11.30	98.28	13.00	97.80	14.67
34	98.69	11.36	98.27	13.05	97.78	14.73
36	98.68	11.42	98.25	13.11	97.76	14.79
38	98.67	11.47	98.24	13.17	97.75	14.84
40	98.65	11.53	98.22	13.22	97.73	14.90
42	98.64	11.59	98.20	13.28	97.71	14.95
44	98.63	11.64	98.19	13.33	97.69	15.01
46	98.61	11.70	98.17	13.39	97.68	15.06
48	98.60	11.76	98.16	13.45	97.66	15.12
50	98.58	11.81	98.14	13.50	97.64	15.17
52	98.57	11.87	98.13	13.56	97.62	15.23
54	98.56	11.93	98.11	13.61	97.61	15.28
56	98.54	11.98	98.10	13.67	97.59	15.34
58	98.53	12.04	98.08	13.73	97.57	15.40
60	98.51	12.10	98.06	13.78	97.55	15.45
$C=0.75$	0.75	0.08	0.74	0.10	0.74	0.11
$C=1.00$	0.99	0.11	0.99	0.13	0.99	0.15
$C=1.25$	1.24	0.14	1.24	0.16	1.23	0.18

STADIA REDUCTION TABLE (*Continued*)

M.	9°		10°		11°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	97.55	15.45	96.98	17.10	96.36	18.73
2	97.53	15.51	96.96	17.16	96.34	18.78
4	97.52	15.56	96.94	17.21	96.32	18.84
6	97.50	15.62	96.92	17.26	96.29	18.89
8	97.48	15.67	96.90	17.32	96.27	18.95
10	97.46	15.73	96.88	17.37	96.25	19.00
12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.43	15.84	96.84	17.48	96.21	19.11
16	97.41	15.89	96.82	17.54	96.18	19.16
18	97.39	15.95	96.80	17.59	96.16	19.21
20	97.37	16.00	96.78	17.65	96.14	19.27
22	97.35	16.06	96.76	17.70	96.12	19.32
24	97.33	16.11	96.74	17.76	96.09	19.38
26	97.31	16.17	96.72	17.81	96.07	19.43
28	97.29	16.22	96.70	17.86	96.05	19.48
30	97.28	16.28	96.68	17.92	96.03	19.54
32	97.26	16.33	96.66	17.97	96.00	19.59
34	97.24	16.39	96.64	18.03	95.98	19.64
36	97.22	16.44	96.62	18.08	95.96	19.70
38	97.20	16.50	96.60	18.14	95.93	19.75
40	97.18	16.55	96.57	18.19	95.91	19.80
42	97.16	16.61	96.55	18.24	95.89	19.86
44	97.14	16.66	96.53	18.30	95.86	19.91
46	97.12	16.72	96.51	18.35	95.84	19.96
48	97.10	16.77	96.49	18.41	95.82	20.02
50	97.08	16.83	96.47	18.46	95.79	20.07
52	97.06	16.88	96.45	18.51	95.77	20.12
54	97.04	16.94	96.42	18.57	95.75	20.18
56	97.02	16.99	96.40	18.62	95.72	20.23
58	97.00	17.05	96.38	18.68	95.70	20.28
60	96.98	17.10	96.36	18.73	95.68	20.34
$C=0.75$	0.74	0.12	0.74	0.14	0.73	0.15
$C=1.00$	0.99	0.16	0.98	0.18	0.98	0.20
$C=1.25$	1.23	0.21	1.23	0.23	1.22	0.25

STADIA REDUCTION TABLE (*Continued*)

M.	12°		13°		14°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	95.68	20.34	94.94	21.92	94.15	23.47
2	95.65	20.39	94.91	21.97	94.12	23.52
4	95.63	20.44	94.89	22.02	94.09	23.58
6	95.61	20.50	94.86	22.08	94.07	23.63
8	95.58	20.55	94.84	22.13	94.04	23.68
10	95.56	20.60	94.81	22.18	94.01	23.73
12	95.53	20.66	94.79	22.23	93.98	23.78
14	95.51	20.71	94.76	22.28	93.95	23.83
16	95.49	20.76	94.73	22.34	93.93	23.88
18	95.46	20.81	94.71	22.39	93.90	23.93
20	95.44	20.87	94.68	22.44	93.87	23.99
22	95.41	20.92	94.66	22.49	93.84	24.04
24	95.39	20.97	94.63	22.54	93.81	24.09
26	95.36	21.03	94.60	22.60	93.79	24.14
28	95.34	21.08	94.58	22.65	93.76	24.10
30	95.32	21.13	94.55	22.70	93.73	24.24
32	95.29	21.18	94.52	22.75	93.70	24.29
34	95.27	21.24	94.50	22.80	93.67	24.34
36	95.24	21.29	94.47	22.85	93.65	24.39
38	95.22	21.34	94.44	22.91	93.62	24.44
40	95.19	21.39	94.42	22.96	93.59	24.49
42	95.17	21.45	94.39	23.01	93.56	24.55
44	95.14	21.50	94.36	23.06	93.53	24.60
46	95.12	21.55	94.34	23.11	93.50	24.65
48	95.09	21.60	94.31	23.16	93.47	24.70
50	95.07	21.66	94.28	23.22	93.45	24.75
52	95.04	21.71	94.26	23.27	93.42	24.80
54	95.02	21.76	94.23	23.32	93.39	24.85
56	94.99	21.81	94.20	23.37	93.36	24.90
58	94.97	21.87	94.17	23.42	93.33	24.95
60	94.94	21.92	94.15	23.47	93.30	25.00
$C=0.75$	0.73	0.16	0.73	0.17	0.73	0.19
$C=1.00$	0.98	0.22	0.97	0.23	0.97	0.25
$C=1.25$	1.22	0.27	1.21	0.20	1.21	0.31

STADIA REDUCTION TABLE (*Continued*)

M.	15°		16°		17°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	93.30	25.00	92.40	26.50	91.45	27.96
2	93.27	25.05	92.37	26.55	91.42	28.01
4	93.24	25.10	92.34	26.59	91.39	28.06
6	93.21	25.15	92.31	26.64	91.35	28.10
8	93.18	25.20	92.28	26.69	91.32	28.15
10	93.16	25.25	92.25	26.74	91.29	28.20
12	93.13	25.30	92.22	26.79	91.26	28.25
14	93.10	25.35	92.19	26.84	91.22	28.30
16	93.07	25.40	92.15	26.89	91.19	28.34
18	93.04	25.45	92.12	26.94	91.16	28.39
20	93.01	25.50	92.09	26.99	91.12	28.44
22	92.98	25.55	92.06	27.04	91.09	28.49
24	92.95	25.60	92.03	27.09	91.06	28.54
26	92.92	25.65	92.00	27.13	91.02	28.58
28	92.89	25.70	91.97	27.18	90.99	28.63
30	92.86	25.75	91.93	27.23	90.96	28.68
32	92.83	25.80	91.90	27.28	90.92	28.73
34	92.80	25.85	91.87	27.33	90.89	28.77
36	92.77	25.90	91.84	27.38	90.86	28.82
38	92.74	25.95	91.81	27.43	90.82	28.87
40	92.71	26.00	91.77	27.48	90.79	28.92
42	92.68	26.05	91.74	27.52	90.76	28.96
44	92.65	26.10	91.71	27.57	90.72	29.01
46	92.62	26.15	91.68	27.62	90.69	29.06
48	92.59	26.20	91.65	27.67	90.66	29.11
50	92.56	26.25	91.61	27.72	90.62	29.15
52	92.53	26.30	91.58	27.77	90.59	29.20
54	92.49	26.35	91.55	27.81	90.55	29.25
56	92.46	26.40	91.52	27.86	90.52	29.30
58	92.43	26.45	91.48	27.91	90.48	29.34
60	92.40	26.50	91.45	27.96	90.45	29.39
$C=0.75$	0.72	0.20	0.72	0.21	0.72	0.23
$C=1.00$	0.96	0.27	0.96	0.28	0.95	0.30
$C=1.25$	1.20	0.34	1.20	0.36	1.19	0.38

STADIA REDUCTION TABLE (*Continued*)

M.	18°		19°		20°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	90.45	29.39	89.40	30.78	88.30	32.14
2	90.42	29.44	89.36	30.83	88.26	32.18
4	90.38	29.48	89.33	30.87	88.23	32.23
6	90.35	29.53	89.29	30.92	88.19	32.27
8	90.31	29.58	89.26	30.97	88.15	32.32
10	90.28	29.62	89.22	31.01	88.11	32.36
12	90.24	29.67	89.18	31.06	88.08	32.41
14	90.21	29.72	89.15	31.10	88.04	32.45
16	90.18	29.76	89.11	31.15	88.00	32.49
18	90.14	29.81	89.08	31.19	87.96	32.54
20	90.11	29.86	89.04	31.24	87.93	32.58
22	90.07	29.90	89.00	31.28	87.89	32.63
24	90.04	29.95	88.96	31.33	87.85	32.67
26	90.00	30.00	88.93	31.38	87.81	32.72
28	89.97	30.04	88.89	31.42	87.77	32.76
30	89.93	30.09	88.86	31.47	87.74	32.80
32	89.90	30.14	88.82	31.51	87.70	32.85
34	89.86	30.19	88.78	31.56	87.66	32.89
36	89.83	30.23	88.75	31.60	87.62	32.93
38	89.79	30.28	88.71	31.65	87.58	32.98
40	89.76	30.32	88.67	31.69	87.54	33.02
42	89.72	30.37	88.64	31.74	87.51	33.07
44	89.69	30.41	88.60	31.78	87.47	33.11
46	89.65	30.46	88.56	31.83	87.43	33.15
48	89.61	30.51	88.53	31.87	87.39	33.20
50	89.58	30.55	88.49	31.92	87.35	33.24
52	89.54	30.60	88.45	31.96	87.31	33.28
54	89.51	30.65	88.41	32.01	87.27	33.33
56	89.47	30.69	88.38	32.05	87.24	33.37
58	89.44	30.74	88.34	32.09	87.20	33.41
60	89.40	30.78	88.30	32.14	87.16	33.46
$C=0.75$	0.71	0.24	0.71	0.25	0.70	0.26
$C=1.00$	0.95	0.32	0.94	0.33	0.94	0.35
$C=1.25$	1.19	0.40	1.18	0.42	1.17	0.44

STADIA REDUCTION TABLE (*Continued*)

M.	21°		22°		23°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	87.16	33.46	85.97	34.73	84.73	35.97
2	87.12	33.50	85.93	34.77	84.69	36.01
4	87.08	33.54	85.89	34.82	84.65	36.05
6	87.04	33.59	85.85	34.86	84.61	36.09
8	87.00	33.63	85.80	34.90	84.57	36.13
10	86.96	33.67	85.76	34.94	84.52	36.17
12	86.92	33.72	85.72	34.98	84.48	36.21
14	86.88	33.76	85.68	35.02	84.44	36.25
16	86.84	33.80	85.64	35.07	84.40	36.29
18	86.80	33.84	85.60	35.11	84.35	36.33
20	86.77	33.89	85.56	35.15	84.31	36.37
22	86.73	33.93	85.52	35.19	84.27	36.41
24	86.69	33.97	85.48	35.23	84.23	36.45
26	86.65	34.01	85.44	35.27	84.18	36.49
28	86.61	34.06	85.40	35.31	84.14	36.53
30	86.57	34.10	85.36	35.36	84.10	36.57
32	86.53	34.14	85.31	35.40	84.06	36.61
34	86.49	34.18	85.27	35.44	84.01	36.65
36	86.45	34.23	85.23	35.48	83.97	36.69
38	86.41	34.27	85.19	35.52	83.93	36.73
40	86.37	34.31	85.15	35.56	83.89	36.77
42	86.33	34.35	85.11	35.60	83.84	36.80
44	86.29	34.40	85.07	35.64	83.80	36.84
46	86.25	34.44	85.02	35.68	83.76	36.88
48	86.21	34.48	84.98	35.72	83.72	36.92
50	86.17	34.52	84.94	35.76	83.67	36.96
52	86.13	34.57	84.90	35.80	83.63	37.00
54	86.09	34.61	84.86	35.85	83.59	37.04
56	86.05	34.65	84.82	35.89	83.54	37.08
58	86.01	34.69	84.77	35.93	83.50	37.12
60	85.97	34.73	84.73	35.97	83.46	37.16
$C=0.75$	0.70	0.27	0.69	0.29	0.69	0.30
$C=1.00$	0.93	0.37	0.92	0.38	0.92	0.40
$C=1.25$	1.16	0.46	1.15	0.48	1.15	0.50

STADIA REDUCTION TABLE (*Continued*)

M.	24°		25°		26°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′	83.46	37.16	82.14	38.30	80.78	39.40
2	83.41	37.20	82.09	38.34	80.74	39.44
4	83.37	37.23	82.05	38.38	80.69	39.47
6	83.33	37.27	82.01	38.41	80.65	39.51
8	83.28	37.31	81.96	38.45	80.60	39.54
10	83.24	37.35	81.92	38.49	80.55	39.58
12	83.20	37.39	81.87	38.53	80.51	39.61
14	83.15	37.43	81.83	38.56	80.46	39.65
16	83.11	37.47	81.78	38.60	80.41	39.69
18	83.07	37.51	81.74	38.64	80.37	39.72
20	83.02	37.54	81.69	38.67	80.32	39.76
22	82.98	37.58	81.65	38.71	80.28	39.79
24	82.93	37.62	81.60	38.75	80.23	39.83
26	82.89	37.66	81.56	38.78	80.18	39.86
28	82.85	37.70	81.51	38.82	80.14	39.90
30	82.80	37.74	81.47	38.86	80.09	39.93
32	82.76	37.77	81.42	38.89	80.04	39.97
34	82.72	37.81	81.38	38.93	80.00	40.00
36	82.67	37.85	81.33	38.97	79.95	40.04
38	82.63	37.89	81.28	39.00	79.90	40.07
40	82.58	37.93	81.24	39.04	79.86	40.11
42	82.54	37.96	81.19	39.08	79.81	40.14
44	82.49	38.00	81.15	39.11	79.76	40.18
46	82.45	38.04	81.10	39.15	79.72	40.21
48	82.41	38.08	81.06	39.18	79.67	40.24
50	82.36	38.11	81.01	39.22	79.62	40.28
52	82.32	38.15	80.97	39.26	79.58	40.31
54	82.27	38.19	80.92	39.29	79.53	40.35
56	82.23	38.23	80.87	39.33	79.48	40.38
58	82.18	38.26	80.83	39.36	79.44	40.42
60	82.14	38.30	80.78	39.40	79.39	40.45
$C=0.75$	0.68	0.31	0.68	0.32	0.67	0.33
$C=1.00$	0.91	0.41	0.90	0.43	0.89	0.45
$C=1.25$	1.14	0.52	1.13	0.54	1.12	0.56

STADIA REDUCTION TABLE

M.	27°		28°		29°		30°	
	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.	Hor. dist.	Diff. elev.
0′......	79.39	40.45	77.96	41.45	76.50	42.40	75.00	43.30
2.......	79.34	40.49	77.91	41.48	76.45	42.43	74.95	43.33
4.......	79.30	40.52	77.86	41.52	76.40	42.46	74.90	43.36
6.......	79.25	40.55	77.81	41.55	76.35	42.49	74.85	43.39
8.......	79.20	40.59	77.77	41.58	76.30	42.53	74.80	43.42
10.......	79.15	40.62	77.72	41.61	76.25	42.56	74.75	43.45
12.......	79.11	40.66	77.67	41.65	76.20	42.59	74.70	43.47
14.......	79.06	40.69	77.62	41.68	76.15	42.62	74.65	43.50
16.......	79.01	40.72	77.57	41.71	76.10	42.65	74.60	43.53
18.......	78.96	40.76	77.52	41.74	76.05	42.68	74.55	43.56
20.......	78.92	40.79	77.48	41.77	76.00	42.71	74.49	43.59
22.......	78.87	40.82	77.42	41.81	75.95	42.74	74.44	43.62
24.......	78.82	40.86	77.38	41.84	75.90	42.77	74.39	43.65
26.......	78.77	40.89	77.33	41.87	75.85	42.80	74.34	43.67
28.......	78.73	40.92	77.28	41.90	75.80	42.83	74.29	43.70
30.......	78.68	40.96	77.23	41.93	75.75	42.86	74.24	43.73
32.......	78.63	40.99	77.18	41.97	75.70	42.89	74.19	43.76
34.......	78.58	41.02	77.13	42.00	75.65	42.92	74.14	43.79
36.......	78.54	41.06	77.09	42.03	75.60	42.95	74.09	43.82
38.......	78.49	41.09	77.04	42.06	75.55	42.98	74.04	43.84
40.......	78.44	41.12	76.99	42.09	75.50	43.01	73.99	43.87
42.......	78.39	41.16	76.94	42.12	75.45	43.04	73.93	43.90
44.......	78.34	41.19	76.89	42.15	75.40	43.07	73.88	43.93
46.......	78.30	41.22	76.84	42.19	75.35	43.10	73.83	43.95
48.......	78.25	41.26	76.79	42.22	75.30	43.13	73.78	43.98
50.......	78.20	41.29	76.74	42.25	75.25	43.16	73.73	44.01
52.......	78.15	41.32	76.69	42.28	75.20	43.18	73.68	44.04
54.......	78.10	41.35	76.64	42.31	75.15	43.21	73.63	44.07
56.......	78.06	41.39	76.59	42.34	75.10	43.24	73.58	44.09
58.......	78.01	41.42	76.55	42.37	75.05	43.27	73.52	44.12
60.......	77.96	41.45	76.50	42.40	75.00	43.30	73.47	44.15
$C=0.75$	0.66	0.35	0.66	0.36	0.65	0.37	0.65	0.38
$C=1.00$	0.89	0.46	0.88	0.48	0.87	0.49	0.86	0.51
$C=1.25$	1.11	0.58	1.10	0.60	1.09	0.62	1.08	0.64

The subdividing of land into building lots; the re-survey of lots; keeping records; filing notes and records; setting monuments, etc., are all fully dealt with in the author's book "Engineering Work in Towns and Cities." ($3.00.)

In making re-surveys a random line must first be run; sometimes several such lines. If the compass and chain were employed on the original survey, it is advisable to run the random lines with the needle and employ green hands to measure with an old-fashioned chain. In this way the locations of monuments will be discovered more readily than if modern methods are employed. When the corners are located the true lines should be run with the transit and a steel tape used by skilled helpers. The new notes should then be recorded.

MAKING THE MAP

A complete map should contain all the data necessary to enable a competent surveyor to re-trace the lines. Not less than two permanent monuments should be shown on the map, connected to the lines of the survey, with tie lines to reference points.

The following items should be shown on a map and the list is given as a sort of "specification reminder" for the surveyor and draftsman.

1. Scale of map.
2. Meridian line with declination of needle.
3. A short title with the date of survey.
4. Monuments: Location; description; tie lines.
5. Lengths of all lines.
6. Bearings of all lines.
7. Angles of intersection of all lines.
8. Name of owner of record.
9. Names of adjoining owners of record placed on their land with location of common corners.
10. Names of all recognized landmarks within the area embraced in the map.
11. Statement over signature of the surveyor that he has carefully checked the map, and that it truly represents the survey made by him, and that he set the monuments described and drove a stake at each corner.

If the map is of an addition to a village, town or city, or is of a subdivision of a tract of land into small parcels or lots, it should contain all the foregoing and in addition the following:

12. The number of each block and lot.
13. The width and name of each thoroughfare.
14. The dedication, acknowledged before a qualified legal officer, of the public thoroughfares to the use of the public.

Every owner whose property is shown on the map as being included within the areas dedicated to public use must sign the dedication.

Mr. S. N. Howard of Chicago, in the *Proceedings of the Illinois Society of Engineers and Surveyors*, gave the following list of items to be shown on maps of city lot surveys:

"Lines, grades, angles, location and elevation of sewer, elevation of walks in front of lot and adjoining the same and elevation of ground; and sometimes in addition is the size and location of water, gas and electric mains; the location of point where water, gas and electric service pipes come through the curb; location of house drains; the location, height and plumb of adjoining buildings; thickness of party walls at the several stories, and location and depth of the foundations of same."

City lot surveys are usually made for building purposes and an architect needs all the above data.

Maps of transit surveys may be plotted like maps of compass surveys with protractors. (See Chapter on Compass Surveying.)

When for any reason the protractor angles are not accurate enough, angles may be set out to the nearest minute by using either a table of chords or a table of natural tangents. If the surveyor has no book containing a table of chords he can use natural sines. *Twice the sine of half the angle is the chord of the whole angle.*

First lay off a base ten inches (or units) long. The tables express the values of the functions as decimal fractions, so this length of 10 = unity (1). The chord (or natural tangent) is measured with the scale used for the base.

In Fig. 204 let AC = base = unity and from A as a center describe the arc BC. Set the compass to scale the chord BC of the angle BAC, and from C as a center with radius = chord, intersect arc BC at B. Draw a straight line from A through B, thus forming the angle BAC.

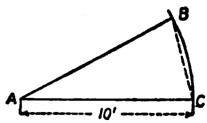


FIG. 204.

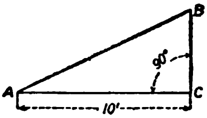


FIG. 205.

In Fig. 205 let AC = base = unity, and at C erect the perpendicular BC. On the perpendicular lay off the tangent of the angle A. The line is then drawn from A through B to form the angle BAC.

The angles may be set off from the ends of lines, thus resembling deflection work, or two lines may be drawn normal to each other in the middle of the sheet and a circle drawn with radius = 10 units of scale, with the intersection as the center of the circle. Chords may be set off on the circumference or tangents measured from the end of the radius for all the angles. The lines can then be transferred by using triangles and straight-edge.

If the error in the survey is large enough to show on the map a closure must be "fudged," no matter how accurately the angles are drawn. If the survey is balanced so the errors are distributed it will be necessary to calculate new bearings and distances if angles are to be plotted. This however is not right, besides being very laborious. The "latitude and departure method" enables a draftsman to make an accurate map on which the lines will close and it is used by all experienced men who are noted for careful work.

The draftsman must remember that the field work has been done with all possible care. In spite of this there is an error of closure, the amount of which depends on the value of the land. The error is probably distributed throughout all the courses so the bearings and distances recorded by the instrument man must be placed on the map. The error of closure is discovered by the computa-

tions, the field work not disclosing it. The same conditions must appear on the map and inaccuracies in plotting added to the field error may prevent the lines closing by a quite appreciable amount. To plat the work by using latitudes and departures, the lines must close, and furthermore the computations give the exact amount of error and the surveyor knows whether it lies within proper limits.

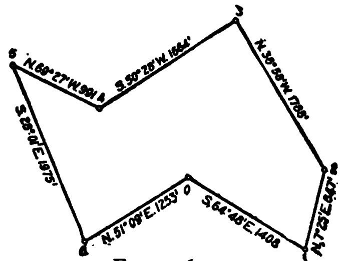


FIG. 206.

The computations for the field shown in Fig. 206 are given in table on the following page.

The latitudes and departures are tabulated, the error found and distributed. The balanced latitudes and departures are then algebraically as follows:

Latitudes	Departures
− 599.51	+ 1273.96
+ 839.89	+ 109.28
+ 240.38	+ 1383.24
+ 1428.58	− 1075.26
+ 1668.96	+ 307.98
− 1059.22	− 1283.46
+ 609.74	− 975.48
+ 347.89	− 927.94
+ 957.63	− 1903.42
− 1743.62	+ 927.66
− 785.99	− 975.76

The starting point has no latitude or departure. In going around a field the travel north equals the travel south and the travel east equals the travel west, so the algebraic latitude and departure for the last station must equal the balanced latitude and departure with the sign reversed. This checks the summation.

The method of plotting is shown in Fig. 207. Through the point selected for Sta. O draw the horizontal line OX, OX' and the vertical line OY, OY'. Divide the sheet into squares 100 ft. by 100 ft. Number them as shown and in each square in which a corner will be located measure the

Sta.	Bearing.	Length in ft.	Latitudes.		Departures.		Balanced. Latitudes.		Balanced. Departures.		Total latitude.	Total departure.
			N+	S−	E+	W−	N+	S−	E+	W−		
0	S 64° 48′ E	1408	....	599.5	1274.0	.....	.....	599.51	1273.96	.....	±0.0	±0.0
1	N 7° 25′ E	847	839.9	....	109.3	.....	839.89	.....	109.28	.....	−599.51	+1273.96
2	N 36° 58′ W	1788	1428.6	....	.....	1075.2	1428.58	.....	.....	1075.26	+240.38	+1383.24
3	S 50° 28′ W	1664	....	1059.2	.....	1283.4	.....	1059.22	.....	1283.46	+1668.96	+307.98
4	N 69° 27′ W	991	347.9	....	.....	927.9	347.89	.....	.....	927.94	+609.74	−975.48
5	S 28° 01′ E	1975	....	1743.6	927.7	.....	.....	1743.62	927.66	.....	+957.63	−1903.42
6	N 51° 09′ E	1253	786.0	....	975.8	.....	785.99	.....	975.76	.....	−785.99	−975.76
			3402.4	3402.3	3286.8	3286.5	3402.35	3402.35	3286.66	3286.66		
			3402.3		3286.5							
			0.1		0.3							

fractional parts, draw the intersecting latitude and departure lines and locate the corner. Finish the outline of the field by connecting the corners. When the outlines and all

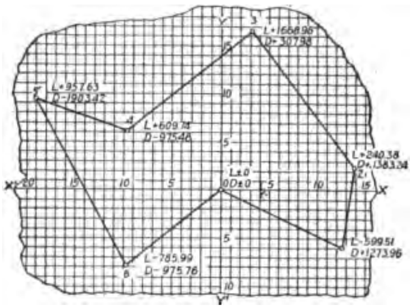


FIG. 207. Platting by latitudes and departures.

bearings and lengths are inked in, the co-ordinate lines are erased, for they are drawn with a sharp, hard lead pencil.

Maps expected to be in fairly constant service (working maps) should be drawn on the best quality of cloth mounted paper and the co-ordinate lines should be very fine red ink lines. These maps never leave the surveyor's office, tracings being made of such portions as may be wanted by clients. In many cities maps are drawn on sheets with co-ordinate lines drawn in ink. The co-ordinates (latitude and departure) are marked in figures at each monument and lot corner. The bearing and distance between any two points may then be obtained readily by trigonometry. It is an admirable way to keep records and working maps. Steel straight-edges should be used in drawing the lines. To project a line too long to be drawn with the straight-edge, drive a fine needle at one end to which fasten a fine silk thread. Stretch the thread exactly over the line and at points which may be included within the length of the straight-edge make fine marks on the paper. Draw the extended line through these marks.

REPRODUCING MAPS

For reproducing drawings smaller than 12 ins. by 18 ins. hektographs or clay process pads are good. The hektograph is a pad of gelatine in a shallow pan, the clay pad

being a substitute for gelatine and in appearance resembling putty. The drawings are made with specially prepared hektograph inks on a smooth hard paper. This is laid face down on the pad and rubbed to remove wrinkles and air bubbles. It is left in place for three minutes. After removal plain pieces of paper, preferably not so hard as the original, are laid on the pad in rapid succession and rubbed smooth. Each sheet receives a print, the impressions becoming gradually more faint. Fifteen to twenty good impressions is the limit. The inks can be had in a number of colors. If only a few copies are wanted, less than six or seven, the original drawing may be made with copying pencils, also to be obtained in several colors.

For larger drawings tracings are used. Tracing cloth is costly, and medium thick "Parchment," "Vellum," "Colonna" or "Ionic" tracing papers are well suited to the use of the surveyor. When tracings are to receive much handling or are expected to be used for many years they should be on tracing cloth. The drawing is made on paper and traced in ink on the transparent cloth or paper. To save time on unimportant work the drawing is made directly on tracing paper or on the rough side of tracing cloth in lead pencil and then inked in. Tracing cloth and paper are greatly affected by moisture in the atmosphere so it is best to make accurate maps first on paper. Surveyors should use a good quality of paper for maps.

When several copies of a drawing are wanted the tracing is blue printed. Blue-print papers are sold by all dealers in surveying supplies. In a frame holding a sheet of plate glass, the tracing is placed on the glass. The sensitized surface of a sheet of blue-print paper is laid on the tracing. On top is placed a piece of felt or blanket, and the whole is covered with a wooden cover held in place by springs and clamps. All bubbles of air and wrinkles in the tracing, the blue-print paper and the felt must be eliminated. The glass is exposed to the direct rays of the sun for several minutes, depending on the sensitiveness of the prepared paper. The blue-print paper is afterwards washed in clean water until white lines appear on a blue ground, and then hung on a line to dry.

Men who have occasionally to make blue prints find it

convenient to prepare paper for their own use, for this material does not have good keeping properties. Take one-half ounce each of potassium ferrid-cyanide and citrate of iron and ammonia; dissolve in from 6 ozs. to 8 ozs. of clean water. Put in a bottle and shake thoroughly until the chemicals are dissolved, which process requires about 10 minutes. The mixture does not keep well so a fresh lot should be prepared for each job of printing. Only chemically pure (C. P.) materials should be used and they can be bought in crystal form from any druggist. Keep in a light-proof bottle with ground-glass stopper.

Eight ounces of the mixture will coat about 100 square feet of paper. Lay in a shallow dish a piece of thin cotton or linen cloth and pour in the mixture. Then lift up the cloth to allow the liquid to strain through so all undissolved lumps will be removed. The work should be done in a room lighted by a red or orange light. Use a broad flat brush for coating the paper, which should be a good bond paper with smooth surface. Hang in the dark until dry and keep in the dark until used.

White lines on a blue ground are not satisfactory when a map is to be colored, but with special inks or erasing fluids white, red and yellow lines may be drawn on blue prints. Several manufacturers sell paper which is used like blue-print paper, the lines coming out blue, black or brown on a white ground. There is also on the market a thin paper which, used like blue-print paper, shows white lines on a brown ground. Such a print may be used with ordinary blue-print paper, as a photographic negative, the result being a print with blue lines on a white ground.

PRACTICAL ASTRONOMY

An ephemeris is a table giving the place of a planet for a number of successive days. The solar ephemeris is of value to surveyors and engineers, for by observations on the sun the local time at a place may be ascertained, the latitude found and the true meridian determined.

Practically all instrument makers issue annually small books containing the solar ephemeris for the year, together with much other useful information. These vest pocket

books are sold for a small price, usually ten cents. From the 1912 edition of the twenty-five cent book issued by Wm. Ainsworth & Sons, Denver, Colo., the following concise description of practical astronomical work has been taken.

The Sun is the center of the solar system, remaining constantly fixed in its position, although often spoken of as in motion around the earth.

The Earth makes a complete revolution around the sun in three hundred and sixty-five days, five hours, forty-eight minutes and forty-six seconds.

It also rotates about an imaginary line passing through its center, and termed its axis, once in twenty-three hours, fifty-six minutes and four seconds, mean time, turning from west to east.

The Poles are the extremities of the earth's axis. The pole in our own hemisphere, known as the North Pole, if produced indefinitely toward the concave surface of the heavens, would reach the North Pole of the heavens a point situated near the Polar star.

The Equator is an imaginary line passing around the earth, equidistant from the poles, and in a plane at right angles with the axis.

If the plane of the equator be produced to the heavens, it forms what is termed the Celestial Equator.

The Orbit of the earth is the path in which it moves in making its yearly revolution. If the plane of this orbit were produced to the heavens, it would form the Ecliptic, or the sun's apparent path in the heavens.

The earth's axis is inclined to its orbit at an angle of about 23° 27′, making an angle of the same amount between the earth's orbit and its equator, or between the Celestial Equator and the Ecliptic.

The Equinoxes are the two points in which the Ecliptic and the Celestial Equator intersect one another.

The Horizon of a place is the surface which is defined by a plane supposed to pass through the place at right angles with a vertical line, and to bound our vision at the surface of the earth. The horizon, or a horizontal surface, is determined by the surface of any liquid when at rest, or by the spirit-levels of an instrument.

The Zenith of any place is the point directly overhead, in a line at right angles with the horizon.

The Meridian of any place is a great circle passing through the zenith of a place and the poles of the earth.

The Latitude of a place is its distance north or south of the Equator, measured on a Meridian. At the equator the latitude is 0°, at the poles 90°.

Refraction. — By reason of the atmosphere, the rays of light from the sun are bent out of their course, so as to make its altitude appear greater than is actually the case.

The amount of refraction varies according to the altitude of the body observed, being zero when it is in the zenith, about one minute when midway from the zenith to the horizon, and almost thirty-four minutes when in the horizon.

The Longitude of a place is its angular distance east or west of a given place taken as the starting point or first meridian; it is measured on the equator or on any parallel of latitude and usually from Greenwich, England.

As the earth makes a complete rotation upon its axis once a day, every point on its surface must pass over 360° in twenty-four hours, or 15° in one hour, and so on in the same ratio. And as the rotation is from west to east, the sun would come to the meridian of every place 15° west of Greenwich just one hour later than the time given in the Ephemeris for apparent noon at that place.

To an observer situated at Denver, Colo., the longitude of which is, in time, seven hours, the sun would come to the meridian seven hours later than at Greenwich, and thus when it was 12 M. at that place it would be but 5 A.M. in Denver.

TIME

A Solar Day is the interval of time between two successive upper transits of the sun across the same meridian. Solar days are of unequal length. A mean solar day is the average for a year.

A Sideral Day is the interval of time between two successive upper transits of a fixed star across the same meridian; it is invariable and is equal to twenty-three hours, fifty-six minutes, four and nine-hundredths seconds

of mean solar time. The earth makes one complete rotation on its axis in a sideral day.

Mean Noon. — A clock keeps mean solar time when it divides a mean solar day into twenty-four equal parts or hours. Noon as shown by such a clock is mean noon.

Apparent Noon for any place is the time of the upper transit of the sun across the meridian of that place; it may occur several minutes earlier or later than mean noon.

Equation of Time. — The column headed "Equation of Time" in the Ephemeris shows the quantity to add to or subtract from mean time to obtain the corresponding apparent time.

Standard Time. — Since November, 1883, in the United States, the mean solar time of the meridians 60, 75, 90, 105, and 120 west of Greenwich is standard time. The time spaces are known respectively as Colonial, Eastern, Central, Mountain and Pacific time. Each differs from the next in time by one hour. Instead of employing the local mean solar time, the time used is the mean solar time at the nearest of the standard meridians.

Hour Angle. — The number of hours from the meridian.

To Set a Watch to Mean Local Time by the Sun. — Set up the transit and adjust the telescope to the true meridian, then note the exact time that the center of the sun's image crosses the vertical cross-hair. This is apparent noon, and at this instant, set the minute hand to as many minutes before 12 as the equation of time for the given day shows is to be added to mean time, or to as many minutes after 12 as it shows is to be subtracted. The correction may be noted in case it is not desired to set the watch.

DECLINATION

The Declination of the sun is its angular distance north or south of the celestial equator; when the sun is at the equinoxes, that is, about the 21st of March and the 21st of September of each year, its declination is 0, or it is said to be on the equator; from these points its declination increases from day to day and from hour to hour, until on the 21st of June and the 21st of December it is 23° 27′ distant from the equator.

It is the declination which causes the sun to appear so much higher in summer than in winter, its altitude in the heavens being about 46° 54′ more on the 21st of June than it is on the 21st of December.

The Ephemeris gives the sun's declination for mean noon at Greenwich for each day in the year. The declination of the sun at any place for any hour of the day is determined from the Ephemeris as follows:

1. Divide the longitude of the place (reckoned from Greenwich) by 15 to obtain the corresponding difference of time in hours.
2. Find the corresponding Greenwich time by adding the difference of time to the mean time at the given place, when west from Greenwich, and subtracting when east.
3. Multiply the difference for one hour, as found in the table opposite the given day of the year, by the number of hours from noon by Greenwich time.
4. This product is the change in declination to be applied as indicated in the following expression:

$$\left.\begin{matrix}\textit{Declination for}\\ \textit{given time}\\ \textit{and place}\end{matrix}\right\} = \left\{\begin{matrix}\textit{apparent dec.}\\ \textit{for given day}\\ \textit{of year}\end{matrix}\right\} \pm \left\{\begin{matrix}\textit{change in}\\ \textit{declina-}\\ \textit{tion.}\end{matrix}\right.$$

The sign of the last term is + for time after noon, Greenwich, when declination is increasing, and for time before noon when declination is decreasing.

The − sign is to be used for time after noon, Greenwich, when declination is decreasing and for time before noon when declination is increasing.

An inspection of the Ephemeris will show whether the declination is increasing or decreasing from day to day.

N in the column of apparent declination indicates north, and S indicates south declination.

Example. — Required the declination at 10 A.M., Aug. 10, 1911, at Denver, Colo., U. S. A., latitude 39° 46′ 31″ north, longitude 105° west.

1. Diff. of time $= \frac{105}{15} = 7$ hrs.

2. As Denver is west from Greenwich, add the diff. of time, obtaining 10 A.M., Denver time $= 10 + 7 = 17$, or 5 P.M., Greenwich time.

3. Change in dec. for 1 hr. $= 43''.32$
   Change for 5 hrs. $= 43''.32 \times 5 = 0° 03' 36''.60$
4. Sun's apparent dec. at
   Greenwich, mean noon $= N\ 15° 49' 53''.60$
   Dec. at 10 A.M., Denver $= N\ 15° 46' 17''$

The change is subtracted as the time is afternoon and dec. is decreasing, since the dec. the next day is less.

LATITUDE

Determination of Latitude by Direct Observation of the Sun. — Carefully level the transit a few minutes before apparent noon, and if it is not provided with solar hairs, bring the horizontal cross-hair tangent to the upper limb of the sun and keep it tangent by the slow motion screw until the sun ceases to rise, then read the vertical angle, and from this angle subtract the semi-diameter of the sun, as given in the Ephemeris for the proper month of the year, also the refraction corresponding to the observed angle. The resulting angle will be the true altitude of the sun's center. Calculate the sun's declination for noon, apparent time, for place of observation as described above. If the declination is *N*, *subtract*, but if it is *S*, *add* it to the true altitude of the sun. The result is the co-latitude of the place of observation and the lat. = 90° − co.-lat.

Note. — In direct solar observations it is necessary to protect the eye by a darkened glass, which should be used at the eye end of the telescope unless both its surfaces are true and parallel. When the altitude is high, a diagonal eyepiece will be found convenient, or an image of the sun and cross-hairs may be formed on a screen, a card or a blank page of a notebook held a few inches from the eyepiece. If this method is used no darkener is required. An average of several observations is preferable to a single observation. When more than one observation is made, the alternate ones should be taken with the telescope reversed in order to eliminate instrumental errors. When great accuracy is not essential, standard time may be used in computing the declination. As the difference between standard and local time is seldom more than 30 min. and the greatest hourly change in declination is about one

minute of an arc, the maximum error in declination due to using standard time would not be greater than 30 seconds.

Example. — On April 17, 1910, an observation was made of the sun to determine the latitude of Denver. The horizontal cross-hair was kept tangent to the lower limb of the sun until it ceased to rise, then the vertical circle reading was 60° 20′ 00″.

Apparent alt. lower limb	= 60° 20′ 00″
Refraction for 60° 20′ 00″	= 00° 00′ 34″
True alt. lower limb	= 60° 19′ 26″
Add sun's semi-diameter	= 00° 15′ 58″
True alt. sun's center	= 60° 35′ 24″
Subtract declination	= 10° 21′ 55″
Co-lat.	= 50° 13′ 29″
Latitude	= 39° 46′ 31″

Latitude may be determined by reference to an accurate map, from which the latitude of a neighboring point may be found and then due allowance made for the distance north or south to the station.

The table on page 275 gives the length, in feet, of one minute of arc and the number of minutes of arc in a mile for latitude and longitude, from 0° to 60° latitude by one-degree intervals.

MEAN REFRACTION.

(To be Subtracted from Observed Altitude, in Direct Solar Observations.) Barometer 30 Inches; Thermometer 50° F.

Altitude.	Refraction.	Altitude.	Refraction.
10°	5′ 19″	20°	2′ 39″
11°	4′ 51″	25°	2′ 04″
12°	4′ 27″	30°	1′ 41″
13°	4′ 07″	35°	1′ 23″
14°	3′ 49″	40°	1′ 09″
15°	3′ 34″	45°	58″
16°	3′ 20″	50°	49″
17°	3′ 08″	60°	34″
18°	2′ 57″	70°	21″
19°	2′ 48″	80°	10″

TABLE SHOWING FEET PER MINUTE (ARC) AND MINUTES (ARC) PER MILE OF LATITUDE AND LONGITUDE

Latitude.	Length of 1 Min. in Feet.		No. of Min. in 1 Mile.	
	Latitude.	Longitude.	Latitude.	Longitude.
0°	6045	6087	0.8734	0.8674
1°	6045	6085	0.8734	0.8677
2°	6045	6083	0.8734	0.8679
3°	6045	6078	0.8734	0.8687
4°	6045	6071	0.8734	0.8697
5°	6045	6063	0.8734	0.8708
6°	6045	6053	0.8734	0.8722
7°	6046	6041	0.8733	0.8740
8°	6046	6027	0.8733	0.8760
9°	6046	6012	0.8733	0.8782
10°	6047	5994	0.8731	0.8808
11°	6047	5975	0.8731	0.8836
12°	6048	5954	0.8730	0.8867
13°	6048	5931	0.8730	0.8902
14°	6049	5907	0.8728	0.8938
15°	6049	5880	0.8728	0.8979
16°	6050	5852	0.8727	0.9022
17°	6050	5822	0.8727	0.9069
18°	6051	5790	0.8725	0.9119
19°	6052	5757	0.8724	0.9171
20°	6052	5721	0.8724	0.9229
21°	6053	5684	0.8722	0.9280
22°	6054	5646	0.8721	0.9351
23°	6054	5605	0.8721	0.9420
24°	6055	5563	0.8720	0.9491
25°	6056	5519	0.8718	0.9566
26°	6057	5474	0.8717	0.9645
27°	6058	5427	0.8715	0.9729
28°	6059	5378	0.8714	0.9817
29°	6060	5327	0.8712	0.9911
30°	6061	5275	0.8711	1.0009
31°	6061	5222	0.8711	1.0111
32°	6062	5166	0.8709	1.0220
33°	6063	5109	0.8708	1.0334
34°	6064	5051	0.8707	1.0453
35°	6065	4991	0.8705	1.0579
36°	6066	4930	0.8704	1.0709
37°	6067	4867	0.8702	1.0848
38°	6068	4802	0.8701	1.0995
39°	6070	4736	0.8698	1.1148
40°	6071	4669	0.8697	1.1308
41°	6072	4600	0.8695	1.1478
42°	6073	4530	0.8694	1.1655
43°	6074	4458	0.8692	1.1846

TABLE SHOWING FEET PER MINUTE (ARC) AND MINUTES (ARC) PER MILE OF LATITUDE AND LONGITUDE (*Continued*)

Latitude.	Length of 1 Min. in Feet.		No. of Min. in 1 Mile.	
	Latitude.	Longitude.	Latitude.	Longitude.
44°	6075	4385	0.8691	1.2040
45°	6076	4311	0.8689	1.2247
46°	6077	4235	0.8688	1.2467
47°	6078	4158	0.8687	1.2698
48°	6079	4080	0.8685	1.2941
49°	6080	4001	0.8684	1.3196
50°	6081	3920	0.8682	1.3469
51°	6082	3838	0.8681	1.3757
52°	6084	3755	0.8678	1.4061
53°	6085	3671	0.8677	1.4383
54°	6086	3586	0.8675	1.4723
55°	6087	3499	0.8674	1.5090
56°	6088	3413	0.8672	1.5470
57°	6089	3323	0.8671	1.5888
58°	6090	3233	0.8669	1.6331
59°	6091	3142	0.8668	1.6804
60°	6092	3051	0.8667	1.7305

ERRORS IN AZIMUTH FOR 1 MINUTE ERROR IN DECLINATION OR LATITUDE

Hour Angle	For 1 Min. Error in Declination.			For 1 Min. Error in Latitude.		
	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 30°.	Lat. 40°.	Lat. 50°.
H M	Min.	Min.	Min.	Min.	Min.	Min.
0 30	8.85	10.00	12.90	8.77	9.92	11.80
1 00	4.46	5.05	6.01	4.33	4.87	5.80
2 00	2.31	2.61	3.11	2.00	2.26	2.70
3 00	1.63	1.85	2.20	1.15	1.30	1.56
4 00	1.34	1.51	1.80	0.67	0.75	0.90
5 00	1.20	1.35	1.61	0.31	0.35	0.37
6 00	1.15	1.30	1.56	0.00	0.00	0.00

By the use of the above table the amount of the azimuth error, resulting from the use of erroneous declination or latitude at the different hours of the day, may be determined.

If the South Polar Distance used be *too great*, the observed meridian falls to the *right* of the *true south point* in the *forenoon* and to the *left* in the *afternoon* and *vice versa* if too small.

If the *latitude* used be *too great*, the observed meridian falls to the *left* of the *true south point* in the *forenoon* and to the *right* in the *afternoon* and *vice versa* if too small.

DETERMINATION OF THE MERIDIAN

By Direct Solar Observation.—The best time of day for a solar observation is from 8 to 10 A.M., and from 2 to 4 P.M. Observations greater than five or less than one hour from noon should not be relied upon. Use colored glass over eye end.

The transit should be accurately adjusted and carefully leveled. Set the limb at zero when the telescope is directed to some convenient mark, then with the lower motion clamped, point the telescope to the sun and bring the vertical and horizontal cross-hairs tangent to its image, say in the lower left-hand quarter; read the vertical circle and the limb. Note the time, quickly reverse the telescope in altitude and azimuth and again bring the cross-hairs tangent to the image, but in the opposite quarter; read the vertical circle and the limb.* The mean of the vertical circle readings will give the apparent altitude of the center of the sun. The mean of the readings on the limb will give the angle between the sun and the selected mark.

Formula A. — Let Z = the angle between the sun and the meridian, then its value may be obtained from the equation

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{\sin \tfrac{1}{2} S \times \sin (\tfrac{1}{2} S - \text{co-dec.})}{\sin \text{co-alt.} \times \sin \text{co-lat.}}}$$

where $$S = \text{co-dec.} + \text{co-alt.} + \text{co-lat.}$$

* When the transit has a vertical arc instead of a full circle the intersection of the cross-hairs may be directed to the center of the sun's image. In fact this method is common even with full circles for the error thus introduced, if any, is small. — McC.

Example. — The following notes are of an observation made at Denver at 10 A.M., April 17, 1910. Latitude 39° 46′ 31.″

Telescope.	Horizontal angle.	Vertical angle.	Position of sun's image.
Direct.................	89° 17′ R	+50° 14′	┿
Reversed.............	89° 07′ R	+51° 07′	┿

Reduction of notes. — 10 A.M., Denver time = 10 + 7 = 17 or 5 P.M., Greenwich time.

$$\begin{array}{lll}
\text{Change in dec. since noon} & & \\
\text{Greenwich} \qquad = 5 \times 53''.02 & = & +00^\circ\, 04'\, 25''.1 \\
\quad\text{Dec. at noon Greenwich} & = & \text{N } 10^\circ\, 15'\, 44''.4 \\
\quad\text{Dec. at 10 A.M. Denver} & = & \text{N } 10^\circ\, 20'\, 09''.5 \\
\qquad\qquad\text{co-dec.} & = & 79^\circ\, 39'\, 50''.5 \\
\text{Apparent alt} = \dfrac{(51^\circ\, 07') + (50^\circ\, 14')}{2} & = & 50^\circ\, 40'\, 30'' \\
\quad\text{Subtract refraction correction} & = & 00^\circ\, 00'\, 49'' \\
\qquad\text{True-alt.} & = & 50^\circ\, 39'\, 41'' \\
\qquad\text{Co-alt.} & = & 39^\circ\, 20'\, 19'' \\
\qquad\text{Lat.} & = & 39^\circ\, 46'\, 31'' \\
\qquad\text{Co-lat.} & = & 50^\circ\, 13'\, 29''
\end{array}$$

$$\tfrac{1}{2}S = \tfrac{1}{2}[(79^\circ\, 39'\, 50''.5) + (39^\circ\, 20'\, 19'') + (50^\circ\, 13'\, 29'')] = 84^\circ\, 36'\, 49''.2$$

Substituting in equation for $\cos \frac{1}{2} Z$

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{(\sin 84^\circ\, 36'\, 49''.2) \times (\sin 4^\circ\, 56'\, 58''.7)}{(\sin 39^\circ\, 20'\, 19'') \times (\sin 50^\circ\, 13'\, 29'')}}$$

$$\log \cos \tfrac{1}{2} Z = \tfrac{1}{2}\begin{bmatrix} +9.9980780 \\ +8.9359106 \\ -9.8020222 \\ -9.8856776 \end{bmatrix} = 9.6231444$$

$$\tfrac{1}{2} Z = 65^\circ\, 10'\, 18'',\quad Z = 130^\circ\, 20'\, 36''.$$

Interpretation of result.—Fig. 208 shows the relative positions of the sun, the meridian, and the point of reference.

Before noon Z is to the left of the line to the sun and to the right after noon.

The true bearing of the reference point is N 41° 08′ 36″ E.

Formula B. — Identical results will be obtained by the use of the following equation and it may be preferred on account of its containing the direct angles instead of the co-angles, but it is necessary to pay careful attention to the algebraic signs when it is used,

$$\cos Q = \frac{\sin dec.}{\cos lat. \times \cos alt.} - \tan lat. \times \tan alt.$$

The sign of the first term of the right-hand side of the equation is negative when declination is S; the second term is positive where the latitude is S. If the algebraic sign of

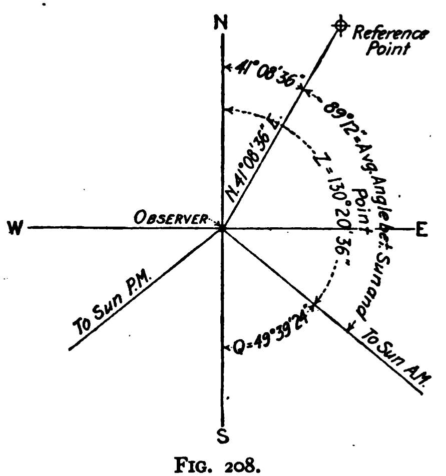


FIG. 208.

the result is positive, Q is the angle between the sun and the north point, but if it is negative, it is the angle between the sun and the south point.

Example.—The solution of the above example by this equation is as follows:

$$\cos Q = \left\{ \begin{array}{l} + \dfrac{\sin 10^\circ\ 20'\ 09''.5}{(\cos 39^\circ\ 46'\ 31'') \times (\cos 50^\circ\ 39'\ 41'')} \\ - (\tan 39^\circ\ 46'\ 31'') \times (\tan 50^\circ\ 39'\ 41'') \end{array} \right\}$$

$$\begin{array}{lll}
\log \sin 10^\circ\ 20'\ 09''.5 & = +\ 9.2538706 & \\
\log \cos 39^\circ\ 46'\ 31'' & = +\ 9.8856775 & \\
\log \cos 50^\circ\ 39'\ 41'' & = -\ \underline{9.8020222} & \\
 & \quad\ 9.5661709 & = \log 0.368274 \\
\log \tan 39^\circ\ 46'\ 31'' & = +\ 9.9203518 & \\
\log \tan 50^\circ\ 39'\ 41'' & = +\ \underline{0.0863893} & \\
 & \quad\ 0.0067411 & = \log \underline{1.015640} \\
 & \cos Q & = -\ 0.647366 \\
 & \therefore\ Q & = 49^\circ\ 39'\ 24''
\end{array}$$

Interpretation of result. — Q is the angle between the sun and the south point, since the declination is N, the latitude is N and the algebraic sign of the right-hand side of the equation is negative. The true bearing of the reference point is therefore N 41° 08′ 36″ E, the same as obtained by the first equation; see Fig. 208.

Time may also be determined from the foregoing data by the equation (for a spherical triangle)

$$\sin T = \frac{\sin Z \sin \text{co-alt.}}{\sin \text{co-dec.}},$$

in which T is the hour angle of the sun at the time of observation. This is reduced to time by dividing by 15 and corrected by the equation of time to obtain mean local time and still further corrected by the difference between mean local time and standard time if the latter is desired.

Example. — Using the data of the example already given to find the time.

$$\sin T = \frac{(\sin 130^\circ\ 20'\ 36'')\ (\sin 39^\circ\ 20'\ 19'')}{(\sin 79^\circ\ 39'\ 50.5'')}$$

$$\begin{array}{ll}
\text{Log} \sin 130^\circ\ 20'\ 36'' = & +9.8820570 \\
\text{Log} \sin\ \ 39^\circ\ 20'\ 19'' = & +\underline{9.8020222} \\
 & \quad 9.6840792 \\
\text{Log} \sin 79^\circ\ 39'\ 50.5'' = & -\underline{9.9928945} \\
\text{Log} \sin T = & \quad 9.6911847 \\
T = 29^\circ\ 24'\ 50'' &
\end{array}$$

$$\therefore \text{Time before noon} = \frac{29^\circ\ 24'\ 50''}{15} = 1 \text{ h. } 57 \text{ m. } 39 \text{ s.}$$

Noon	12 h. 00 m. 00 s.
Deduction................	1 h. 57 m. 39 s.
Sun time	10 h. 02 m. 21 s. A.M.
Equation of time to nearest sec.,	−16 s.
Mean local time	10 h. 02 m. 05 s. A.M.

As mean local time and standard time are the same at Denver no further correction is necessary.

According to the above result, the time used was 02 m. 05 s. slow, making an error of about 02″ in the declination which, from the table on page 276, makes an error in the azimuth of about 05″, being well within the allowable limits of error for ordinary field work.

Note. — The signs preceding the logarithms in the three examples above are used to indicate whether the logarithm is to be added or subtracted.

THE MERIDIOGRAPH

Mr. Louis Ross, a civil engineer in San Francisco, California, placed on the market, in 1913, a device by means of which the true north may be found at any time of the day, without any computations or preparation of tables, the only instrumental operation being an observation on the sun with the transit. Between 10 A.M. and 2 P.M. the results are unreliable, for the best time is when the altitude of the sun is from 15° to 25°. Results obtained when the altitude is between 25° and 35° are only fair, while an altitude of more than 45° should not be used. The foregoing remarks apply as well to direct observations on the sun when the solution of a spherical triangle is made as previously described.

The surveyor needs a table of the sun's ephemeris, but instead of going to the labor of making a table for the declination for each hour he merely takes from the table the declination for Greenwich time. A diagram accompanying the meridiograph is then consulted and the declination for the time and place is at once obtained. Another diagram gives the refraction to be subtracted. Declination is *D*.

The latitude of the place may be taken from a good map. If not available measure at noon the sun's altitude H; then, if declination is north, *Latitude* $= 90° - H + D$; if declination is south, *Latitude* $= 90° - H - D$.

"Apparent local time" can be found with the meridiograph, without any added work, by a single setting of the proper scales nearly as fast as by referring to a watch and to an accuracy, so the inventor claims, of one-half minute or less.

From a circular issued by Mr. Ross, the following description is taken, which should be clear to the reader by referring to the illustration.

The meridiograph is a protractor which solves graphically the problem of finding a true meridian by the sun. It consists of two circular scaled disks, 7 in. max. diam., with a reading arm. The names of all scales are on the arm, exactly over the graduations; — this obviates searching for any desired scales. Nearly all graduations are 5 or 10 minutes spaces; angles may therefore be read to an accuracy of 1 minute.

The scales, beginning with the outermost, are:

NUMBER	0.10 to 1.00,	complete circle, for numbers A, B, $(A + B)$.
BEARING	20° to 88° 40′	approx. $1\frac{2}{3}$ loops, for true bearing of sun.
b	10° to 60°	approx. $\frac{1}{4}$ loop, for either alt. or lat.
a	10° to 60°	approx. 1 loop, for either alt. or lat.
a	10° to 60°	approx. 1 loop, for either alt. or lat.
b	10° to 60°	approx. $\frac{1}{4}$ loop, for either alt. or lat.
DECL.	1° to 23° 30′	approx. $1\frac{1}{3}$ loops, for declination.

The transparent celluloid cover prevents the disks from moving accidentally after they have been set and also protects the scales from wear and dirt. The inner disk is rotated through the finger slots above and below.

To find true north, measure sun's altitude, take its declination from the Ephemeris, and take latitude of station from a map. Set these data on the meridiograph by means of the reading arm, thus:

On scales *a* set alt. against lat., opposite index read number A.
On scales *b* set alt. against lat., opposite given declination read number B.

Opposite $(A + B)$ read *true* bearing of sun.

On the two *a* scales the alt. is set against the lat.; it is

immaterial which is set on which as the scales are identical, but set arm on *outer* disk first, then turn inner disk until proper reading is under reading line. The same applies to the two *b* scales. Note that the two *a* scales produce number *A*, while the Decl. with the two *b* scales produce number *B*.

Check solution of example given on face of meridiograph.

FIG. 209. The Meridiograph.

The solar compass was superseded by the solar attachment to the transit but the method of direct observation on the sun is preferred by the majority of surveyors. The computations however are tedious and the amount of figuring required makes many men slight the work somewhat. With the most careful work the true north is found

only within the nearest 1 or 2 minutes of angle. The meridiograph is claimed to give results within this degree of accuracy.

BY OBSERVATIONS OF POLARIS

At Elongation.—A few minutes before elongation, set up the transit and center the plumb-bob over a tack driven into a stake. Level up very carefully and keep the vertical cross-hair on Polaris, using the tangent screw of the vernier plate, until elongation is reached. This is easily recognized since the azimuth then remains practically constant for several minutes.

When elongation has been reached, depress the telescope and carefully fix a stake on line, reverse the telescope on its axis and rotate the instrument 180° on its vertical axis, fix the vertical hair on Polaris, depress the telescope and fix another stake on line, if the vertical hair does not bisect the first one. These two observations must be made before Polaris has appreciably commenced its return motion in azimuth.

When it is necessary to set two stakes,* a third stake midway between them will be in a vertical plane through the plumb line of the transit and Polaris at elongation. By daylight, lay off from this plane the proper azimuth. North is to the right, if Polaris was at western, and to the left, if at eastern, elongation.

In making this and the following observation it is necessary to illuminate the stakes and the cross-hairs. The latter may be accomplished by a suitable lamp held at one side of the transit, so that sufficient light is reflected into the telescope.

At Culmination.—On account of the great difficulties attending this method, it should be used only when the method of elongation is impracticable.

This method is based on the fact that Polaris is very nearly on the meridian when it is in the same vertical plane with the star Delta, in the constellation Cassiopeia, or

* When the transit is in correct adjustment it will be necessary to set but one stake, whose position will correspond with that of the third one, as given above.

Zeta of the Great Dipper, the star at the bend in the handle. It consists in watching either Delta or Zeta until it comes into the same vertical plane with Polaris and then waiting a known interval of time,* until Polaris is on the meridian. The vertical cross-hair must be placed on Polaris precisely at the end of this interval as the motion, in azimuth, is most rapid at culmination. The telescope is now in the meridian, which may be marked in any suitable manner.

Limitations. — On account of the haziness of the atmosphere near the horizon, the lower culminations of Zeta and Delta cannot be used below about 38° north latitude; neither can their upper culminations be used north of about 25° and 30° respectively, on account of their being too near the zenith.

Selecting the Star. — The diagram, Fig. 209, shows Delta Cassiopeia on the meridian below the pole at midnight about April 10. It may therefore be used in the above method for two to three months before and after that date. Likewise Zeta, of the Great Dipper, may be used for two to three months before and after October 10.

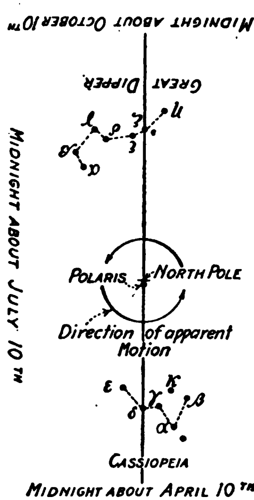


FIG. 210.

Time of Elongation and Culmination. — Fig. 209 shows Polaris near eastern elongation at midnight about July 10, at western at midnight about January 10, at upper culmination at midnight about October 10, and at lower at midnight about April 10. The approximate time of elongation or culmination for other dates may be determined by noting the position of the line adjoining Zeta of the Great Dipper and Delta Cassiopeia. When this line is vertical, Polaris is near culmination and when horizontal it is near elongation. Polaris is on the opposite side of the Pole from Zeta of the Great Dipper, thus furnishing a convenient means

* The waiting time for 1912 is 6 min. 48 sec. for Zeta of the Great Dipper and 7 min. 21 sec. for Delta Cassiopeia.

of distinguishing eastern from western elongation and upper from lower culmination. When Zeta is west, Polaris is east of the pole, and when Zeta is above, Polaris is below the pole.

MEAN POLAR DISTANCE

The azimuth of Polaris at elongation for any year is found from the following table, which gives the mean polar distance. The maximum error in azimuth for any month cannot exceed one minute from the mean polar distance for the year, this being within the limits of permissible error.

The sine of the true azimuth in any latitude is found by the formula.

$$Sin = \frac{Sine\ of\ Polar\ Distance}{Cosine\ Latitude}.$$

1915	1° 09′ 53.51″
1916	1° 08′ 34.97″
1917	1° 08′ 16.45″
1918	1° 07′ 57.94″
1919	1° 07′ 39.45″
1920	1° 07′ 20.98″

In *Engineering News*, April 22, 1915, appeared the following article:

AZIMUTH OBSERVATIONS ON POLARIS BY DAYLIGHT

By Robert V. R. Reynolds*

Polaris may always be found in clear weather as soon as the sun has set, and very frequently for five or ten minutes before sunset or after sunrise. It is stated on good authority that under very favorable conditions an observation has been successful as late as 10 A.M. In the northern parts of the United States the cross-wires may remain visible for a long time after sunset.

For the novice it is often difficult at the first few attempts to see Polaris while the sky is still bright, but having once

* Forest Examiner, U. S. Forest Service, Washington, D. C.

found the star, which appears as a small white dot in the field, he will never thereafter feel in doubt. Tabulated

TABLE TO FIND POLARIS BY DAYLIGHT

Hour Angle of Polaris Approximate* (Use Suitable Interpolations)	Azimuth Setting N E or N W Depending upon Position of Polaris E or W of Meridian	Altitude Setting Latitude Plus or Minus the Tabulated Quantities
0.0 or 12.0	00′	1° 08′
0.5 or 11.5	12′	1° 07′
1.0 or 11.0	24′	1° 06′
1.5 or 10.5	36′	1° 04′
2.0 or 10.0	47′	1° 00′
2.5 or 9.5	57′	55′
3.0 or 9.0	1° 06′	50′
3.5 or 8.5	1° 15′	43′
4.0 or 8.0	1° 22′	35′
4.5 or 7.5	1° 27′	27′
5.0 or 7.0	1° 30′	18′
5.5 or 6.5	1° 33′	09′
6.0 hours	1° 35′	00′

Add to latitude for hour angles less than 6. Subtract from latitude for hour angles exceeding 6.

* The hour angle used as the argument in this table needs only to be approximate. If it is correct within 5 min. of time, sufficiently accurate settings will be indicated provided interpolation is made. Hence, there is no need of correcting for longitude until the surveyor has made the observation and is preparing to enter the table of Azimuths of Polaris.

The table is computed for a mean latitude of 42°, but purposely modified slightly to make it more useful along the 49th parallel, where much of the Forest Service work is being done. It is accurate enough to bring Polaris within the field of an ordinary transit between latitudes of 10° and 58° N. It is not for use to determine the true azimuth after the observations.

settings (such as accompany this article) sufficiently accurate to bring the star into the field are required, and there remain several factors which must be given consideration before success can be assured:

1. A slight haziness, which may hardly be obvious to the eye, is sufficient to conceal the star until darkness comes on.

2. The telescope must be in exact focus for celestial objects. This may be accomplished either by focusing at night upon the moon and making a slight scratch upon the objective slide to show the point to which it should be extended, or the surveyor may focus at the time of observa-

tion upon a well-defined object 3 or 4 miles distant, which focus will usually be found sufficiently close. Accurate focusing is one of the most important factors in finding the star.

3. For the purpose of cutting off objectionable light, the sunshade should always be attached. Certainty of finding the star is assured by throwing a coat or other dark cloth over the head when searching through the telescope, as a photographer uses a focusing cloth.

4. An approximate meridian must be had, from which the azimuth settings are turned off. Commonly, the surveyor will already have such a meridian from his backsight. Otherwise, a meridian determination from a solar attachment in reasonable adjustment will suffice. Sometimes, when the magnetic declination is closely known, it will even be possible to turn upon the star from the needle. A reference meridian which is true within 5′ or 10′ will be precise enough to locate the star when the table of approximate settings is used.

Polaris having been found, the angle from the reference mark to the star should be measured twice, the second time with the telescope inverted. The mean time of observation and the mean angle are then used to find the azimuth of the mark by the simplified hour-angle method. There is practically no chance that any other star will be seen and mistaken for Polaris. (Finis.)

BY ANY STAR AT EQUAL ALTITUDES

In high latitudes neither the sun nor Polaris give reliable results. The sun is low and the refraction is uncertain, while, on account of the height of Polaris, the observation is difficult to make and instrumental errors are magnified. In southern latitudes Polaris is not visible at all. Although this method may be used in any latitude, it is particularly applicable under the above conditions.

The method consists in observing a star, when at equal altitudes, east and west of the meridian. The meridian will then be halfway between these two positions of the star. The star selected should be 30° or more from the zenith when on the meridian and, at least, the same dis-

tance from the pole. The observations should be made when the star is three to four hours from the meridian.

Making the observation. — To make the first observation level the transit carefully, direct the telescope to the star, clamp all motions and fix the intersection of the cross-hairs on the star by the slow motion screws, read the star's altitude, unclamp the telescope axis, depress the telescope and fix a point on line.

To make the second observation re-level the transit carefully, set the telescope at the altitude determined by the first observation, clamp the limb and lower motion when the star comes near to the horizontal hair, keeping the vertical hair on the star by means of the slow-motion screw of the vernier plate until it reaches the intersection of the cross-hairs, then unclamp the telescope axis, depress the telescope and fix a point on line. A third point set halfway between these two will mark the meridian through the transit.

If the transit has a vertical circle, the error due to the adjustment of the height of standards may be eliminated by making the first observation with the telescope direct and the second with it reversed in altitude and azimuth. In this method artificial illumination must be used for the cross-hairs and for setting the points on line.

GREEK LETTERS

α	Alpha	ϵ	Epsilon
β	Beta	η	Eta
δ Δ	Delta	ζ	Zeta
γ	Gamma	κ	Kappa

CHAPTER VII

SURVEYING LAW AND PRACTICE

The business of the surveyor is firmly bound up with that of the lawyer, but much of the litigation over land lines would be eliminated if the lawyer could be prevented from interfering with the surveyor in the doing of his work. When it comes to the courts that is another matter, for the decisions of judges based upon precedent, and latterly upon changes in customs and advances in civilization, are generally right. Even if occasionally wrong a decision must stand until a better informed judge has a similar case presented to him. The decisions of courts are based upon a very few common sense principles of law, but it should not be necessary to have cases go into court in order to settle lines and the location of monuments.

The surveyor is presumed to have enough skill to measure angles and lines and make a record of same so competent surveyors can re-trace the work and re-locate boundaries. The average attorney-at-law does not recognize "THE ERROR," a thing that looms up large before every surveyor and which the courts must recognize.

Mr. J. Francis Le Baron, Member of the American Society of Civil Engineers, on page 425, in the Transactions of the American Society of Civil Engineers, Vol. LXXV (1912), presents the following as a sample of instructions for re-surveys, given him by lawyers, instructions that are positively insulting to men of extended experience in such work. They resemble similar instructions which lawyers at times attempted to give the author in the years when his work was closely associated with land surveying.

"I want you to start from the beginning corner, as given in the deed, run the exact course and distance and set a stake there. It will not take you long. I suppose you make an allowance for the variation of the needle. The

needle, you know, does not point exactly north, and you must make an allowance. This deed reads 'due north' so many chains. Now does that mean true north or the way the needle points? I suppose when you chain down hill you make an allowance, don't you, because I think the distance wouldn't come right if you didn't? I don't know how much you allow, but I suppose you have some custom about it. You see this distance reads so many chains and links, so you must measure it with a chain and links, and not any other kind of measure, or I am inclined to think that the Court would reject your survey."

The surveyor is also employed by real estate agents to make surveys for clients and fully ninety-five per cent of these real estate agents demand a commission of no small amount. This is even customary with some attorneys and managers of estates. The surveyor who is compelled to obtain work under such conditions cannot afford to do the work properly and furthermore few competent surveyors will accept work on such terms. The foregoing remarks do not indict all lawyers, real estate agents and managers of estates, for many high-minded men occupy positions of trust and jealously safeguard the interests of their clients to a degree that they would not in work for themselves. To work for an intelligent and honest man occupying such a position is an experience so filled with satisfaction that the surveyor often feels a trifle shamefaced in presenting his bill. If he could afford it he would do the work for the mere satisfaction it affords, yet work done for such men is generally the most highly paid. They are high-priced themselves and prefer to have work done by men who set a proper value upon their services. There is no heart-breaking competition in such work, yet it is seldom offered to a man until after he has served an apprenticeship of many years in competitive work and has established a reputation for accuracy and honesty.

To properly re-survey a piece of ground requires considerable preliminary work in obtaining starting points, for in a community every lot is tied to adjoining lots. Sometimes to re-locate one line, means the running of many lines, some at a considerable distance from the one wanted. Many men do not want to pay for such work. Once the

author had to work for ten days before he could definitely locate the line he had been employed to survey, this work consuming only three hours' time. When his bill was presented the lawyer offered him pay for the three hours and told him to go to the other owners, whose land he had first run out, for the remainder of the bill. It took a lawsuit to recover the amount due and some evidence obtained in the suit resulted in the disbarment of the attorney. This leads naturally to the statement that not every attorney is a lawyer and with real lawyers there is seldom any difficulty. The average *attorney*, therefore, is the man who tries to hold the surveyor down to a definite procedure and who is ignorant of THE ERROR. It is difficult however to distinguish between the real lawyer and the pseudo-lawyer, who is really only a licensed attorney-at-law, until some experience is had with the man.

Every student who contemplates following surveying as a vocation should procure a copy of Paper No. 1242, Transactions of the American Society of Civil Engineers, entitled "Retracement Surveys — Court Decisions and Field Procedure," by N. N. Sweitzer, M. Am. Soc. C. E., together with the complete discussion by a number of experienced men. He should also read "Boundaries and Landmarks," by A. C. Mulford ($1.00) and "Descriptions of Lands," by R. W. Cautley ($1.00) in order to obtain the point of view of competent surveyors and hasten the acquirement of knowledge he can obtain otherwise only by years of experience. The surveyor must not forget that his work is not to merely re-trace old lines but to FIND THE LAND. To re-run old field notes exactly as recorded is a simple matter, but to determine the location of the original monuments and lines calls for skill gained only by practical experience. The surveyor's work is as much legal as technical and many times the legal side is the most important. The importance of the legal nature of a surveyor's work is so great that a number of years ago F. Hodgman, then secretary of the Michigan Engineering Society, wrote a book giving the gist of a number of court decisions as a guide for surveyors. This work should be in the possession of every land surveyor. It is entitled "A Manual of Land Surveying" ($2.50). John Cassan Wait also wrote a book entitled

"Law of Operations Preliminary to Construction in Engineering and Architecture," which covers very completely the subject of the law of boundaries.

A number of years ago Chief Justice Cooley, of the Michigan Supreme Court, delivered an address at a meeting of the Michigan Engineering Society on "The Judicial Functions of Surveyors," and no modern textbook on surveying is presumed to be complete unless this address is made a part of the contents.

THE JUDICIAL FUNCTIONS OF SURVEYORS

By Chief Justice Cooley

When a man has had a training in one of the exact sciences, where every problem within its purview is supposed to be susceptible of accurate solution, he is likely to be not a little impatient when he is told that, under some circumstances, he must recognize inaccuracies, and govern his action by facts which lead him away from the results which theoretically he ought to reach. Observation warrants us in saying that this remark may frequently be made of surveyors.

In the State of Michigan all our lands are supposed to have been surveyed once or more, and permanent monuments fixed to determine the boundaries of those who should become proprietors. The United States, as original owner, caused them all to be surveyed once by sworn officers, and as the plan of subdivision was simple, and was uniform over a large extent of territory, there should have been, with due care, few or no mistakes, and long rows of monuments should have been perfect guides to the place of any one that chanced to be missing. The truth unfortunately is, that the lines were very carelessly run, the monuments inaccurately placed, and as the recorded witnesses to these were many times wanting in permanency, it is often the case that when a monument was not correctly placed, it is impossible to determine by the record, by the aid of anything on the ground, where it was located. The incorrect record of course becomes worse than useless when the witnesses it refers to have disappeared.

It is, perhaps, generally supposed that our town plats

were more accurately surveyed, as indeed they should have been, for in general there can have been no difficulty in making them sufficiently perfect for all practical purposes. Many of them however were laid out in the woods; some of them by proprietors themselves, without either chain or compass, and some by imperfectly trained surveyors, who, when land was cheap, did not appreciate the importance of having correct lines to determine boundaries when land should become dear. The fact probably is, that town surveys are quite as inaccurate as those made under authority of the general government.

It is now upwards of fifty years since a major part of the public surveys in what is now the State of Michigan were made under the authority of the United States. Of the lands south of Lansing, it is now forty years since the major part were sold, and the work of improvement began. A generation has passed away since they were converted into cultivated farms, and few if any of the original corner and quarter stakes now remain.

The corner and quarter stakes were often nothing but green sticks driven into the ground. Stones might be put around or over these if they were handy, but often they were not, and the witness trees must be relied upon after the stake was gone. Too often the first settlers were careless in fixing their lines with accuracy while monuments remained, and an irregular brush fence, or something equally untrustworthy, may have been relied upon to keep in mind where the blazed line once was. A fire running through this might sweep it away, and if nothing was substituted in its place, the adjoining proprietors might in a few years be found disputing over their lines, and perhaps rushing into litigation, as soon as they had occasion to cultivate the land along the boundary.

If now the disputing parties call in a surveyor, it is not likely that any one summoned would doubt or question that his duty was to find, if possible, the place of the original stakes which determined the boundary line between the proprietors. However erroneous may have been the original survey, the monuments that were set must nevertheless govern, even though the effect be to make one half-quarter section ninety acres and the one adjoining seventy; for

parties buy, or are supposed to buy, in reference to these monuments, and are entitled to what is within their lines, and no more, be it more or less. While the witness trees remain, there can generally be no difficulty in determining the locality of the stakes.

When the witness trees are gone, so that there is no longer record evidence of the monuments, it is remarkable how many there are who mistake altogether the duty that now devolves upon the surveyor. It is by no means uncommon that we find men, whose theoretical education is thought to make them experts, who think that when the monuments are gone, the only thing to be done is to place new monuments where the old ones should have been, and would have been, if placed correctly. This is a serious mistake. The problem is now the same that it was before: To ascertain, by the best lights of which the case admits, where the original lines were. The mistake above alluded to, is supposed to have found expression in our legislation, though it is possible that the real intent of the act to which we shall refer is not what is commonly supposed.

An act passed in 1869 (Compiled Laws, § 593), amending the laws respecting the duties and powers of county surveyors, after providing for the case of corners which can be identified by the original field notes or other unquestionable testimony, directs as follows:

Second. — Extinct interior section corners must be re-established at the intersection of two right lines joining the nearest known points on the original section lines east and west and north and south of it.

Third. — Any extinct quarter-section corner, except on fractional lines, must be re-established equidistant and in a right line between the section corners; in all other cases at its proportionate distance between the nearest original corners on the same line.

The corners thus determined, the surveyors are required to perpetuate by noting bearing trees when timber is near.

To estimate properly this legislation, we must start with the admitted and unquestionable fact that each purchaser from government bought such land as was within the original boundaries, and unquestionably owned it up to the time when the monuments became extinct. If the monu-

ment was set for an interior section corner, but did not happen to be "at the intersection of two right lines joining the nearest known points on the original section line east and west and north and south of it," it nevertheless determined the extent of his possessions, and he gained or lost, according as the mistake did or did not favor him.

It will probably be admitted that no man loses title to his land or any part thereof merely because the evidences become lost or uncertain. It may become more difficult for him to establish it as against an adverse claimant, but theoretically the right remains; and it remains as a potential fact so long as he can present better evidence than any other person. And it may often happen that notwithstanding the loss of all trace of a section corner or quarter stake, there will still be evidence from which any surveyor will be able to determine with almost absolute certainty where the original boundary was between two government subdivisions.

There are two senses in which the word extinct may be used in this connection: One, the sense of physical disappearance; the other, the sense of loss of all reliable evidence. If the statute speaks of extinct corners in the former sense, it is plain that a serious mistake was made in supposing that surveyors could be clothed with authority to establish new corners by an arbitrary rule in such cases. As well might the statute declare that if a man loses his deed, he shall lose his land altogether.

But if by extinct corner is meant one in respect to the actual location of which all reliable evidence is lost, then the following remarks are pertinent:

1. There would undoubtedly be a presumption in such a case that the corner was correctly fixed by the government surveyor where the field notes indicated it to be.

2. But this is only a presumption, and may be overcome by any satisfactory evidence showing that in fact it was placed elsewhere.

3. No statute can confer upon a county surveyor the power to "establish" corners, and thereby bind the parties concerned. Nor is this a question merely of conflict between State and federal law. It is a question of property

right. The original surveys must govern, and the laws under which they were made must govern, because the land was bought in reference to them; and any legislation, whether State or federal, that should have the effect to change these, would be inoperative, because disturbing vested rights.

4. In any case of disputed lines, unless the parties concerned settle the controversy by agreement, the determination of it is necessarily a judicial act, and it must proceed upon evidence, and give full opportunity for a hearing. No arbitrary rules of survey or of evidence can be laid down whereby it can be adjudged.

The general duty of the surveyor in such a case is plain enough. He is not to assume that a monument is lost until after he has thoroughly sifted the evidence and found himself unable to trace it. Even then he should hesitate long before doing anything to the disturbance of settled possessions. Occupation, especially if long continued, often affords very satisfactory evidence of the original boundary when no other is attainable; and the surveyor should inquire when it originated, how, and why the lines were then located as they were, and whether a claim of title has always accompanied the possession, and give all the facts due force as evidence. Unfortunately, it is known that surveyors sometimes, in supposed obedience to the State statute, disregard all evidences of occupation and claim of title, and plunge whole neighborhoods into quarrels and litigation by assuming to "establish" corners at points with which the previous occupation cannot harmonize. It is often the case that where one or more corners are found to be extinct, all parties concerned have acquiesced in lines which were traced by the guidance of some other corner or landmark, which may or may not have been trustworthy; but to bring these lines into discredit when the people concerned do not question them, not only breeds trouble in the neighborhood, but it must often subject the surveyor himself to annoyance and perhaps discredit, since in a legal controversy the law as well as common sense must declare that a supposed boundary line long acquiesced in is better evidence of where the real line should be than any survey made after the original monuments have dis-

appeared. (Stewart *v.* Carleton, 31 Mich. Reports, 270; Diehl *v.* Zanger, 39 Mich. Reports, 601.) And county surveyors, no more than any others, can conclude parties by their surveys.

The mischiefs of overlooking the facts of possession most often appear in cities and villages. In towns the block and lot stakes soon disappear, there are no witness trees and no monuments to govern except such as have been put in their places, or where their places were supposed to be. The streets are likely soon to be marked off by fences, and the lots in a block will be measured off from these without looking farther. Now it may perhaps be known in a particular case that a certain monument still remaining was a starting point in the original survey of the town plat; or a surveyor settling in the town may take some central point as the point of departure in his surveys, and assuming the original plat to be accurate, he will then undertake to find all streets and lots by course and distance according to the plat, measuring and estimating from his point of departure. This procedure might unsettle every line and every monument existing by acquiescence in the town; it would be very likely to change the lines of streets, and raise controversies everywhere. Yet this is what is sometimes done, the surveyor himself being the first person to raise the disturbing questions.

Suppose, for example, a particular village street has been located by acquiescence and used for many years and the proprietors in a certain block have laid off their lots in reference to this practical location. Two lot owners quarrel, and one of them calls in a surveyor, that he may make sure his neighbor shall not get an inch of land from him. This surveyor undertakes to make his survey accurate, whether the original was so or not, and the first result is, he notifies the lot owners that there is error in the street line, and that all fences should be moved, say one foot to the east. Perhaps he goes on to drive stakes through the block according to this conclusion. Of course, if he is right in doing this, all lines in the village will be unsettled; but we will limit our attention to the single block. It is not likely that the lot owners generally will allow the new survey to unsettle their possessions, but there is always a

probability of finding some disposed to do so. We shall then have a lawsuit; and with what result?

It is a common error that lines do not become fixed by acquiescence in a less time than twenty years, in fact, by statute, road lines may become conclusively fixed in ten years; and there is no particular time that should be required to conclude private owners, where it appears that they have accepted a particular line as their boundary, and all concerned have cultivated and claimed up to it. Public policy requires that such lines be not lightly disturbed, or disturbed at all, after the lapse of any considerable time. The litigant, therefore, who in such a case pins his faith on the surveyor, is likely to suffer for his reliance, and the surveyor himself to be mortified by a result that seems to impeach his judgment.

Of course nothing in what has been said can require a surveyor to conceal his own judgment, or to report the facts one way when he believes them to be another. He has no right to mislead, and he may rightfully express his opinion that an original monument was at one place, when at the same time he is satisfied that acquiescence has fixed the rights of parties as if it were another. But he would do mischief if he were to attempt to "establish" monuments which he knew would tend to disturb settled rights; the farthest he has a right to go, as an officer of the law, is to express his opinion where the monument should be, at the same time that he imparts the information to those who employ him, and who might otherwise be misled, that the same authority that makes him an officer and entrusts him to make surveys, also allows parties to settle their own boundary lines, and considers acquiescence in a particular line or monument, for any considerable period, as strong if not conclusive evidence of such settlement. The peace of the community absolutely requires this rule. It is not long since, that in one of the leading cities of the state an attempt was made to move houses two or three rods into a street, on the ground that a survey, under which the street has been located for many years, had been found on a more recent survey to be erroneous.

From the foregoing it will appear that the duty of the surveyor where boundaries are in dispute must be varied

by the circumstances. 1. He is to search for the original monuments, or for the places where they were originally located, and allow these to control. (If he finds them, unless he has reason to believe that agreements of the parties, express or implied, have rendered them unimportant.) By monuments in the case of government surveys we mean of course the corner and quarter-stakes; blazed lines or marked trees on the lines are not monuments; they are merely guides or finger posts, if we may use the expression, to inform us with more or less accuracy where the monuments may be found. 2. If the original monuments are no longer discoverable, the question of location becomes one of evidence merely. It is merely idle for any state statute to direct a surveyor to locate or "establish" a corner, as the place of the original monument, according to some inflexible rule. The surveyor, on the other hand, must inquire into all the facts, giving due prominence to the acts of the parties concerned, and always keeping in mind, *first*, that neither his opinion nor his survey can be conclusive upon parties concerned; and, *second*, that courts and juries may be required to follow after the surveyor over the same ground, and that it is exceedingly desirable that he govern his actions by the same lights and the same rules that will govern theirs.

It is always possible, when corners are extinct, that the surveyor may usefully act as a mediator between parties, and assist in preventing legal controversies by settling doubtful lines. Unless he is made for this purpose an arbitrator by legal submission, the parties, of course, even if they consent to follow his judgment, cannot, on the basis of mere consent, be compelled to do so; but if he brings about an agreement, and they carry it into effect by actually conforming their occupation to his lines, the action will conclude them. Of course, it is desirable, that all such agreements be reduced to writing; but this is not absolutely indispensable if they are carried into effect without.

Meander lines. — The subject to which allusion will now be made, is taken up with some reluctance, because it is believed the general rules are familiar. Nevertheless, it is often found that surveyors misapprehend them, or err in their application; and as other interesting topics are some-

what connected with this, a little time devoted to it will probably not be altogether lost. The subject is that of meander lines. These are lines traced along the shores of lakes, ponds, and considerable rivers, as the measures of quantity when sections are made fractional by such waters. These have determined the price to be paid when government lands were bought, and perhaps the impression still lingers in some minds that the meander lines are boundary lines, and that all in front of them remains unsold.

Of course this is erroneous. There was never any doubt that, except on the large navigable rivers, the boundary of the owners of the banks is the middle line of the river; and while some courts have held that this was the rule on all fresh-water streams, large and small, others have held to the doctrine that the title to the bed of the stream below low-water mark is in the state, while conceding to the owners of the banks all riparian rights. The practical difference is not very important. In this state the rule that the center line is the boundary line, is applied to all great rivers, including the Detroit, varied somewhat by the circumstance of there being a distinct channel for navigation, in some cases, with the stream in the main channel, and also sometimes by the existence of islands.

The troublesome questions for surveyors present themselves when the boundary line between two contiguous estates is to be continued from the meander lines to the center line of the river. Of course, the original survey supposes that each purchaser of land on the stream has a water front of the length shown by the field notes; and it is presumable that he bought this particular land because of that fact. In many cases it now happens that the meander line is left some distance from the shore by the gradual change of course of the stream, or diminution of the flow of water. Now the dividing line between two government subdivisions might strike the meander line at right angles, or obliquely; and, in some cases, if it were continued in the same direction to the center line of the river, might cut off from the water one of the subdivisions entirely, or at least cut it off from any privilege of navigation, or other valuable use of the water, while the other might have a water front much greater than the length of the line crossing it at right

angles to its side lines. The effect might be that, of two government subdivisions of equal size and cost, one would be of very great value as water-front property, and the other comparatively valueless. A rule which would produce this result would not be just, and it has not been recognized in the law.

Nevertheless it is not easy to determine what ought to be the correct rule for every case. If the river has a straight course, or one nearly so, every man's equities will be preserved by this rule: Extend the line of division between the two parcels from the meander line to the center line of the river, as nearly as possible at right angles to the general course of the river at that point. This will preserve to each man the water front which the field notes indicated, except as changes in the water may have affected it, and the only inconvenience will be that the division line between different subdivisions is likely to be more or less deflected where it strikes the meander line.

This is the legal rule, and it is not limited to government surveys, but applies as well to water lots which appear as such on town plats. (Bay City Gas Light Co., *v.* The Industrial Works, 28 Mich. Reports, 182.) It often happens, therefore, that the lines of city lots bounded on navigable streams are deflected as they strike the bank, or the line where the bank was when the town was first laid out.

When the stream is very crooked, and especially if there are short bends, so the foregoing rule is incapable of strict application, it is sometimes very difficult to determine what should be done; and in many cases the surveyor may be under the necessity of working out a rule for himself. Of course his action cannot be conclusive, but if he adopts one that follows, as nearly as the circumstances will admit, the general rule above indicated, so as to divide as near as may be the bed of the stream among the adjoining owners in proportion to their lines upon the shore, his division, being that of an expert, made upon the ground and with all available lights, is likely to be adopted as law for the case. Judicial decisions, into which the surveyor would find it prudent to look under such circumstances, will throw light upon his duties and may constitute a sufficient guide when peculiar cases arise. Each riparian lot owner ought to

have a line on the legal boundary, namely, the center line of the stream proportioned to the length of his line on the shore and the problem in each case is, how this is to be given him. Alluvion, when a river changes its course, will be apportioned by the same rules.

The existence of islands in a stream when the middle line constitutes a boundary, will not affect the apportionment unless the islands were surveyed out as government subdivisions in the original admeasurement. Wherever that was the case, the purchaser of the island divides the bed of the stream on each side with the owner of the bank, and his rights also extend above and below the solid ground, and are limited by the peculiarities of the bed and the channel. If an island was not surveyed as a government subdivision previous to the sale of the bank, it is of course impossible to do this for the purpose of government sale afterward, for the reason that the rights of the bank owners are fixed by their purchase; when making that they have a right to understand that all land between the meander lines, not separately surveyed and sold, will pass with the shore in the government sale; and having this right, anything which their purchase would include under it cannot be taken from them. It is believed, however, that the federal courts would not recognize the applicability of this rule to large navigable rivers, such as those uniting the great lakes.

On all the little lakes of the state which are mere expansions near their mouths of the rivers passing through them — such as the Muskegon, Pere Marquette and Manistee — the same rule of bed ownership has been judicially applied as applied to the rivers themselves; and the division lines are extended under the water in the same way. (Rice *v.* Ruddiman, 10 Mich., 125.) If such a lake were circular, the lines would converge to the center; if oblong or irregular, there might be a line in the middle on which they would terminate, whose course would bear some relation to that of the shore. But it can seldom be important to follow the division line very far under the water, since all private rights are subject to the public rights of navigation and other use, and any private use of the lands inconsistent with these would be a nuisance, and punishable as

such. It is sometimes important, however, to run the lines out for considerable distance, in order to determine where one may lawfully moor vessels or rafts, for the winter, or cut ice. The ice crop that forms over a man's land of course belongs to him. (Lorman *v.* Benson, 8 Mich., 18; Peoples Ice Co. *v.* Steamer Excelsior, recently decided.)

What is said above will show how unfounded is the notion, which is sometimes advanced, that a riparian proprietor on a meandered river may lawfully raise the water in the stream without liability to the proprietors above, provided he does not raise it so that it overflows the meander line. The real fact is that the meander line has nothing to do with such a case, and an action will lie whenever he sets back the water upon the proprietor above, whether the overflow be below the meander lines or above them.

As regards the lakes and ponds of the state, one may easily raise questions that it would be impossible for him to settle. Let us suggest a few questions, some of which are easily answered, and some not:

1. To whom belongs the land under these bodies of water, where they are not mere expansions of a stream flowing through them?
2. What public rights exist in them?
3. If there are islands in them which were not surveyed out and sold by the United States, can this be done now?

Others will be suggested by the answers given to these.

It seems obvious that the rules of private ownership which are applied to rivers cannot be applied to the great lakes. Perhaps it should be held that the boundary is at low-water mark, but improvements beyond this would only become unlawful when they became nuisances. Islands in the great lakes would belong to the United States until sold, and might be surveyed and measured for sale at any time. The right to take fish in the lakes, or to cut ice, is public, like the right of navigation, but is to be exercised in such manner as will not interfere with the rights of shore owners. But so far as these public rights can be the subject of ownership, they belong to the state, not to the United States; and so, it is believed, does the bed of the lake also. (Pollord *v.* Hagan, 3 Howard's U. S. Reports.)

But such rights are not generally considered proper subjects of sale, but like the right to make use of the public highways, they are held by the state in trust for all the people.

What is said of the large lakes may perhaps be said also of many of the interior lakes of the state; such, for example, as Houghton, Higgins, Cheboygan, Burt's, Mullet, Whitmore, and many others. But there are many little lakes or ponds which are gradually disappearing, and the shore proprietorship advances *pari passu* as the waters recede. If these are of any considerable size — say, even a mile across — there may be questions of conflicting rights which no adjudication hitherto made could settle. Let any surveyor, for example, take the case of a pond of irregular form, occupying a mile square or more of territory, and undertake to determine the rights of the shore proprietors to its bed when it shall totally disappear, and he will find he is in the midst of problems such as probably he has never grappled with, or reflected upon before. But the general rules for the extension of shore lines, which have already been laid down, should govern such cases, or at least should serve as guides in their settlement.

Where a pond is so small as to be included within the lines of a private purchase from the government, it is not believed the public have any rights in it whatever. Where it is not so included, it is believed they have rights of fishery, rights to take ice and water, and rights of navigation for business or pleasure. This is the common belief, and probably the just one. Shore rights must not be so exercised as to disturb these, and the states may pass all proper laws for their protection. It would be easy with suitable legislation to preserve these little bodies of water as permanent places of resort for the pleasure and recreation of the people, and there ought to be such legislation.

If the state should be recognized as owner of the beds of these small lakes and ponds, it would not be owner for the purpose of selling. It would be owner only as trustee for the public use; and a sale would be inconsistent with the right of the bank owners to make use of the water in its natural condition in connection with their estates. Some of them might be made salable lands by draining; but

the state could not drain, even for this purpose, against the will of the shore owners, unless their rights were appropriated and paid for.

Upon many questions that might arise between the state as owner of the bed of the little lake and the shore owners, it would be presumptuous to express an opinion now, and fortunately the occasion does not require it.

I have thus indicated a few of the questions with which surveyors may now and then have occasion to deal, and to which they should bring good sense and sound judgment. Surveyors are not and cannot be judicial officers, but in a great many cases they act in a *quasi* judicial capacity with the acquiescence of parties concerned; and it is important for them to know by what rules they are to be guided in the discharge of their judicial functions. What I have said cannot contribute much to their enlightenment, but I trust will not be wholly without value. (Finis.)

Commenting upon the famous address of Judge Cooley it is right to inform the surveyor that in his dealings with owners and their attorneys he will find there is a thing known as "common law" and another thing known as "statute law." The common law governs in the majority of states until a statute is passed to settle if possible uncertainties which lead to lawsuits. These statutes are passed by state legislatures, in which the majority of members usually are attorneys (by courtesy, lawyers) and whenever the legislature meets, one of the first committees appointed is a committee to pass upon the constitutionality of all proposed legislation. The statute referred to, which purported to enable surveyors to "re-establish" corners, was the product of a legislature in which the usual percentage of membership was lawyers and having the customary committee to scan all proposed laws. Such statutes have been passed in a number of states but when interested owners object and take a case to the Supreme Court the statute is held to violate the common law doctrine of rights of property and to be unconstitutional. The reasons are well stated by Judge Cooley. Yet many surveyors have been compelled by attorneys to follow the letter of the law, even while these same attorneys knew how the case would go if carried to the proper court. Many sur-

veyors made mistakes through believing every law on the statute books to be good law, but it was not customary fifty years ago for legal points to be touched upon in text-books on surveying.

In his reference to lot lines, Judge Cooley had particular conditions in mind. On the points he brings up there is considerable room for differences of opinion. Rights never run against the public. That is, when a man encroaches upon a public highway with a fence or building the public still has every right it was given so possession in such a manner does not give title. However, if the man was permitted to place any part of his building on public property he gains an easement thereby *for that structure.* He cannot be compelled to remove it, without being compensated for whatever damages he may thereby suffer, but the public may declare its right in the ground occupied and forbid him making repairs so the structure may remain indefinitely. The public may also, when the structure is razed, compel the new structure to be placed where it properly belongs, thus terminating the easement. This again is governed by the conditions under which the public rights in the highway were acquired. The street may have been given outright to the public. It may have been dedicated merely for highway purposes and nothing said about reversion in case of abandonment. Conditions of reversion may have been stated at the time of dedication. The ownership of adjacent lots may be limited to the exact boundaries of the lots and the edge of the highway; it may extend to the center line of the highway.

Assume a town site to have been carefully surveyed and well monumented. Stakes in some way became lost and the people who took possession were too penurious to pay five dollars or ten dollars for each lot survey. In course of time every fence line and many building lines are not in the position indicated by the records. Some street improvements are started and the encroachments discovered. The encroachment may be a fence in such a state of dilapidation that the necessary street grading causes it to fall, whereupon the city seizes the property thus abandoned. The lot owner finds he has lost a strip of land and proceeds to demand it from his neighbor, the trouble running

through the block, until at the far end a surplus is discovered in the last lot. This owner proposes to fight for the continued possession of the surplus. The monuments are uncovered and a careful re-survey made with the result that it is found every person in the town may have all the land his, or her, deeds called for and the city may also have all the width in each street. Nobody loses but everyone is put to considerable expense and annoyance. This is typical and all the surveyor can do is to show the facts. There will be plenty of lawyers to take either side in a controversy and depending upon the cleverness of the lawyers and the wealth, or poverty, of the litigants a crop of decisions will be given in the lower courts which will disgust intelligent jurists. If the general rule laid down by Judge Cooley be followed everyone should be permitted to stay put where found, but his rule referred to carelessness in the original surveys and the subsequent disappearance of stakes and monuments.

Possession implies two things. Physical possession and payment of taxes. If the deed calls for Lot 5 in Block B, the owner will be taxed for Lot 5 in Block B. If his fence encroaches on Lot 6 he obtains an easement so long as the owner of Lot 6 does not object, but the complaisance of that owner gives the encroacher no title to a part of Lot 6 without the payment of taxes. He has a sort of squatter claim as it were, modified by the law of the state. It is under some such rule that the case of the carefully surveyed town site with the owners too stingy to make surveys must be considered. It is not believable that all the people can be compelled to get off the land of their neighbors after a long undisturbed possession of their neighbor's land, but if they remove the encroachment they cannot again encroach. Also if the owner of a lot wishes to fully occupy it he has the right to excavate up to his line and if the encroachment thereby falls the owner of the encroachment cannot claim damages, but if the structure on the property he actually owns is damaged, then he can stand in court. The foregoing is again to be modified by circumstances, all of which become matters of evidence and thereby give all parties concerned some reason for taking their troubles to court. If it were the custom to employ a board of

three arbitrators consisting of an intelligent surveyor, a genuine lawyer and a real estate agent of good reputation and long experience, such a board probably would settle all disputes in exactly the way they would be settled in the Supreme Court of a state where the Supreme Court is indifferent to the dotting of an i or the crossing of a t, provided the case has been sensibly presented to the court and substantial justice is desired. Common law is the crystallization of the common sense of the people for unnumbered generations. Statute law is not always on a plane with common law, many times representing the opinions of men elected to serve the people, but really representing some special interest which may be a corporation; and may be an exasperated community.

The general rule about length of possession, as well as the rule regarding supposed acquiescence, may be often capable of modification. Both cases fail when the undisturbed possession or acquiescence may be the result of fraud, or of a mistake made many years before. Fraud vitiates all contracts and acquiescence is the performance of a contract or understanding, a contract being a mutual agreement enforceable at law. The surveyor cannot therefore be too critical of lawyers whose sole interest is that of their clients. The surveyor's interest is merely to ascertain the facts and report them. He may, if a man of considerable experience, be of great assistance in smoothing the feelings of property owners and can suggest means for avoiding actions at law. The general criticism against lawyers is that, as a rule, they are averse to seeing that a surveyor is properly paid for his work, while charging good fees themselves, and they too often try to tell a surveyor how his work should be done and object when he attempts to show that to recover a line it may be frequently necessary to run many lines and take many days to ascertain the facts. When a man requires the services of a physician or surgeon he makes no attempt to secure bids and award the work to the lowest bidder, but rather he secures as soon as possible a man with a good reputation. He acts similarly when engaging a lawyer, proceeding upon the theory that expense cannot be spared when he is compelled to go into court. When the employment of an engineer or surveyor

is necessary attempts are made to employ the man who will work for the least money and too often the selection is left to the lawyer, who is generally inclined to employ the cheapest workman. Young men are preferred as a rule because it is supposed they can get over more land in a given time than older men. Surveyors in the course of years acquire a fund of valuable experience, and many times lawyers are employed on property line disputes only once or twice in a lifetime. It is irksome to competent surveyors to be employed under the direction of such men, hence the friction between the two professions.

Occasionally a surveyor becomes so interested in the legal phases of his work that he studies law and is admitted to practice. John Cassan Wait of New York City is an example of this class, for he made a good reputation as a civil engineer and then became a lawyer whose dictum in such matters is considered conclusive. William E. Kern, Attorney-at-law and Civil Engineer, wrote an article on the legal side of surveys which was printed in *Engineering News*, Vol. 48, and the author obtained permission from his widow to reprint the article in this book.

A Brief Discussion of the Law of Boundary Surveys

By William E. Kern, C. E.

A surveyor may enhance his reputation for piety by being able to cite from the Bible, Deut. XXVII: 17 and XIX: 14; Job XXIV: 2; Prov. XXII: 28 and XXIII: 10, the verses mentioned referring to the sinfulness of removing land marks and the penalty therefor. Josephus has something to say on the same subject, and in his first volume of "Antiquities of the Jews," Chapter II, he intimates that the first murderer was also the first surveyor. In the 12th book of the "Æneid," Virgil recites an unusual use of a monument. The esteem of the ancients is shown by their provision of a special deity, Terminus, to preside over boundary matters, all land marks of stone being regarded as monuments to his godship.

So much for ancient history. The modern surveyor is often called upon to decide boundary disputes, where the best results would flow from a union of his efforts with those of a counsellor at law. In view of this condition and the many questions arising in the practice of a surveyor, some systematic knowledge of the subject on his part would seem desirable.

Several works upon surveying contain a number of syllabi of cases involving boundary disputes. These syllabi are condensed statements of the law applicable to a particular combination of circumstances; and have limited utility for the purpose for which they are inserted in such textbooks. All laymen must be cautioned against making a general application of an abstract statement of law, as determined in some special case. To illustrate this, the following is quoted from a textbook on surveying: "Seventy acres lying and being in the southwest corner of a section is a good description, and the land will be in a square. W. *v.* R. 2 Ham. Ohio 327." This is sufficient reference for a lawyer, as he would look up all the facts; but for a surveyor's enlightenment it should be explained that the description read "Seventy acres lying, and being in the southwest corner of," a certain section, etc., "of the lands sold at Steubenville," and nothing further. The piece in question was part of a larger lot, and had never been marked; nor was there anything to indicate the intention as to its shape. The court disagreed with the views on both sides and held it must be surveyed as a square.

The term "boundary" is used in two senses, and may be defined as: A series of lines forming the perimeter of a specific tract of land; or, an object, parcel of land, or body of water contiguous to a specific tract of land. "Land" in legal parlance usually means a portion of the earth's surface, whether dry or covered with water. Every tract of land with which we may be called upon to deal, can be considered as having at some prior time formed part of a larger tract. Incident to the severance was the creation of a new boundary along the line of division. Every boundary in dispute may be conceived as having had such an origin. After its inception, however, neighbors on either side may, by friendly agreements between each other; or

by long-continued hostile encroachments by one, with neglect to assert rights by the other; or by other acts; affect the legal status of the line so as to alter its original position.

The data for determining the legal status of a boundary may be divided into two classes: (1) Circumstances connected with the severance from a parent tract at the time the line in question first became a boundary, and (2) conduct of neighbors on either side of the line, which may affect their relative status thereto.

EXTENT OF GRANT

The first class above referred to may be designated as Extent of Grant. In discussing this subdivision, it is almost superfluous to say that the rules stated may be limited in their application in specific cases by the principles falling under the second class. In locating a boundary as determined by extent of grant, we must seek the intention of the parties to the primal conveyance.

Let us suppose that A, being the owner of a large plot of ground, divides it into two portions by a line XY running north and south, and sells all east of XY to B, and subsequently all on the west to M. B then sells to C, the latter to D, and D to E; M conveys to N. At a time when N is owning on one side and E on the other, it becomes necessary to define the line XY. This must be based upon the description used in A to B, it being the one which created the line XY. No conveyance in the chain of A to N can affect the rights acquired by E through the conveyance of A to B. The intermediate deeds should be examined, however, as they frequently throw light upon the original intention, as for example: If M took in 1821, N in 1860, B in 1820, C in 1840, D in 1860, and E in 1880, the monuments called for in 1820 may be gone in 1900, but a surveyor in 1820 might have found vestiges and restored them, or created new marks in correct positions; and another surveyor in 1860, finding the marks of 1820, may have made an accurate mathematical description and set two or more imperishable monuments.

It may be urged that the original deed cannot be found,

or is too indefinite to be of any service. The attempt, however, ought always be made; if it fails we must do the next best thing. In a recent case the mesne conveyances of a plot within the limits of a large city differed about 100 ft., in frontage on a street. An examination of the original deed disclosed a call for the edge of a fast land along the side of a swamp; a city plan, showing topography, fixed the position of the water line and solved the problem.

The intention of the parties is generally to be gathered from the description in the instrument of conveyance. Description we will define as a statement designed to identify a specific tract of land. Every specific tract of land is included between lines of definite length, making equally definite angles with each other and with the meridian, and containing a definite area; and any angle point may be conceived as located at a definite distance and direction from some well-known and fixed point. Some specific tracts are marked at angle points or elsewhere by natural or artificial monuments; some are known by particular or distinguishing names. A complete description would contain all of these elements so far as they exist.

MONUMENTS

Monuments are objects established or used to indicate the boundary of a specific tract of land. They may be natural or artificial, the distinction being too obvious to require definition. Under this head are included varieties of land, as swamps, meadows, forests, pasture, etc.; bodies of water and water courses; hillocks, cliffs, trees, rocks, buildings, walls, fences, stones, stakes, pits, etc.

Ambiguity in a description may arise from conflict between elements of a different sort, or between those of the same sort. Monuments may disagree with the mathematical description, or with each other, or be missing; the dimensions may disagree with each other or with the area. When discrepancies arise, the cardinal rule to follow is, *effectuate the intention of the original parties*. Where there is sufficient evidence to explain the conflict and point out the mistake, the line must be run upon the reformed data whether it cause the rejection of a monument or dimension.

It most frequently happens, however, that no such evidence is at hand, in which event the following rules are to be applied:

Monuments are the best evidence of location of a boundary, and natural monuments are to be preferred to all other forms of description. This rule is so important that a list of cases is added containing one from nearly every state in the Union. A perusal of any one case will show the reason for the rule, and probably, in addition, refer to other cases in the same state which may be looked up, as the scope of this article will not permit of stating all the authority for the rules stated.*

This rule, of course, applied only where the monuments were adopted as such, either by mention in the description or by reference to or use of a survey wherein they are established or used (Beckman *v.* Davidson, 162 Mass. 347). In such cases it is not necessary that they be seen by the grantor to bind him. No matter how exactly the survey closes or agrees with the content, if the monuments do not agree therewith, the former falls. If a line is marked on the ground in any manner, though described as straight, it must follow the breaks in the marked line if the latter is not straight. An instance of this kind frequently occurs in running up a creek; several reaches are often combined in one course; in such case the line must follow the water. This occurred in Spring *v.* Hewston, 52 Cal. 442.

In Esmond *v.* Tarbox, 7 Me. 61, a survey was made by

* Ayers *v.* Watson, 137 U. S. 584; Wright *v.* Wright, 34 Ala. 194; Stoll *v.* Beecher, 94 Cal. 1; Nichols *v.* Turney, 15 Conn. 101; Nivin *v.* Stevens, 5 Harr. (Del.) 272; Andreu *v.* Watkins, 26 Flo. 390; Harris *v.* Hull, 70 Ga. 831; Bolden *v.* Sherman, 110 Ill. 418; Shepherd *v.* Nave, 125 Ind. 226; Yocum *v.* Haskins, 81 Iowa 436; Bruce *v.* Morgan, 1 B. Mon. (Ky.) 26; Lebeau *v.* Bergeron, 14 La. Ann. 494; Oxton *v.* Groves, 68 Me. 371; Wood *v.* Ramsey, 71 Md. 9; Woodward *v.* Nims, 130 Mass. 70; Bruckner *v.* Lawrence, 1 Doug. (Mich.) 19; Newman *v.* Foster, 3 How. (Miss.) 383; Harding *v.* Wright, 119 Mo. 1; Johnson *v.* Preston, 9 Neb. 474; Cunningham *v.* Curtis, 57 N. H. 158; Kalbfleisch *v.* Oil Co., 43 N. J. L. 259; Thayer *v.* Finton, 108 N. Y. 394; Redmond *v.* Stepp, 100 N. C. 212; Hare *v.* Harris, 14 Ohio 529; Anderson *v.* McCormick, 18 Oreg. 301; Morse *v.* Rollins, 121 Pa. 537; Faulwood *v.* Graham, 1. Rich. L. (S. C.) 491; Lewis *v.* Oakley, 19 Heis (Tenn.) 483; Wyatt *v.* Foster, 79 Tex. 413; Grand Trunk Co. *v.* Dyer, 49 Vt. 74; Coles *v.* Wooding, 2 P. & H. (Virg.) 189; Teass *v.* St. Albans, 38 W. Va. 1; Miner *v.* Brader, 65 Wis. 537.

one surveyor and marked with monuments, and a plan made by another; the deed recited the latter only. A disagreement between the two having been found, it was held that the monuments governed. In Wilson *v.* Bass, 6 Tenn. 110, a course called for a known point, and then crossing a river at 200 poles continued 213½ poles to its terminus. It was found that the true distance to the river was 213 poles. It was held that as the parties had agreed that it was 200 poles to the river, the line must extend 13 poles beyond the same. In Frey *v.* Baker, 7 Ky. L. Rep. 663, testator divided a tract into two pieces, and described the dividing line as beginning at a point at a fence, and running thence west along the fence. It was shown that the fence did not run east and west, and that testator was ignorant of that fact. It was held that the fence and not an imaginary line due west was the proper line.

Wendell *v.* Jackson, 8 Wend. 183, is an early but interesting New York case. A patent had been issued for a tract described as beginning at the easterly corner of township No. 20, etc., and proceeding by various courses and distances, Fig. 211. The third course ran S 50° E 153 chains to the side of Schroon Lake, with content as 3500 acres. The actual distance to the lake on the third course *FG* was less than one-half of the distance stated. A survey for an adjacent tract, made seven days later, began at a clump of rocks on the side of Schroon Lake, at a corner of the preceding patent, thence followed the same around N 50° W 153 chains, S 40° W 105.5 chains S 31° 15′ E 330 chains to corner of town 20, etc. It was necessary to locate the first patent to determine the title to the land northwest of *EF* and southeast of the line *IJ*. The township corner used as a beginning was well marked by a stake and stones, and was at a point in a swamp difficult of access. It was argued on one side that the call for the beginning at *D* should be disregarded, that the second survey following the first so closely, and having been made so short a time before, should throw light on the latter, and that the first should start at *A* on the side of the lake, and, reversing the courses, run backwards until arriving at *A*. This would

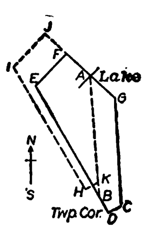


FIG. 211.

locate the land at *AJIHK*. It was held by a divided court of 14 to 5 that the beginning *D* being a known point must be used and the line run to the side of the lake at *A*, thence to beginning, or to *B*. The point being, the supremacy of the call for a natural monument on the "side of a lake." This case might have been decided otherwise if many members of the court had ever had any practical experience at surveying. It is quite probable that the first survey was begun at a point *H*, and the line between *H* and *D* not run, owing to the difficulty of getting over a bog. The adding of an estimated distance *HD* was forgotten, and *AK* and *HK* calculated to make the figure close.

On the other hand, in White *v*. Leming, it was held that where rejecting the call for one monument would reconcile the other parts of the deed and leave enough to identify the land, the rule was not applicable.

As between natural and artificial monuments the former will have the greatest weight. Stakes, being perishable and so easily moved, are regarded as very inferior marks; in one case, Huffman *v*. Walker, 83 N. C. 411, the identification of the location was considered impossible. Where a monument called for is lost, its original position must be ascertained if possible. This may be done by the testimony of witnesses as to its former location, by indications on the ground, or, in the absence of these, by such methods as appear to be the nearest to certainty.

RESTORING LOST SECTION CORNERS

In restoring lost section or quarter corners, various and conflicting rules have been stated by the land office and courts. In order to understand how to restore corners of the public land surveys, some knowledge of the method of making the original surveys should first be had. The details of the latter may be learned from several textbooks, or better still, from the manual of instructions issued by the government to deputy land surveyors. It may be summarized briefly as follows: In several different parts of the western country, initial points have been established from which base lines have been run towards all four points of the compass. The north and south line, being straight

its entire length, is referred to as the principal meridian for its section of the domain. The east and west line is called the principal base, and is likewise straight its full distance. From points on the meridian, at intervals which are multiples of six miles, lines are extended east and west and are known as standard parallels. On the base and parallels, similar multiples of six miles are laid off, and from their termini guide meridians are extended north to the adjacent parallels. The blocks thus formed are subdivided into six-mile rectangles by lines due east and west and north and south, these blocks being called townships. Sections of one mile square are next surveyed by beginning at a point on the south boundary of a township one mile west of its south east corner, and running slightly west of north, so that each mile point shall be one mile from the east boundary of the township, all error being thrown towards the north and west. Each half-mile point is marked by a "quarter corner."

In 1885 the General Land Office issued a pamphlet entitled "Restoration of Lost and Obliterated Corners." This seems to be a very good article except, perhaps, in the method it directs for finding the center of a section. Where a government corner is missing, and after diligent inquiry its former position cannot be ascertained, the surveyor should proceed as follows: If the point was on a base, parallel or meridian, it must be restored to a position on a line between the nearest corners on the same parallel or base, and at a distance which is a proportional part of the whole distance between the known corners. To illustrate, suppose that the intersection of a meridian with a parallel is marked, and the nearest corner south is a section corner two miles away. Three marks are missing. The distance measures 159.2 chains, and the field notes of the original survey show the distances going north to be 40 chains to first-quarter corner, 40 to the section corner, 40 to the next quarter, and 39 chains to close. The excess of 0.2 must be equated all along the line so that the distances become 40.05, 40.05, 40.05, 39.05, respectively, and corners are set at these distances on a line between the two known corners.

In the same manner, points on the boundaries of a township should be set at equated distances, and on straight

lines between the nearest marks on the same boundary, except in the case of the township corners when on a parallel or base. Owing to the system of completing each township by itself, its corners are not necessarily on straight lines. To re-locate a township corner, not on a base or parallel, a trial line should be run between the nearest corners north and south and the distance measured and compared with the field notes. A point is then marked on the trial line at the equated distance. From this point the distances to nearest points east and west are to be measured, added and compared with the field notes, and an equated distance ascertained. The point first fixed is to be moved to the east or west to suit its equated distance in that direction.

The same course is to be followed in re-locating section corners. Let us suppose (Fig. 212) the section corner *A* and quarter corners *B* and *C* are lost and we wish to re-locate *A*. *D*, *E*, *F* and *G* are found marked. A trial line is run from *E* to *G* and the distance measured, temporary corners being marked at *A* and *B*, *B* being placed at 39 chains from *E* and *A* 40 chains from *B*, these being the distances in the original field notes. The whole length having been reported as 119 chains is found to measure but 118.70 a deficiency of 3.30 chains. This being equated between *EB*, *BA* and *AG*, *B* must be moved N 0.10 chains and *A* placed 0.20 chains north of their first temporary positions, *D* to *F* is then measured and found 111.54 chains, as compared with 110 chains on the official plat. This is an excess of 1.54 chains, and duly equated would make *DC* 30.42, *CA* and *AF* each 40.56. *A* is then to be set 40.56 chains from *F* and 39.90 chains from *G*. The quarter corners are to be set on a straight line between their section corners, hence *B* is to be located on a line from *A* to *E* 39.90 chains from *A*, and quarter corner *C* on a line between *D* and *A* and 40.56 from *A*.

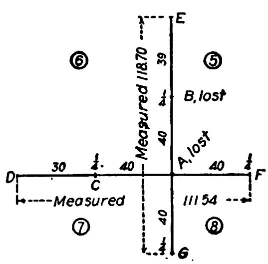


FIG. 212.

FINDING CENTER OF SECTION

Two methods have been mentioned for finding the center of a section. The pamphlet above referred to suggests that it be placed at the intersection of lines run from the opposite quarter corners. Another method was to place it at a point equidistant from the quarter corners, except in the north and west tiers of sections, where it would be located as described above for section corners, using the quarters as a basis. From a legal standpoint the following would probably be preferable:

When completely marked, the exterior of a section has eight monuments. From carelessness, these frequently disagree with the field notes. Quarters are often out of line and nearer one corner than the other. In spite of these defects, when set, the marks must not be altered in position. Again, fractional subdivisions are described as "quarter sections," indicating the idea of quantity; hence a quarter section should be surveyed as one-fourth of the area of the section as physically defined. This should be done by first making a careful survey of the entire section and calculating its actual area. A point should then be fixed so that lines run from it to the quarter corners east, west and south would include exactly one-fourth of the entire calculated area. This would determine a boundary for the S E and S W quarters. A second point on the line between the E and W quarters at such position, that a line drawn from it to the quarter corner north would divide the north half equally, should then be fixed.

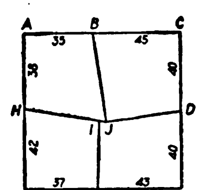


FIG. 213.

The section might then have two centers, as it were, but usually one point would answer the requirements. Fig. 213 will illustrate a possible condition. Suppose the field notes show a section 80 by 80, with quarter corners all 40 chains from the section corners, and a measurement shows an area of 640 acres, but the quarter corners at incorrect distances as indicated. A point I is found by calculation, so that by joining H, I, FI and DI the S E and S W quarters are each 160 acres. A second point J is

found, so that by joining BJ the N E and N W quarters are equal to each other and the other two. The patentee of the N W quarters cannot complain at his irregular lines, as he receives more land than by running straight lines between the quarters corners, as indicated by the land-office circular.

FAULTY DESCRIPTION

Where a description by course and distance does not close in itself, courses may be reversed if necessary, and clearly erroneous, as for example: A description read, "Beginning at the center of a railroad at its intersection with the *B* road, thence S 20° W along said center of said railroad 150 ft., thence N 70° W 50 ft. to a point, thence N 20° E 150 ft. to a point, thence N 70° W 50 ft. to beginning." Here the fourth course was plainly in error, and was read as S 70° E, as there was evidence to show that the fourth and not the second course was erroneous. A similar error occurred in Brown *et al v.* Hage, 21 How. 320, where a patent read, "Beginning at a sycamore standing on the edge of the Shanadoah River, and extending thence down the said river (N 48° W 200 ch.), etc., etc." The title to a large area depended on the retention or rejection of the words in the parenthesis, and the court said it was clear that to go down the river would not be northwesterly, it being a matter of common knowledge that the river ran in the opposite direction at this point, and the words were rejected.

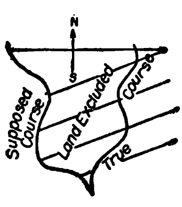


FIG. 214.

Where, however, a natural monument is called for by mistake, such calls will be rejected. Thus in Land Co. *v.* Thompson, 83 Tex. 169, several grants began "on the side of Devil's River," under a mistaken apprehension as to its real position, no survey having been made on the ground. It was held that the surveys must be made by course and distance, recourse having been had to other established marks on the east, and the land between the true and false position of the river was excluded from the grants calling for it. The two positions are shown in the annexed sketch, Fig. 214.

Patents calling for fractional lots along rivers do not take the land between the meander survey line and the river in Nebraska, but this state seems to differ from the others, in decisions on the point. In Jefferis *v.* Land Co., 134 U. S. 178 and Ayers *v.* Watson, the contrary was held, and United States statutes of 1796, 1800, 1805, 1820, 1832, and Sec. 2395, Rev. Stat. U. S. cited to sustain the doctrine that the land between the meander lines and river does pass with the fractional lots, even though the river be navigable. Quantity is usually subordinated to both course and distance as being the least certain element. Occasionally it may govern, however. In one case a deed called for the side of a certain highway, which at the time of the dispute was obliterated. The rear line was defined on the ground. Several positions for the lost highway were found. It was held that the one agreeing most nearly with the area described should be preferred, as none agreed with the course and distance.

Calls for adjoining surveys or tracts must be observed. This question most frequently arises in patents from state governments. In the same manner a plan or plat when referred to must be considered a part of the description. But where the plan conflicts with monuments of an actual survey on the ground, the monuments govern; unless the survey was subsequent to the plan, when, in such case, the latter controls.

The term more or less is construed as importing inexactness in all quantities to which it is annexed. It might seem ridiculous to describe a lot in feet, inches and thirty-seconds of an inch, and add to each length "more or less," but if adjacent properties or other monuments are called for, and the distances to such marks disagree with those of the description, the statement of the indefiniteness of the distances in the description becomes of some value, but, at the most, of very slight account, as the monuments would control as well without the words more or less as with them. Cases where they are of importance will seldom arise.

When course and distance disagree with each other, that will prevail which under all the circumstances and evidence appears to be the most certain.

The cases generally arise in this way: The opposite ends of two adjoining courses being agreed upon as definitely determined and located, and one course being extended to meet the other reversed to run from its opposite terminus, the point of intersection is found to disagree with the distances. The general rule is that courses prevail over distances. Thus, in Curtis *v.* Aaronson, 49 N. J. L. 68, 1886, both parties agreed until the 16th course was run. This was described as S 24° E 29 chains, thence S 65° 15′ W 151.5 chains to pine on east side of Shoal Branch. The position of this pine was not disputed, nor was the initial point of the 16th course; but by running the latter S 24° E to intersect the 17th course extended N 65° 15′ E from the pine, the 16th distance would be lengthened to 96.5 chains in lieu of 29 chains as stated. The surveyors in re-running the entire survey where the lines were not disputed, found the courses all approximately correct, but the distances very erroneous. This fact, among others determined the court to hold that the disputed courses must be intersected and the disputed distances ignored.

On the other hand, in a somewhat similar case in Kentucky in 1821, the discrepancy between accuracy of courses as compared with distances did not appear, and the court directed the distances to be maintained, as by intersecting the courses the distances would be very greatly distorted.

Areas are regarded as the least certain of the elements of a description. So where courses and distances do not include the same area as that stated, the latter must yield, unless intention is clearly expressed that an area of specific quantity shall be conveyed, in which latter case the area and not the distances should control in construing the description of the land. Sanders *v.* Golding, 45 Iowa 463.

Plats or maps referred to in a deed are to be considered as a part of the description. The following illustrates their status: In Vance *v.* Fore, 24 Cal. 435, a deed referred to another deed, previously made, for a description of the premises granted, and also to a map not contemporaneous with the last deed. The description in the older deed called for no monument at the initial point, nor could any be

identified with accuracy. The first course terminated at "the base of the mountain, thence running at right angles, following down the base of the mountains." The character of the country was such that witnesses might reasonably differ in their location of the lines. The map referred to showed all the natural and artificial monuments found on the ground, such as streams, buildings and roads. The descriptions as set forth in the old deed and on the map conflicted. It was held that the description shown on the plat was the most certain under the circumstances, and the least likely to be affected with mistakes, hence to be followed in making the re-survey.

There is an apparent conflict in the cases as to the relative precedence of plats, and the elements of course, distance, etc., but, as explained before, all cases must be considered from the standpoint of principle involved. The principle in this case being that that which is shown to be the most certain will prevail.

In Heaton *v.* Hodges, 14 Me. 66, no survey could be ascertained to have been made, and the monuments shown on the plan could not be identified; certain distances on the plan disagreed with measurements of established lines. It was held that the disputed lines were to be first scaled from the plat and the lengths so ascertained, altered in such proportion as the length of the accepted lines bore to their dimensions on the plat.

In Lampe *v.* Kennedy, 45 Wis. 23, several deeds were made from a plat. The courses and distances did not agree with the plat, but the latter prevailed. In the same manner, plats were held to control quantity, in 109 Ill. 46, and Hathaway *v.* Power, 5 Hill (N. Y.) 453.

In Beaty *v.* Robertson, 130 Ind. 589, it was said that where the plat and field notes of a government survey conflicted, the former showed the lines as fixed by the surveyor-general, and were those by which the land was sold, hence to be taken.

The last method of describing land is by a particular name. In earlier times this was necessarily the most common. In such cases disputed boundaries are established by the evidence of witnesses familiar with the oldest physical location of the lines, or the admissions of inter-

ested parties. This class would include, however, description by lot and block number as per a plat. Town-site plats usually show in addition to the dimensions of the lots, stone monuments set at intersection of specified offset range lines at street corners. The monuments are frequently found to be inaccurately set, and various expedients have been adopted by later surveyors to eliminate the error. In some cases the total error of each block has been thrown in the adjacent street; in still others, the streets have been made the proper width and the error equated through the lots. Probably the best method by analogy with the decisions on lost government corners, would be to measure the distance between monuments and compare the same with the distance between monuments and compare the same with the distance between the same points as shown on the plat. The difference should then be equated for every foot alike, whether street or lots.

STANDARD OF MEASUREMENTS

Congress has never exercised its constitutional right to fix a standard of linear measurement, but an official standard having been adopted by the United States surveyors, a so-called United States standard foot has arisen, and several states have enacted statutes adopting this standard, which, in the absence of Congressional action, is lawful for the respective states so adopting them, and all standards used in contravention of such state standards are illegal in these states. This gives rise to some important questions, which will now be considered.

Where a piece of ground is conveyed and the description used is an illegal standard, that standard must be used to establish its boundaries; but if it cannot be shown that the illegal standard was adopted by the parties, then the construction must be based upon the legal standard of the state. To make the case more concrete: If A owns 500 ft. front of unimproved land and sells 100 ft. from one end, this must be measured in legal standard; but if he owns one lot, described as either 20 or 100 ft., but by an illegal standard, and the further words are added, "being the same premises," etc., then he would take the same premises,

or all that his grantor took in the recited conveyance, whether more or less. In other words, it is a question of intention; if both parties meant the same thing, then that is what they contracted for, or else nothing.

PROPERTY ABUTTING ON ROADS AND STREETS

Where land is conveyed by side of a road, street or alley, the usual practice is that, by implied grant, half the road is conveyed with the abutting lot. In some states the fee of the streets is in the public; in such cases the abutter stops at the side of the road. In a few isolated cases the law has been held to be that a conveyance did not pass title to the center by implication.

Thus, in Union Cemetery *v.* Robinson, 5 Wh. (Pa.) 18, the general principle that a conveyance passed a fee to the center of the road was not doubted, but it was said that as in the case at bar the street described was only on paper, and unopened, and further, that the description being in feet, inches and fractions of an inch, much stress should be laid upon the minuteness of the measurements, as showing an intention to limit the conveyance to the side of the road or street. This case has very properly been overruled in the same and other states, both as respects the importance of the refinement of measurements and as to the fact that the street was unopened. The leading case on the subject being Paul *v.* Carver, 26 Pa. 223. The principle that a conveyance along a road carries the boundary to the center of that road has been followed in practically every state of the Union, although there have been instances where courts have been swayed by exceptional circumstances to hold that an intention was manifest to exclude the road. The words "by the side of" have been given that effect; also a grant of right of way over the strip included in the street. The following from Paul *v.* Carver is a forcible statement of the reason of the rule: "The rule had its origin in a regard to the nature of the grant. Where land is laid out in town lots, with streets and alleys, the owner receives full consideration for the streets and alleys in the increased value of the lots. The understanding always is that houses may be erected fronting on the streets with

windows and doors, doorsteps and vaults. If a right of property in the streets might under any circumstances be exercised by the grantor, he might deprive his grantee of the means of entry into or exit from his house, and of all the enjoyments of light and air, and might thereby deprive him of the means of deriving any benefit from his purchase."

Where, however, the original line between two adjacent tracts existed before the road was laid out, and was not coincident with the center line of the road; then it, and not such center line, is the boundary.

Where the conveyance is along the margin of a river or other water way, the decisions are conflicting, and no precise rule can be stated. In the majority of the states, however, a conveyance by the side of a tidal or navigable river or sea carries the fee to high-water mark, and on unnavigable rivers and ponds to their center. The side lines are projected at right angles with the thread of the stream, unless otherwise provided for in the deed.

BOUNDARIES FIXED BY AGREEMENT OR AQUIESCENCE

Boundaries may be fixed by agreement of the owners affected, and in such cases bind those who take from them. But if either party was led to agree by a mistake in a measurement made for the purpose, he may repudiate his agreement if he does so within a short time after discovery of the mistake. Coon *v.* Smith, 29 N. Y. 392. An agreement to fix a boundary where it is indefinite need not be in writing. Turner *v.* Baker, 64 Mo. 218.

In each state of the Union statutes exist prescribing a length of time during which various suits may be brought, and upon failure to bring such suit the cause is lost. The periods of these several "Statutes of Limitations," vary in the several states. A boundary, though erroneous, if acquiesced in by the parties for a term beyond that of the appropriate statute, becomes fixed, and as suit could not be brought to rectify it, it should be surveyed as maintained.

Estoppel is a legal principle by which one who has led another to believe that certain conditions are true, is after-

wards precluded from showing such conditions are false, where the innocent party would thereby be injured. So boundaries may become fixed by estoppel. Thus, in Sheridan *v.* Barret, one led another to believe that their boundary was in a certain location, and to build a wall in a position based upon that fact. The first was compelled to adopt the boundary which he had falsely represented as being correct. See also New York Co. *v.* Gardner, 25 S. W. 737; Jordan *v.* Deaton, 23 Ark. 704.

In re-marking a boundary, we are first to satisfy ourselves that no line has been acquiesced in, agreed upon or maintained by adverse possession for the statutory period. If not, the deeds are to be examined and the line traced on the ground as originally run; if no vestiges of the latter remain, then the elements of description are to be observed in the order named, bearing in mind that that which is most certain will prevail over the less certain. (Finis.)

GOVERNMENT SURVEYS

When a district was established for the disposal of public lands an Initial Point — of which there are 31 in the United States — was selected within the district. Through this point was run a north and south line called the Principal Meridian; and an east and west line called the Principal Base.

At intervals of 24 miles on the Principal Meridian, measured from the initial point, Standard Parallel or Correction Lines run east and west parallel with the Principal Base. Beginning from the Initial Point they are known as 1st, 2nd, 3rd, etc., Standard Parallel North (or South).

From the Principal Base and Standard Parallels, true north lines called guide Meridians run at intervals of 24 miles. Thus the land is divided into "checks" approximately 24 miles square.

The standard parallels were called Correction Lines because the Guide Meridians being true Meridians converged on account of the spherical shape of the earth, so a "check" is narrower on the north than on the south. On each standard parallel two stakes not far apart are set for each guide meridian, which has caused many lawsuits because the

first settlers, their attorneys and surveyors knew little or nothing of astronomy and geodesy.

Each check is divided into townships measuring six miles north and south and six miles wide on the south end. The north and south township boundaries are parallel but the east and west boundaries being true meridians converge, hence there are again two stakes marking meridians where parallels intersect them.

Each township is divided into sections one mile square (approximately) and in surveying the sections (subdividing a township), monuments were supposed to be set every half mile. The corners are called "section" corners and the half-mile corners are called "Quarter Corners," because they divide the section into quarters. No stakes were set in the interior of a section.

The original surveys were made by contract, all contracts being awarded to the lowest bidders. Many ignorant men obtained contracts and so much trouble arose that after a time farcical examinations were held to license surveyors. Political influences vitiated this attempt to improve matters. The business of making government surveys assumed large proportions and many frauds were perpetrated by dishonest contractors, some of whom turned in field notes for surveys that were not made.

The intelligent men in the land department tried to check fraud by having each survey examined before paying the contract price. The work of examining surveys was let also by contract to the lowest bidders, with results any man of common sense could have foretold. At the present time examiners hold office for life with a fair salary and are selected by means of competitive civil service examinations.

Cheap work was done; dishonest work was done; corners were seldom made of permanent material; early settlers were careless; many surveyors who made the first re-surveys were very ignorant; state legislatures unfortunately thought corners could be "re-established"; the government altered methods a number of times; on all surveys prior to 1864 an inaccurate method was used to indicate courses on true lines; these were a few of the many causes for trouble over "government lines," and the Land Department was compelled to issue instructions to guide

surveyors in making re-surveys in states in which the land was "sectionized."

The pamphlet is revised from time to time and may be procured free of cost from the General Land Office, Washington, D. C. The title is "Restoration of Lost or Obliterated Corners and Subdivisions of Sections." It should be owned and carefully studied by every land surveyor.

The government "Manual of Surveying Instructions for the Survey of the Public Lands of the United States and Private Land Claims," is supplied by the Government Printing Office, Washington, D. C., for seventy-five cents, postpaid. Instructions for surveying and marking mining claims may be obtained from the Surveyor-General for the district in which the claims lie.

The student after reading this chapter is ready to appreciate the statement that most of the trouble over boundary lines is caused by the ignorance of the men who make the first re-surveys. They are disposed to follow field notes rather than monuments; or possession when monuments cannot be found. Field notes and maps are often in error, when compared with modern standards of accuracy, but nevertheless must be used as a guide in re-tracement surveys, and they always stand in court until evidence is introduced to destroy their credibility. The surveyor must refuse to accept work from clients, or their attorneys, who attempt to dictate as to how a survey must be conducted. *A single line cannot fix the location of an obliterated monument.*

To subdivide a section all the exterior lines must be retraced and correct bearings and lengths obtained. Then, and not until then, smaller parcels may be cut out and safely monumented. The plat and field notes should show this.

CHAPTER VIII

ENGINEERING SURVEYING

Surveyors and civil engineers use the same instruments and in schools where surveying is taught as a branch of mathematics instruction is given in the use of instruments and the computations connected therewith. From this point surveying is divided into:

1. Land surveying.
2. Engineering surveying.

Land surveying, strictly, is concerned only with the dividing of land; to determine areas and to re-locate missing monuments and boundaries. Leveling is really engineering work but it is a part of the work performed by all land surveyors because they are surveyors.

Engineering surveying consists in making surveys for railways, highways, canals, ditches, and the setting out of work, such as bridges, dams, large buildings, tunnels, etc., together with the computation of quantities of materials. Surveys for the purpose of making contour maps, selecting the gradients for roads, surveying for the location of sewers and computing the amount of earthwork falls naturally to local surveyors who specialize in land surveying, because such work may be done by anyone familiar with the use of instruments, surveying methods and drafting. Mining surveying is engineering work which may be equally well done by all qualified land surveyors. Surveys made for the purpose of preparing good maps belong both to the land surveyor and the civil engineer. In this chapter it is proposed to present some fundamental information relative to good practice in engineering surveying.

MINING SURVEYS

Surveys made to determine the boundaries of mining claims on the surface of the ground are called surface surveys. No further instructions are required for such work

than have been given for land surveys. The work is done with a transit and steel tape with the greatest possible accuracy. When the location of a mining claim is made it must be done in accordance with instructions issued by the Commissioner of the General Land Office, Washington, D. C., and the local rules and regulations of the United States Surveyor General within whose district the claim is located.

Underground surveys are made to show the workings of mines. The top of the shaft, or the mouth of the tunnel where either appears on the surface, is located and tied to some known corner or permanent object on the surface. A meridian line is chosen and set out on the surface for some distance and the ends marked with good stakes, so a long backsight may be obtained. If a tunnel is to be surveyed, a point on this line is set opposite the mouth of the tunnel and the line of the tunnel is run out, and tied to the meridian. The line is carried through the workings by backsight and foresight, all angles being double centered and all tack points being set by double centering. When a survey is made on the surface, it is possible to run around the land and make a closed survey, thereby obtaining a check. When the survey is run underground this opportunity to make a check does not occur, except when a point may be projected up through a shaft, or be carried through another tunnel to the surface. The greater number of mining surveys, however, take the lines well underground and unless much care is used a mistake may be made in setting down *R* for *L*, or vice versa, and a map made from such incorrect notes will be considerably in error.

Double centering is always advisable, but if the transit is in perfect adjustment and the surveyor is very careful, it is well to run the line in with azimuths first. Then follow with a carefully run double centered line. On account of the presence of ores or of tracks and metal tools the needle cannot check underground surveys. In coal mines the needle may sometimes be used to advantage in the approximate location of rooms for placing same on working maps, but the cases are few when the needle may be used.

The preservation of points is a vexing question in mine surveys. When the ground is squeezing and sinking it is impossible to preserve points for any length of time and much of the work of a mining surveyor is concerned with the preservation of lines. Usually it is best to drive nails in the timber over the tunnels, these nails having in them holes through which a plumb-bob string is tied. Flat-headed horseshoe nails with holes drilled in the heads are commonly used. To set up a transit it is necessary to have points underneath on the ground so temporary points are used. These temporary points are often made of pointed nails driven through a piece of lead made in the shape of a shallow cone and weighing several pounds. The point of the nail is at the top. A plumb-bob is suspended from the nail in the roof and the temporary hub placed on the ground so the point of the plumb-bob is directly above the point of the nail. The bob is then hung on one side and the transit placed over the point. Backsights and foresights are taken on the nails in the roof. On account of strong currents of air in mine workings all lines should be as thin as possible and plumb-bobs should weigh not less than two pounds. If they swing too much they should be immersed in buckets of oil with the lower end free of the bottom of the bucket.

Various forms of plumb-bobs are made for use in mines having lamps in the shank, so sights may be taken on the flame. The writer found in his experience that the best sight is a candle about two inches or less in length. When lighted the candle is to some degree transparent and the wick showing dimly through the luminous wax or paraffine makes a very good point on which to sight. These lights may be set on the ground by using the suspended plumb-bob in the manner mentioned for setting temporary transit points. For reading the verniers it was formerly customary to have tallow candles, but today small pocket electric lamps are used, many instrument makers now supplying reading lamps made especially for use in mines and tunnels.

The methods described are those used in all ordinary tunnel work, the driving of very costly and important tunnels for railways being work that is entrusted only to men having good experience in this particular specialty.

In mining work and in the driving of ordinary tunnels the surveyor is interrupted often by the passing of cars, laborers, etc., and as he is not permitted to interrupt the regular work by his operations, they have the right of way. This calls for the exercise of considerable patience and necessitates rapid work. Mistakes, however, are not forgiven, interrupted work being considered no excuse for mistakes. When the surveyor can choose his own time to do the work he works on Sunday, provided the mine shuts down on that day, something seldom done, or he works when the mine is shut down, which is a frequent occurrence in some lines of mining, especially in a dull season. Usually, however, a mine is run night and day and the surveyor is the least considered of all the laborers, but his work nevertheless must be kept up.

Underground surveys are made for the purpose of keeping up the office maps and also for guiding the miners. When done to keep up the maps a line is run down the tunnels, and stakes set opposite openings in the face, or opposite entrances to rooms, at regular stations, 50 to 100 ft. apart. From these stakes approximate measurements (to the nearest half foot) are made forward and back on line to the edges and a measurement taken normal to the line in to the face at each end and at the station. If closer results are wanted closer measurements are made. The mine maps, however, are on a scale that will hardly show a measurement of less than 2 ft. anything less being estimated. When the survey is made for the purpose of guiding the miners, it is necessary to set three points on a line pointing the direction in which the work is to proceed. The foreman, or boss, then hangs three plumb-bobs to these points and uses them for sighting purposes. He will carry his work in quite a distance, setting points ahead from which to project his line, when he gets too far away to see the three lines plainly that were left by the surveyor. When he feels there is danger that he may be working to one side the surveyor is sent for to set three more points, one close in to the face.

Careful work is all that is necessary in ordinary underground work, much care being necessary to prevent injury to person and instrument. The best mining surveyors

are men who have worked as miners before studying surveying.

The greatest work of a mining surveyor is the carrying of a line from the surface to the bottom of a shaft. Perhaps a shaft is being sunk in a certain place and it is intended to make an upraise to complete the job, or an upraise is to be started to reach a certain point on the surface. The traverse cannot be closed until the shaft, or upraise, is finished, and a small error in running the lines may cost thousands of dollars.

First the point on the surface is located and a line run carefully to the shaft, two points being set at the shaft, one on either side and carefully marked with a tack. All angles are double centered. The sight on the surface may of course be several hundred feet but the line projected down into the tunnel cannot be wider than the shaft, and from this short base in the bottom the underground lines must be run with such a degree of accuracy that the hole at the starting point will be dug in the proper place. Having marked the base line on the surface set the transit over one of the points at the edge of the shaft and sight down the shaft, placing a hub and tack there. For this purpose it is often necessary to have an extra telescope on the transit, swinging clear of the edge of the plate. When the hub is set below, take the transit to the bottom and from a backsight on the tack above, project the line through the tunnel.

Work, such as that just described, cannot always be accomplished because of the depth of the shaft. It then becomes necessary to set points in the bottom of the shaft by using plumb-bobs. These bobs should weigh not less than ten pounds and be suspended by fine piano wires instead of cord, provided with reels at the top to keep the points off the ground. A cord is stretched across the top of the shaft from one point to the other. The heavy bobs are then let down on each side of the shaft, as close as possible to the sides without touching same. The bobs must be immersed in buckets of oil to prevent vibration and swinging. When the lines are perfectly still the surveyor goes back into the tunnel and gets himself in line with the plumb lines, which are illuminated by his helpers. His

transit is set up and leveled and a sight taken on the wires. If he sees both, he moves the transit to one side and tries again. By trial he finally gets in a position where one line only is seen, the other being hidden behind it. He reverses the telescope and tries again, proceeding in this manner until he is satisfied the center of the transit is exactly in line with the two wires and that the instrument is level. He then sets two points, one ahead and one back of the instrument as points on his underground base.

The student is referred for more information on mining surveys to the best American books on the subject:

"Underground Surveying," by L. W. Trumbull, E. M. ($3.00) and "Mine Surveying," by Edward B. Durham, E. M. ($3.50.)

The best English book was written by the late Bennett H. Brough.

HYDROGRAPHIC SURVEYS

Hydrographic surveys are surveys of rivers, lakes and harbors. The survey of the shore line is made in the customary manner of traverse surveys, the hydrographic work being the survey of the land under the water. Sometimes in making surveys of rivers a method of triangulation may be used as shown in Fig. 215.

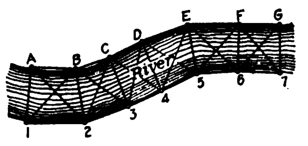


FIG. 215. Triangulation survey of river banks.

Assistants first set stakes on each side of the river at distances approximately equal to the width of the river. The distance AB is carefully measured with a steel tape and the transit is set on each stake on that side of the river. From A, with a foresight on B, angles are taken to stakes 1 and 2. The transit is then set on B and from a backsight on A, angles are read to stakes 1, 2, 3 and C. The transit is then set on C and from a backsight on B angles are read to stakes 2, 3, 4 and D. Proceeding in this manner for a mile or so another distance is carefully measured with a steel tape. By plane trigonometry the lengths of all the lines are computed and the new base line is also computed as a check on the work. The work is carried forward from

the measured length of the new base line, these measured bases being introduced at fairly regular intervals to keep errors within reasonable limits. The limit of error is first fixed for the work and if the error at any base line is too great it may be wise to have the measured bases at closer intervals. In the figure EF is a measured base. The distance to the shore line is measured on offset lines from the numbered stakes, or the helpers on the banks may have stadia rods and the shore line be located thus from the transit.

Soundings may be made from boats, men in the boats holding stadia rods which are read from a station on shore. This is only possible in still water. For the proper sounding and survey of lakes and harbors along the shores of lakes or the sea, several methods are in vogue.

In one the boat containing the men making the soundings keeps as closely as possible to a line by means of three stakes set on shore while two observers at stations on shore read angles from a base line to the boat with transits or sextants. Sometimes the boat is rowed at a certain rate on a line determined by sighting on range stakes, and soundings are taken at definite intervals of time. Sometimes an observer in the boat takes angles with a sextant to three points on shore, thus introducing the "Three Point Problem," one of the standard problems inserted in textbooks paying considerable attention to the mathematical side of surveying.

Problem. — Given three points in a triangle and the distances between them $AB = 317$ ft., $AC = 308$ ft., and $BC = 478$ ft.; also the angles at a point D which these distances subtend in the same plane with them, i.e., $ADB = 24°\ 50'$, and $ADC = 27°\ 44'$; to find the distance of the station D from each of them.

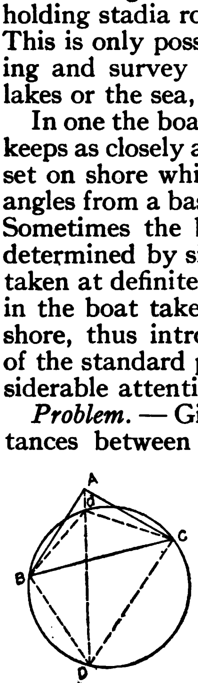


FIG. 216. Three-point problem.

Construct the triangle ABC and on the line BC, set off at C the angle $BCd = ADB = 24°\ 50'$; and at B set off the angle $CBd = ADC = 27°\ 44'$. Point d is located by the intersection of the lines from B and C. Through the three points B, d, C draw a circle. From A draw a line through d and produce it

to an intersection with the circumference, thus locating the point D. Draw lines from D to B and C.

Three sides being known in the triangle ABC, the angle $B = 39° 25' 14.6''$; then $ABd = ABC + dBC = 67° 9' 14''$, when A and d are on different sides of BC, or $= 11° 41' 14.6''$, when A and d are on the same side of BC as in the present case.

In the triangle BCd are given the side BC and the angles B and C. Then the side $Bd = 252.7$ ft.

In the triangle ABd are given the sides AB and Bd with the included angle ABd. Then the angle $AdB = 131° 53' 53''$, and $BAd = 36° 25' 53''$.

Then in the triangle ABD are given the angles and the side AB. We find $BD = 448.066$ ft., and $AD = 661.738$ ft. In the triangle DBC are given the angles and the side BC. We find $DC = 591.563$ ft.

If the triangle ABC is reversed so the point A is the point nearest D, the angle $BAd = 46° 47' 32.2''$; then $BD = 550.153$ ft., $AD = 282.25$ ft. and $CD = 528.4$ ft.

1. If D be within the triangle, as at d, make the angles BCD and CBD = supplements of BaA and AdC.

2. When D is in one of the sides, describe a segment on BC containing the given angle.

3. If A and B be in a straight line with D, then BC and CA subtend the same angle BDC. Solve the triangle DBC after finding the angle at B.

4. If the three points A, B, C lie in a straight line, the first operation will not be required. The other operations are unchanged.

When making soundings of a river an approved method is to stretch a cord across, having knots at definite intervals and row a boat along the line, taking soundings at each knot. This is possible only in rivers with sluggish current. Another method is to set stations along the bank at distances about equal to the width of the river and row a boat across at an angle in each of the squares thus formed, taking soundings at as nearly regular intervals as possible. Each square has two diagonals and when the notes are plotted it is assumed that the spaces between soundings are equal to the length of the diagonal divided by the number of soundings. In addition to the diagonal

lines, soundings are taken on a straight line normal to the banks of the river from the stake on one bank to the stake on the opposite bank. When studying a navigable river for the purpose of locating a bridge, the government requires an accurate map of the banks for half a mile below and one mile above the proposed bridge site. Soundings must be shown at approximately 100 ft. intervals for the entire distance on the map. The right-hand bank of the river is the bank on the right hand of the observer when traveling with the current.

Current observations must also be given in hydrographic surveys of rivers. These are best obtained by measuring a definite base line along one bank of the river and sighting across the river perpendicularly at each end. At the upper end floats are released and timed as they cross the upper line, being timed again as they pass the lower line. Bottles partly filled with water or sand and having in them small rods carrying flags, make good floats for the purpose. One bottle with a white flag should be traced down the middle of the river, one with a red flag close to the right bank and one with a blue flag close to the left bank. To avoid confusion let the middle float go and after the recorder makes his entry of the time, release a second float, and when that record is made, the third. The rate of flow is generally given in feet traveled per second. To determine accurately the amount of water flowing in a stream it will be necessary to use more than three floats to get the stream lines, and in each line of floats there must be three: One to give the velocity near the surface, one the velocity at about half the depth and one to give the velocity close to the bottom, so that for such work it will be necessary to first make soundings.

To obtain the velocity at different depths the floats will consist of very thin rods — wire is best — with flags at the upper ends and weights at the bottom, regulated to keep at a definite depth, which is not a difficult matter. Sounding lines are prepared so they will not stretch when wet, by wetting and wrapping around a tree or post as tightly as possible and leaving there until dry. This operation is repeated several times and takes out all the stretch, after which they may be measured off in five-foot lengths, with

colored cloths for markers and small tags to indicate the length. The weight on the sounding lines should be very heavy so the line will be taut and there may be a reasonable certainty that the depth obtained is vertical and not inclined. Tags at five-foot intervals are all that are necessary, for lesser intervals may be measured with a rule by the observer in the sounding boat.

Soundings in water are wanted for a number of reasons but an engineer or surveyor in private practice usually does such work to ascertain the yardage in dredging operations. The bottom must be first surveyed and after the dredging is completed, or when a progress estimate is wanted, must be again surveyed. A method used for such work is to make soundings from a scow, the positions occupied by the scow being obtained from the shore.

Fig. 217 is a diagrammatic representation of a sounding device used for such work. Several graduated rods are attached to the scow by passing through holes in boards projecting over the edges, the lower ends of the rods being fastened to inclined pieces. These inclined pieces are hinged at the upper end where they are fastened to the scow and the lower end of each drags on the bottom. Every rise and fall in the bottom is read by an observer on the scow as the area is "swept" over.

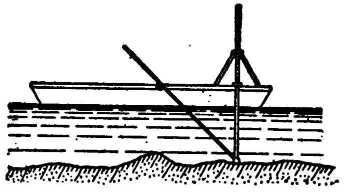


FIG. 217. Sounding from barge.

For complete information on hydrographic work the student is referred to "Hydrographic Surveying," by Samuel H. Lea ($2.00).

ROUTE SURVEYS

Route surveys are made for the location of railways, highways, canals, pipe lines, etc., where a study must be made of the possibilities for obtaining a proper grade. With railways a one per cent grade (1 ft. rise in 100 ft. of distance) is steep. For common highways a 5 per cent grade is about the limit on good roads, although grades of 10 per cent are not uncommon. The grade of an electric

road may be considerably higher than that of a steam road, but pulling trains and wagons up a grade is expensive, amounting to a vertical lift in a given distance equal to the difference in elevation between the ends. The grade for drainage ditches should be as great as possible in order to carry the water without washing the banks and cutting gullies in the side of the hill. This is true also of ditches used in mining operation. Open earth ditches have a very steep grade when it exceeds 10 ft. in one mile and not all soil will stand as much grade. For drainage ditches it is frequently difficult to get a grade greater than 2 ft. in one mile. In irrigation work the problem is to take the water to the highest point on the tract to be irrigated and when this tract is reached to keep it high to serve the greatest possible area. If the grade of an irrigation ditch is too light it will silt up rapidly and aquatic plants will grow and interfere with the flow. The problem is then to nicely adjust the grade to keep the ditch, or canal, on high ground and yet have a rapid enough flow to prevent undue silting and growth of grass. Three feet in one mile is a big grade for an irrigation ditch and some have a fall of only 8 ins. to the mile.

A simple method of survey for roads, ditches and canals consists in setting grade pegs according to methods given in the chapter on leveling. The line follows very closely the contours of the surface and is necessarily crooked. A better way is to set the grade pegs farther apart, 100 to 200 ft., and follow this line with a transit line regularly stationed. Elevations of the ground at each station are taken and also side slopes so contours may be placed on the map. A grade line is then selected to give as long straight sections and as easy curves as may be advisable. The grade line is plotted on profile paper and the cuts and fills balanced. The line is then staked out on the ground from the field notes picked out on the map.

A railway survey proceeds upon practically the same system but the locating engineer goes ahead and picks out the route by eye without running preliminary levels, using a hand level to keep him close to grade. On this line selected by the chief of party the transitman directs the chainmen in setting stakes at stations one hundred

feet long and takes angles at all changes in direction. The leveler follows and gets the elevations at each stake. The topographer — slopeman he is called on some roads — takes the side slopes and the draftsman plats all the information on a map, the leveler making a profile each evening of the day's work. On the map the chief of party picks out the line, which is then run in and "located" on the ground.

For full information on field and office methods of making route surveys the student is referred to "Railroad Location Surveys and Estimates," by F. Lavis, M. Am. Soc. C. E. ($3.00).

CONTOURS

Many surveys are made for the purpose of obtaining information about the shape of the surface of the ground. This is called topographical surveying. The common method today for showing the surface of the ground is by means of contours and every surveyor should know how to make surveys for the purpose of preparing contour maps and should know how to use such maps. A surveyor given a tract of land to subdivide should first prepare a contour map in order that he may lay the roads on proper grades and take care of sewerage and drainage. Properly made contour maps may be used also in earthwork calculations.

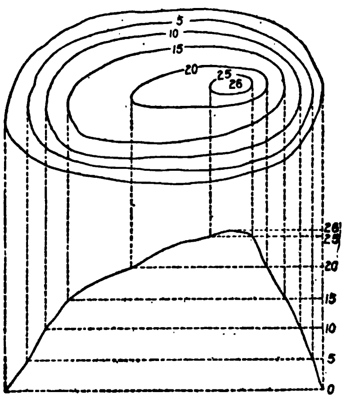
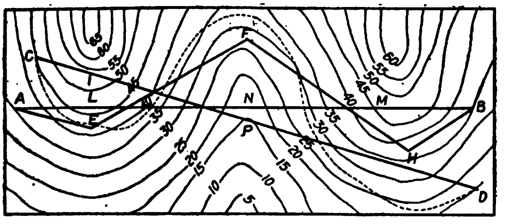


FIG. 218. Contours and profile of hill.

In the upper part of Fig. 218 is shown a contour map of a hill and the lower part shows a side view of the hill with the contour lines dotted across the face. Contours are level lines defining the shape of the surface of a hill, or hollow. Assume that the hill in Fig. 218 was submerged by water which stood at a depth of 26 ft. long enough to leave a mark on the hillside at that height. The water

then receded to a depth of 25 ft. and left a mark. It receded 5 ft. at a time, standing at each level long enough for débris to collect on the hillside at each step, so that the respective elevations were marked on the surface. Water surfaces are always level so each line of débris marked a level line around the hill at the elevations shown. A side view shows the lines as level, while the plan, or map, gives the shape of the hillside.

In the upper part of Fig. 219 is shown a map of a hillside, having two projections. The straight lines show lines



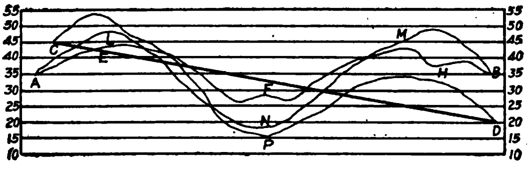


FIG. 219. Selecting grades on contours.

connecting points *A* and *B* and *C* and *D*. The contours are marked on the map and it is desired to obtain a uniform grade connecting *C* and *D*. First draw a straight line from *C* to *D* and on this line note the elevations of the points *I* and *P*. On the profile paper below plot these points, and also the elevations of intermediate points and the wavy line *C*, *I*, *P*, *D* is obtained. A straight line therefore will not give a uniform grade. Following the dotted line and plotting it on the profile a uniform grade is obtained so that it is represented by the straight line *CD* on the profile.

The dotted line is obtained in actual practice by drawing

the straight line on the profile and then projecting the points where it intersects the horizontal lines on the profile paper up to an intersection with the contour lines on the map. The intersection being obtained with the contour line in each case, a dotted line is drawn from one contour to the next as shown.

The line *AB* is similarly studied. First the straight line was drawn on the map resulting in the very wavy line *A*, *L*, *N*, *M*, *B*. Next a crooked line on the map was selected resulting in the wavy profile *A*, *E*, *F*, *H*, *B*, which has smaller depressions and less marked elevations than the first line. To obtain a proper grade line on the map a straight line should be drawn on the profile from *A* to *B* and this line projected up to the map.

In making route surveys all data are obtained which are necessary to show the contours for some distance either side of the surveyed line. A grade is picked out on the profile made by the leveler and the point where the grade crosses each contour is marked by stations. These points are laid off on the contour map and a line sketched in to indicate the line of uniform grade. The chief engineer, or chief of party, then lays a line as close to this as possible so cuts and fills will be fairly equalized and the line be as free as possible from great curvature and abrupt changes in direction.

Fig. 220 is reproduced from an article by Dr. W. G. Raymond, in Vol. VI, Transactions of the Technical Society of the Pacific Coast, p. 72 (1889). The description is as follows: "A small reservoir is to be built on a hillside, and will be partly in excavation and partly in embankment. The contours are spaced 5 ft. apart. The top of the wall — shown by the full lines making the square — is 10 ft. wide and at an elevation of 660 ft. The reservoir is 20 ft. deep, with inside and outside side slopes of 2 to 1, making the bottom elevation 640 ft. and 20 ft. square, the top being 100 ft. square on the inside. The dotted lines are contours of the finished surfaces inside and outside the reservoir. The areas in fill all fall within the broken line marked *abcdefghik*, and the cut areas all fall within the broken line marked *abcdefgo*. These broken lines are grade lines. The areas of fill and cut are readily traced by following

the closed figures formed by contours of equal elevation, thus:

At 640 ft. level the area in fill is *pst.*
At 650 ft. level the area in fill is *lmnuvxl.*
At 650 ft. level the area in cut is 1, 2, 3 *uxl.*

The other areas are as easily traced. The closed figures formed by the intersecting contours of equal elevation are

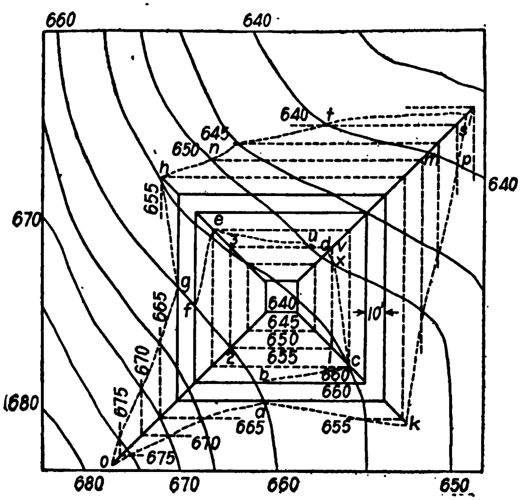


FIG. 220. Computing earthwork from contour map.

horizontal areas of cut or fill separated by the common vertical and perpendicular distances between successive contours. These areas may be measured and the quantities intercepted between computed by the prismoidal, or other formula, used in earthwork computations. Where the grade contours do not intersect natural contours of equal elevation, but themselves form closed areas, those areas are to be measured.

The author worked for men who actually ran out the contours on the ground for the purposes of map making. A level was used to insure the lines being kept at the proper elevation and stakes were set on these level lines at regular

intervals. A transit party followed the level party taking angles and making measurements to stakes so the lines could be plotted on paper. This is a slow method and seldom used.

A method often followed is to set stakes in rectangles or squares as already described, taking the elevation of the ground at the stake. This information is platted in the office and contours drawn. On extensive surveys the rectangles may have sides several hundred feet in length, while for surveys of small tracts, such as city lots, small reservoirs, etc., the dimensions may be from 10 to 50 ft. It is not necessary, however, to set many stakes in making topographical surveys.

PLANE TABLE WORK

One of the oldest map-making methods is that of the plane table. It is in high favor for certain classes of work today and a number of government engineers and surveyors and men with government experience display great sensitiveness if plane tabling is criticized.

The author at one time was a fairly good "plane tabler" but must confess a strong preference for the transit and stadia for making topographical surveys. The transit is a universal surveying instrument, while the plane table costs as much and is good for one kind of work only, work that does not come often enough to the average engineer, or surveyor, to justify the expenditure provided he has a transit equipped with stadia wires. The author has never been able to quite see where the plane table possesses any advantage over the transit and stadia, especially since modern plane tables have telescopes equipped with stadia wires, and depend less than formerly upon the principle of intersection.

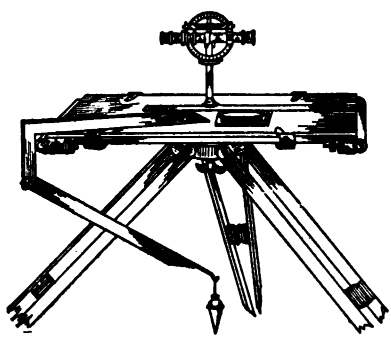


FIG. 221. Plane table.

The plane table is illustrated in Fig. 221. A sheet of paper is fastened to the board and the map drawn as rapidly as the sights are taken. Damp weather and very dry, hot, sunshiny weather affects the paper and the working hours during the day are not long. The map is accurate only for the scale to which it is drawn. A reduction of course increases the apparent, but not the relative, accuracy while all errors are multiplied by an enlargement of the map.

When a topographical survey is made with transit and stadia, notes are placed in a book with such sketches as are advisable and from these notes maps may be made to any scale. The degree of accuracy is independent of the scale and depends on the care with which the notes are taken and recorded. The work may be done in any weather when the graduations can be read on the rods and for more hours in the day than are possible with a plane table. For most of the work of this nature transit work is far more rapid than plane table work.

For complete information on plane table work, as well as on all kinds of topographical surveys the student is referred to "Topographic, Trigonometric and Geodetic Surveying," by Herbert M. Wilson, M. Am. Soc. C. E. ($3.50).

STADIA WORK

The writer assumes that every surveyor has a transit equipped with stadia wires, so a description of methods is in place in this book. The principles underlying the use of stadia wires were explained in a preceding chapter.

To make a stadia survey for topographical purposes, a number of points are selected at which the transit is set and from each point side shots are taken to locate the inequalities of the ground, natural objects, fences, buildings, etc. The first step is the selection of the instrument stations. The maximum length of sight from station to station should not exceed 1000 ft., and the best average length for sights is less than 700 ft. The instrument stations therefore should be selected with these limits in mind and each should be on a point from which as many side shots as possible may be taken. After the instrument sta-

tions are selected it is a good plan to run a line of levels with a good level and obtain the ground elevation at each point. This then leaves the angular elevations as merely a check. If time is an object and the survey is not very important the leveling may be omitted and the elevations obtained by means of vertical angles.

After each point is selected and a stake set, which does not require a tack in the top, the transit is set up on each station precisely as though a closed farm survey was being made. The distance is read forward and back on each course, also the vertical angles. The horizontal angles are usually azimuths and when this survey is completed the survey is closed in the usual way and the errors distributed. Then each point is given its co-ordinate numbers so it may be plotted by latitudes and departures. When the framework is platted the side shots from each station are platted. In addition to the readings taken with the stadia rods intersection angles are read from each station to as many other stations as can be seen, simply to check the work.

From the foregoing paragraph the student must not imagine that the work is gone over twice; once for the framework and once for the side shots. When each station is occupied and the instrument oriented all the sights possible from that station are read and recorded. When the office work is done the reductions for distance and elevation are made first for the station framework and the results plotted before the work of reducing and plotting the side shots begins.

The instrument is first oriented. This means the line through 0° and 180° lies in the selected meridian, which may or may not be the true meridian. It is convenient, however, to use the true meridian so angles may be checked by the needle and to assist the man who makes the map to keep himself located. People being trained from infancy to look to the north as a starting point, the average man is usually unable to read a map until it is oriented, a habit the surveyor loses with experience. The instrument properly oriented, a sight is taken to the station ahead, the azimuth read, the rod read and the vertical angle read, all being recorded in the field book. Keeping the lower clamp tight the instrument is swung to right and left and the rod read

as it is held on different points by the rodman, or men, for on some ground an instrument man can keep two rodmen busy. When he has three or four rodmen busy he must have a recorder at his side to whom he can call off the readings on the rod and the angles. When all the sights are taken which are thought necessary from this station the plates are set to the angle first read to the station ahead and the telescope pointed properly. It will be found frequently that the instrument has been slightly disturbed by the intermediate sighting so this must be corrected by the lower tangent screw, thus not disturbing the angle. However, if the plate is set to read the correct azimuth it will not be necessary to touch the lower tangent screw, for the instrument is then carried ahead to the next station.

Set the instrument over the next station. Level carefully and reversing the telescope sight back on the station just left. Without in any way disturbing the angle on the plate get an exact sight by means of the lower clamp and tangent screws. Having clamped the transit in position, read the angle, and if any change has taken place, bring the plate and vernier to indicate the correct reading and then re-set the line of sight on the last station. When this is done and the telescope transited to read in a forward direction it will lie in the line of sight between the two stations with the proper azimuth indicated on the plates. Now unclamp the plate, read the azimuth, vertical angle and rod to the next station, record the readings, and take the side shots as was done from the previous station. Some writers do not advise the reversing of the telescope, thinking it best to set the plate on vernier *B* to read an angle 180° different from the forward azimuth from the previous station. With this angle set on the plate the backward sight is taken on the previous station with the telescope erect, after which the plate clamp is loosened and the instrument may be revolved on its vertical axis to take sights in all directions. The author could never see a good reason for this. It increases the number of operations; it renders mistakes possible because the instrument man might get mixed up in adding the 180°, and it causes loss of time. To keep the azimuth set on the plate and reverse the telescope introduces an automatic check.

All the line readings should be taken forward and back and all the elevations checked by the positive and negative vertical angles. The side shots do not require such repeating and the instrument man must be constantly on guard to avoid setting down wrong readings. The side shots are taken to every object it is desired to locate on the map, fence corners, corners of buildings, bridges, etc., and it is well to make a sketch of these on the opposite page. The principal side shots, however, are taken to inequalities of the surface of the ground to obtain their location and elevation. An experienced instrument man with experienced rodmen will find 300 shots a day about an average day's work, except in such rare cases where more than 50 shots can be taken from one station. The average number of sights from one station in average farming country with the average number of natural objects is about 20 to a station, the number from some stations being less than 5. A fault with beginners is to take too many sights to obtain ground elevations. This is better, however, than to take so few that the map has to be pretty thoroughly "guessed out" when the contours are platted. The rule is to have the rod held at each point where there is a decided change in slope. An experienced rodman who has also helped to make maps can cut down the working time of a party one-half. Plentiful use of sketches is advisable.

The author for many years used loose sheets in his field book having two lines ruled at right angles and crossing in the middle of the sheet. From this as a center ten circles were drawn with intervals between the lines of one-tenth of an inch. Using the vertical line as the meridian when a sight was taken to any fence, building, etc., and the rod read, a ruler was laid from the center of the circle at as nearly as possible the same angle with the meridian and a point made at the distance indicated by the rod reading, the interval between two circles being assumed as 100 ft. After all these sights to objects were marked a sketch was made showing each in its relative position. The same method was pursued when getting the edges of steep banks, large boulders, etc. Sheets, such as the author used, can be drawn on tracing cloth by any surveyor and printed by the direct process, showing dark lines on a

white ground, or they may be duplicated by the mimeograph, the hektograph or the clay process. The writer had a zinc etching made showing three sets of circles, somewhat smaller than recommended above, on one page with heavy division lines, so sights from one station would not be confused with sights from the other. From this cut he had several thousand sheets printed. The sheets being held in the fold of the book by a rubber band each could be removed as fast as it was filled and the center of the circle used at each station was numbered with the num-

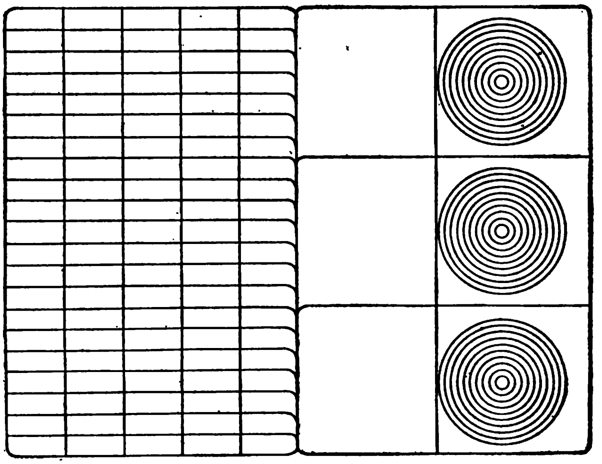


FIG. 222. Stadia survey note book.

ber of the station. A copyrighted field book on a similar plan is sold by instrument dealers, but as the author and several of his friends used the method for a number of years before the copyright was granted for the book mentioned, no surveyor need fear to make his own sheets. A variation on this was used by one surveyor, who had a rubber stamp made with the concentric circles and the intersecting normal lines. This stamp was used on each page where it was desired to make sketches, as it is not necessary to make sketches from each station. The stamp was in a thin metal case containing an ink pad and had a folded handle on

top, so it could be carried in the vest pocket. A blotter was necessary to prevent smudging of notes.

For ordinary work a Philadelphia leveling rod with alternate hundredths black may be used. For long sights graduations of a more pronounced character should be used. The form of mark on the face of a rod may be pointed or square, opinion today tending towards the discarding of angular points and favoring square block-like graduations. The author favored a combination of acute and square marks a few years ago, but now prefers the square marks.

The graduation shown in Fig. 223 was devised by Prof. W. H. Burger of Northwestern University. The graduations for even feet are placed on the left and those for odd

FIG. 223. Stadia rod.

numbered feet on the right. Opposite each half foot is placed a dot. As with all stadia rods it is optional with the user whether the feet are indicated by figures, it being an advantage to omit them, for it will then make no difference if the rodman accidentally holds the rod upside down. The instrument man can readily direct the middle hair to the height of the telescope and read the number of graduations intercepted between the stadia wires. Some men prefer to have foot marks on the rods and in such event they are one-tenth of a foot high, the bottom touching the division line, the figure being placed in the white space.

To make a rod use a piece of clear white pine, well seasoned, three inches wide by one or $1\frac{1}{8}$ ins. thick, surfaced and sandpapered. Give it a thin filler coat and on top of that three coats of pure white paint. Let each coat dry thoroughly and sandpaper it before applying the next coat. With a steel tape carefully measure the rod and make cuts at the foot divisions. On the ends of the rod place iron or brass strips to prevent undue wear and abrasion. Make the pattern for one foot in length on tough drawing paper and cut it out with a sharp knife. Lay

this stencil on the foot division marks and with a fine-pointed pencil trace the outline of the figure on the painted surface. Use good black oil paint and a fine camel's hair brush and fill in the marks. A steady hand is required. On the back of the rod may be hung some device to insure its verticality or the rodman may carry a rod level, this being as good a device as can be used. In stadia work the judgment of the rodman cannot be of service, for the rod must be as nearly vertical as it is possible to get it and it cannot be waved as in leveling operations. The length

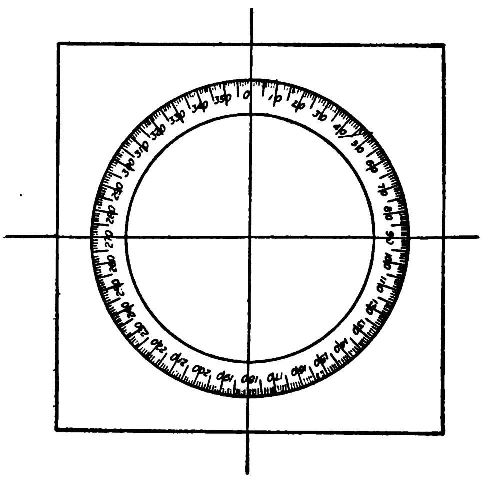


FIG. 224. Protractor for platting stadia survey.

of the rod depends upon the work to be done, the writer having 8-ft. and 12-ft. rods. Long rods may be made convenient for carrying by hinging them. Good stadia rods may be purchased.

The notes are reduced in the office very rapidly, it being best to use two men, one to call off and the other to look up the tables, or use the diagram, and reduce the rod bearing to horizontal distance and obtain the difference in elevation. The difference in elevation is added to, or subtracted from (according to the positive or negative sign

of the vertical angle), the elevation of the ground at the instrument station. When the instrument is set up the surveyor measures with a tape or rod the height of the axis of the telescope above the ground and directs the middle wire to this height on the rod. The line of sight is then parallel to a straight line drawn from the hub to the foot of the rod. In the office the last two columns in the field book are filled.

PLATTING STADIA WORK

To plat the work, the instrument stations are first placed on paper by any approved method, the author doing this work by computing the latitude and departure of each point and platting by co-ordinates. Through each station draw a fine line for the meridian and a perpendicular through it.

Take a 14-in. paper protractor, untrimmed, and cut out the interior with a radius of $5\frac{1}{2}$ ins., making the diameter of the cut-out portion 11 ins. The figured graduations go from 0° to 360°, clockwise as on the transit plate. Draw a fine ink line from 0° to 180° and from 90° to 270°, the lines terminating at the edges of the interior cut-out space. When the protractor is placed on the perpendicular lines through a station so the lines coincide with those above mentioned, the station is at the center of the graduated circle. The protractor is to be oriented so angles read on it will coincide with the same angles read on the transit. Fasten the protractor in place with two thumb tacks near the upper edge, so it may be flopped over without disturbing the adjustment, when plotting long sights.

Having decided upon the scale, glue to the bottom of the scale at the zero end projecting beyond the edge from $\frac{1}{8}$ in. to $\frac{1}{4}$ in. a piece of tough paper $\frac{1}{4}$ in. wide. Put a fine needle through this projecting strip at the zero graduation and push the point through the station. The scale can now be swung around the circle freely.

An assistant calls off the azimuth from the field book and the draftsman swings the scale to this reading on the graduated circle. The assistant calls off the horizontal distance, the draftsman scaling it, and putting a dot on the paper. The draftsman then writes down the elevation

read to him by the assistant. The shots are plotted in this manner at each station. Sketches are transferred from the field book to the map and the protractor moved to the following station. When the plotting is completed each point occupied by a rod shows on the map with the ground elevation, and all buildings, fences, etc., are shown in light lines. The next step is to put in the contours.

INTERPOLATING CONTOURS

In Fig. 225 is shown a method described by the author in *Engineering News*, May 10, 1900, while Fig. 226 illustrates a method described by H. F. Bascom, C.E., in *Engineering News*, June 21, 1900.

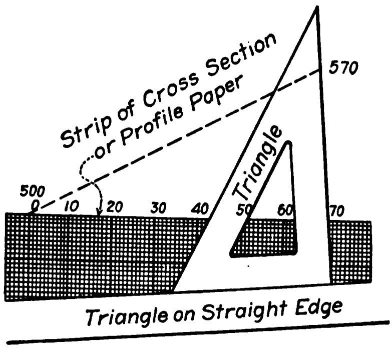


FIG. 225. Interpolating contours.

In Fig. 225 two points are shown with the elevations. A fine line is drawn connecting them. A piece of graduated paper is placed at an angle with the line with the graduations corresponding to the lower elevation coinciding with it. A triangle is then placed so one edge will touch the other point and also the graduation on the squared paper corresponding to the elevation of the point. A straight-edge is now placed under the triangle. The triangle is moved along the straight-edge and as it successively reaches the numbered graduations a point is marked at the intersection of the edge of the triangle with the line connecting the plotted points.

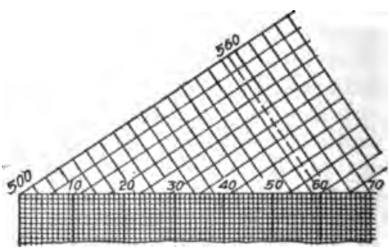


FIG. 226. Interpolating contours.

In Fig. 226 the triangle and straight-edge are not used.

A strip of ruled paper is placed with the edge touching the two plotted points. A second strip with graduations is placed over it as shown. From each figured graduation the intersecting line on the under piece of paper is followed to the edge and the contour point marked. Contours are seldom drawn at closer intervals than 5 ft. in height, depending on the slope. On easy slopes the interval may be less than 5 ft., while on very steep slopes the interval may be as great as 25, or even 50, ft. When all the contour points are marked the draftsman draws in the contours in lead pencil. The instrument man and his assistants, having a good knowledge of the shape of the surface of the land, look over the map and suggest corrections before the contours are inked in. Contours are usually drawn in light brown or red lines.

PHOTOGRAPHIC TOPOGRAPHY

The author, on May 5, 1893, read before the Technical Society of the Pacific Coast, a paper with the above title. The paper is here given with the final paragraph only omitted. In the intervening 22 years the camera has been used on many surveys and its use is increasing. The principles, however, are the same as when the paper was prepared. The best book on the subject is "Phototopographic Methods and Instruments," by J. A. Flemer, a recent work ($5.00).

The object of this paper is to present a method of surveying which will be a valuable auxiliary to plane-table and stadia work, and in some cases is extremely useful alone. It is by no means new, but is not very well known, and its advantages are so great that engineers who have much topographic surveying to do should understand it.

The principle depends upon the art of projecting perspective views upon a horizontal plane, and was first used by French naval officers in the beginning of this century in the survey of coast lines. Perspective drawings were made of certain places from two or more positions, and sextant angles taken to several objects in the landscape, and the angles recorded on the drawings. These drawings were afterwards used in the mapping of the shore line, and

the accuracy of the work depended upon the skill with which the sketches were made.

The invention of photography made it possible to do better work, and Colonel Laussedat of the French army made experiments covering a period of years to perfect the system. He published a work on the subject in 1865, and no improvements have been made since, unless multitudinous patents for cameras can be styled improvements.

The system is extensively used in Europe, and is very little known in this country, where, of all places, one would think it most valuable from an economical standpoint.

Instances can be given of the time in which various surveys have been made by this method, but such records are of no use unless the ground were known, but the French engineers generally considered that a topographical survey and map, when a camera with horizontal and vertical circles is used, can be made in one-third the time required by other methods.

Any camera may be used provided it is perfectly level when the view is taken, and the smallest size adapted for the work is one with a 5 by 8-in. plate. Smaller cameras may be used in the same manner as a sketching pad, to carry away unimportant details, but are of little practical use unless most excellently made. All lenses must be good. Although an ordinary camera may be used, still it is better to have one for the purpose, provided with two levels on top at right angles, and four leveling screws beneath. The box should be solid, and focusing done by means of the objective slide. If the camera has a compass, or a horizontal limb, and a vertical limb, so that up or down sights may be taken, it will be complete.

Glass negatives are the most accurate to use, but films on account of portability are more convenient. The weight of the glass in the field is a drawback.

There are several adjustments of the camera which must not be neglected. The first is called "the test for register." The film on the sensitive plate must exactly re-place the surface of the ground glass. To do this set the instrument up and focus for a distant view. Make a scratch to show the relative positions of the plate and tube. Take out the ground glass, and put in one with a transparent film. Focus

on this, and make another mark. In actual work this difference must be allowed for by changing the focus after the removal of the ground glass, so the film on the plate will be in the right position. The instrument maker should see to the register, but it is as well to test his work.

As everything depends upon the focal distance this must be accurately determined. Lens makers usually state the focal distance, but as it is liable to vary, the operator had better determine it himself. For simple convex lens, double or plano convex lens, measure from optic center to surface of ground glass. For double compound lenses proceed as follows (Fig. 227):

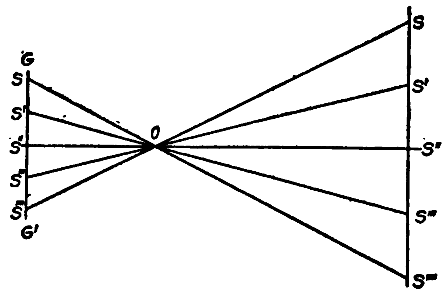


FIG. 227.

Set up several stakes in the ground distant from O about two or three hundred feet as $SS'S''S'''S''''$. With transit at O, measure angles SOS'', etc. Set up the camera at O, level it carefully, make the image S'' coincide with a vertical line through the center of the plate, and photograph the stakes. The greater the distance apart on the plate of the stakes the more accurate will be the determination of the focal length. GG' represents the plate, OP the focal length. Measure $s''s''''$ on plate, then

$$OP = \frac{s''s''''}{\tan a} = s''s'''' \cot a$$

$$SOS'' = a.$$

For a test of distortion of the lens, with OP just found, compute the angles SOS', SOS'', etc., and if they agree with angles taken with the transit the lens is free from distortion.

Next, the horizon of the view must be found. Find the center of the ground glass and draw a vertical and horizontal line through it. Level the instrument carefully, and set beside it an engineer's level, with the telescope at same height as the lens of the camera. With the level find some object in the distance. Turn the camera to this object and move the object slide up and down until the object is exactly at the intersection of the lines on the ground glass; the object slide is then in its normal position, and a scratch on the slide will determine that position for all time. This scratch should be marked zero, and graduations should extend above and below it. The lower graduations should have a minus sign before their numbers. The plate holder should have four fine needles, so inserted in the frame that their shadows will be photographed. When the picture is developed, lines scratched on the plate and connecting these points will occupy the same positions as the lines drawn on the ground glass. The maker can fix these needles in place.

The horizontal line represents the horizon of the picture, and is the trace of a line on a level with the center of the instrument. The object of graduating the vertical movement of the object slide is to provide for a changing of the horizon when necessary to limit sky views. By noting the number on this index when the view is taken, the actual horizon of the picture is set off from the horizon of the instrument when plotting.

One more adjustment and we are done. This is to measure the field of view. Half the length of the plate, divided by the focal length gives the tangent of half the horizontal angle. The horizontal angle is the field of view we require, and dividing 360° by this angle, gives us the number of views needed to go around the circle.

Half the width of the plate, divided by the focal length, gives us the tangent of half the vertical angle. As a general proposition it may be stated that the greater the focal length, the smaller the field of view and the greater the accuracy in the work. The smaller the focal length, the greater the field of view, greater rapidity (because fewer views) and less accuracy.

To take a view, set up the camera and level it carefully.

Adjust to focal length and set the object slide to the most favorable position and note index number for fixing horizon in views. Then adjust the stop, set in the plate holder and verify the leveling. The levels are apt to get a little out during all the handling. When all is ready, take the picture.

THE PLATTING

In the test for distortion, the whole idea of the method for using the proof is given. The proofs are conical projections, and the optic center the point of view. The objects represented are so far distant that their images are formed on the same focal plane, and the point of sight remains constant as in perspective drawing.

On the plate draw the horizontal line (the horizon) and the vertical line, from the shadows of the points of the needles. If the objective was above or below the horizon, then instead of drawing it, draw a line parallel to it above or below, as indicated by the index number observed. From the vertical line measure to the right or left to the object you wish to locate, and divide this distance by the focal length, this will give the tangent of the horizontal angle from the line of sight. From the horizontal line, which is a trace of the plane of the optic center, measure up or down, as the case may be, to the object, and divide this distance by the focal length to obtain the tangent of the vertical angle.

Every point located on the map must show in two views at least. These views are taken from points previously fixed by the triangulation or by direct measurement. The points from which the views are taken must be plotted, and from these points lines drawn on the bearings given in the field notes when the view was taken. On these bearings lay off the focal distance and at the end of this line draw one at right angles to represent the plate. On the line representing the plate, lay off on either side the distances from the vertical to the object, and from the point of view draw lines through these points. The lines through two plates produced to an intersection locate the objects.

Fig. 228 illustrates well the method of plotting. *OO'* represent the points from which the views were taken. *GG'*

and $G''G'''$ the plates, OP and $O'P'$ the line of direction of sight. ABC, etc., and $A'B'C'$, etc., represent on the plate the objects to be located and their positions on the maps are shown by the points of intersection.

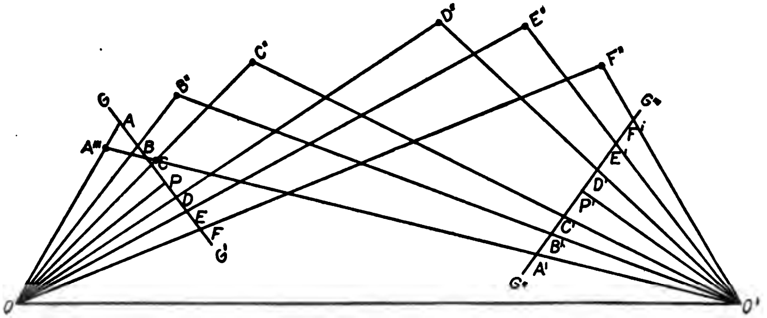


FIG. 228.

To fix the elevation, measure the distance from point of sight to object, and multiply into tangent of vertical angle already found; add to the elevation of the point from which the sight was taken, the height of instrument, and add or subtract, according to whether the point is above or below the horizon, the height above ascertained. This will give the elevation of the object above datum.

Spherical aberration does not interfere with the accuracy of the work, provided the focal length is ascertained by means of a point near the extremity of the plate in the horizon. (See Fig. 227.)

FIELD WORK

The field work may be performed in one of three ways, or a combination of all.

1. The ground may be triangulated with the transit and views taken from the triangulation points with the camera, the direction of the views to be ascertained by azimuths from the lines between stations. These azimuths are to be taken by a compass, or by means of a horizontal limb.

2. The camera may be used in connection with a pocket compass, the work starting from a measured base.

3. The work may be done with a camera alone, fitted with a compass, horizontal limb and vertical circle. In this case the triangulation is carried on with the work. With ordinary care either method is good. The camera must be rigid and the plate truly vertical.

Below is a form of record.

Station.	View.			Remarks.
	Number.	Index number.	Bearing.	
A..........	1 2	0 0	$195\frac{1}{2}°$ $227°$	
B..........	1	-2	$84°$	

The index number may be the same or different for all views at the same station. Any time of the year is good for this work, and any hour of the day when the air is clear. Long distances between stations should be chosen as short bases increase error. A few views only are necessary, as sketches may be made of unimportant places, and these views should be well chosen. A little care exercised in selecting positions will save much office work.

OFFICE WORK

Upon the scale used depends the accuracy of the plotting. If the scale is large then very long sights should not be attempted, but if the scale is small then of course the range can be longer. The error in height is in proportion to the distance, and Professor Hardy says that with a focal length of 1.64 ft. this error will not exceed 1 ft. in 550 yds. Colonel Laussedat has shown that with a scale of $\frac{1}{3000}$ with a focal length of 0.5 m. points, 1500 meters distant, may be represented, and with a scale of $\frac{1}{8000}$ the operations could be conducted at 4000 meters.

When the plates are prepared for plotting the office notes are placed in a book in seven columns as follows:

FORM OF RECORD TO REDUCE HEIGHT TO COMMON DATUM

View.	Distance.	Point.	Ref. ft.	Ref. of sta., ft.	True elev., ft.	Remarks.
Va.	....	1	+102	+460	+562	
	....	2	+ 90	....	550	
	....	3	− 25	....	435	

The first column is for the views.
The second column contains the distances to the points.
The third column the names or numbers of the points.
The fourth column the height above or below station.
The fifth column elevation of station.
The sixth column elevation of point.
The seventh column for remarks.

For drawing in contours the fixing of natural and artificial objects on the plan, with their heights noted, will give all the data necessary together with a close inspection of the proofs as the work proceeds. In the case of a bare country, with no buildings, fences or trees, a few painted stakes or flags put in at salient points will serve.

It is best to work directly from the negatives as the paper positives are too much affected by atmospheric changes. Blue prints are as easy to work from as silver prints if positives are used.

Surveying by camera has equal advantages with surveying by plane-table as it is a graphic process, but it is more exact than the plane-table as atmospheric changes have no effect on the records. It is more rapid in the field work, and is accurate for more than one scale in the plotted work.

As compared with transit and stadia it is more rapid in the field, and a little quicker in the office. Like it, the records may be kept, and reproduced at any time to different scales, and it has a further advantage in that all errors of observation are entirely eliminated. Various plans for saving labor in the office work have been suggested, but this paper is already long enough.

Before closing, attention should be called to one point, namely, that the notes may be sent to a draftsman who never was on the work, and a correct map may be drawn by him. No other method approaches it in this particular.

APPENDIX A

ESSENTIALS OF ALGEBRA

The average person whose mathematical training does not extend beyond that given in the grade school arithmetic is afraid of shadows when reading technical books.

Many men declare an expression like

$$V = \pi h\,(R^2 - r^2)$$

or

$$V = \pi r^2 \frac{h_1 + h_2}{2}$$

to be algebra. Not having studied algebra they slur over their studies until finally the art of using symbols is acquired to some degree and they believe they know enough of algebra for practical purposes. What they have learned is the ability to evaluate simple formulas but of algebra they know nothing. The two expressions above given are not algebra but are merely arithmetical rules written in mathematical shorthand.

In ordinary language in a grade school arithmetic the rule for finding the volume of a hollow cylinder appears as follows:

Multiply the radius of the outside of the cylinder by itself (that is, square it) and subtract from the product the product of the radius of the inside of the cylinder by itself. Multiply the difference between these products by the height of the cylinder and this final product is to be multiplied by 3.1416.

Written in mathematical shorthand the rule appears:

$$V = \pi h\,(R^2 - r^2),$$

in which π = 3.1416, the ratio of the circumference to the diameter (that is the circumference is 3.1416 greater than the diameter),

h = height of cylinder,
R = radius of the outside of the cylinder,
r = radius of the inside of the cylinder,
V = volume (cubic contents) of the cylinder.

When h, R and r are in feet, V = cubic feet. When h, R and r are in inches, V = cubic inches. Similarly for units in the metric or any other system.

Enclosing some of the factors in a parenthesis indicates that they are first dealt with, the result being a single factor.

A frustum of a cylinder is shown in Fig. 229. A rule for the volume as written in a school arithmetic is as follows:

Multiply the area of the base by the average height.

Assuming that the student does not know how to obtain the area of the base the rule will be written as follows:

Square the radius of the bottom. Multiply the product by the average height and this result by 3.1416.

Written in mathematical shorthand it appears:

$$V = \pi r^2 \frac{h_1 + h_2}{2},$$

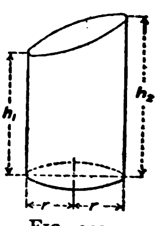


FIG. 229.

in which V = volume of frustum,
π = 3.1416,
r = radius of base,
h_1 = height of one side,
h_2 = height of other side.

(*Note.* If the diameter is used instead of the radius divide 3.1416 by 4 so the rule appears $V = 0.7854\, d^2 \left(\frac{h_1 + h_2}{2}\right)$.)

EXAMPLES.

1. What is the volume of a ring (cylinder) 8 ins. long, inside diameter 6 ins. and outside diameter 7 ins.?

Here $h = 8$; $R = \frac{7}{2}$; $r = \frac{6}{2}$; $R^2 = 12.25$; $r^2 = 9$;
$V = 3.1416 \times 8 \times (12.25 - 9) = 3.1416 \times 8 \times 3.25 = 81.68$ cu. ins.

2. What is the volume of a frustum of a solid cylinder 10 ins. in diameter, having one side 9 ins. long and the other side 15 ins. long?

$$V = 3.1416 \times 5^2 \times \frac{9 + 15}{2} = 942.48 \text{ cu. ins.}$$

No matter how accustomed one may become to handling such expressions the use of letters is generally confusing without some knowledge of algebra, so, for the benefit of students who experience difficulty in understanding the mathematical expressions and using the formulas in this book the author will attempt to present some of the essentials of algebra, which is in fact only an extension of arithmetic.

Arithmetic, as defined in textbooks, is "the science of numbers and the art of computation." Sir Isaac Newton termed Algebra "universal arithmetic," for it enables us to reason out in a general way solutions for many problems which by ordinary arithmetic would involve much cut-and-try work. It is therefore an abbreviated method for solving questions relating to numbers and quantities. In proceeding to solve a problem algebraically the algebraic work is completed when the formula is derived, the solution of the problem from that point being done arithmetically. Briefly stated the reasoning is done by algebra and the work by arithmetic.

All the fundamental operations of algebra depend upon the single principle that quantity bears no relation to order. When a quantity is to be increased, or diminished, by other quantities the result will be the same no matter in what order the work is done, *provided* none of the quantities be neglected.

WHY LETTERS ARE USED IN ALGEBRA

In arithmetic ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are used to represent numbers, there being no limit to the size of a number which may be expressed by a proper arrangement of the characters. These characters, or figures, are in themselves sufficient for use in solving all problems in common arithmetic except those in which three numbers are given and it is necessary to find a fourth. The method used was once known as the "Rule of Three," the problems being termed problems in proportion.

Proportion involves both multiplication and division. During part of the work some character must be used to represent the unknown quantity until it has a numerical value, the letter x being generally used. The work is expressed in the following manner:

$$5 : 10 :: 20 : x,$$

which is read, *As 5 is to 10, so is 20 to x.*

The two end quantities are known as the *extremes;* the two middle quantities (or terms) as the *means.*. The product of the means = the product of the extremes, therefore,

$$\frac{10 \times 20}{5} = 40 \quad \therefore \quad x = 40.$$

Algebraically it is written,

$$\frac{5}{10} = \frac{20}{x},$$

and it is solved as follows:

$$\frac{5x}{10} = 20,$$

$$5x = 10 \times 20 = 200,$$

therefore

$$x = \frac{200}{5} = 40.$$

When two expressions are equal they are written with the sign of equality (=) between them and thus a new complete expression is obtained known as an equation. $\frac{5}{10} = \frac{20}{x}$ is a typical equation; $5/10$ is known as the left side and $20/x$ as the right side. The resolution of an equation is accomplished by collecting all the known quantities on one side, the unknown quantity then standing alone on the other side. Algebra is used in separating the known from the unknown quantities, after which the work is arithmetical and the unknown quantity receives a numerical value. Any analogy, or proportion, may be changed into an equation by making the product of the extremes equal the product of the means.

A knowledge of the rules and methods of algebra and common arithmetic enables one to solve equations without difficulty. Given a problem the mathematician first attempts to form a clear idea of its nature and then tries to express its terms and the relation of its parts in algebraic language, using the letters x, y, z, etc., for the unknown quantities. Usually more time is spent in attempting to express a problem than is spent in solving it.

When a problem is proposed in which there are several unknowns there must be as many independent equations as there are unknowns. If the problem is properly limited the equations may be readily written.

Suppose we have two quantities x and y.
Their sum amounts to 18.
This is written $x + y = 18.$
Their difference is 6; $x - y = 6.$
Their product is 72; $x \times y$ or $xy = 72.$
Their quotient is 2; $x \div y$ or $\frac{x}{y} = 2.$
Expressed as a proportion $x : y :: 4 : 2.$
Expressed as an equation $2x = 4y.$
The sum of their squares is 180; $x^2 + y^2 = 180.$
The difference of their squares is 108; $x^2 - y^2 = 108.$

The values of x and y are not material, the above equations being illustrative only. The values are found by solving two simultaneous simple equations. This may be done by addition or subtraction.

By addition:

$$\begin{array}{rl} x + y &= 18 \\ x - y &= 6 \\ \hline 2x \quad &= 24 \end{array}$$

$$x = \frac{24}{2} = 12$$

$$\therefore\ x = 12 \quad \text{and} \quad y = 6.$$

Here the $-y$ and $+y = 0$.

By subtraction:

$$\begin{array}{r} x + y = 18 \\ x - y = 6 \\ \hline 2y = 12 \end{array}$$

$$y = \frac{12}{2} = 6.$$

Here the signs in the subtrahend were changed from + to − and from − to +.

When the relation of one unknown quantity to another is simple, a letter may be used for one of them and an expression for the other deduced from the relation between them. This shortens the work and, in the language of mathematicians, "renders it more elegant." For example, if the difference between two quantities be 5, let y be the less, then $x = y + 5$; or, let x be the greater and $y = x - 5$.

The work will often be shortened if letters are taken, not for the unknown quantities themselves, but for their sum, difference or any other relation from which their values may be readily found.

A competent mathematician does not consider the solution of a problem to be complete unless it exhibits all the cases which can occur; and the results which flow from contrary suppositions can only be exhibited by expressions such as those to be considered when we demonstrate algebraic multiplication and division. In practical work algebra is applied only when position or other conditions must be assigned to quantities which are frequently to be found under such conditions that they are sometimes to be added and sometimes to be subtracted. The use of the positive (+) sign to indicate when a quantity is to be added, and the negative (−) sign when it is to be subtracted does not alter the meaning affixed to these signs (that is to indicate opposite conditions or states of being) in the algebraic sense, for they are prefixed solely for the purpose of subjecting the quantities to algebraic processes. This will be illustrated when we touch on negative exponents.

USE OF SIGNS

In algebra multiplication is indicated by (×) as in arithmetic. When quantities are represented by letters instead of figures, a dot

is sometimes used, for the multiplication sign might be mistaken for an x. Thus $a \times b$ and $a \cdot b$ have the same meaning.

The dot is not used between figures for it might be mistaken for a decimal point, although in practice when used to indicate multiplication it is written in a higher position than a period or decimal point. It is customary to write letters with no sign or mark between them when they are to be multiplied together, so $a \times b$, $a \cdot b$ and ab have the same meaning. However, if several letters form one quantity when grouped and this is to be multiplied by a letter or another group of letters, the dot is sometimes used to collect the factors. For example $abcde$ means $a \times b \times c \times d \times e$, or $a \cdot b \cdot c \cdot d \cdot e$, but $abc \cdot de$ means that abc form one factor and de another, so a continued product is not wanted, but the product of two factors.

Division is represented in the following ways, in the order in which they are favored by mathematicians, $\frac{a}{b}$, a/b, $a \div b$.

The parenthesis is very common in algebraic work. Numbers or letters enclosed within parentheses are treated as one group. For example $a\,(b + c + d)$ means $ab + ac + ad$, for a is a factor common to the three letters, which may therefore be first added together and the sum multiplied by a.

The sign for addition $(+)$ is termed the plus sign in arithmetic and the sign for subtraction $(-)$ is termed the minus sign. These two signs in arithmetic indicate operations to be performed.

When used in algebra the plus sign is called the *positive* sign and the minus sign is called the *negative* sign. These signs do not indicate operations but represent existing conditions. Whatever is not positive must be negative, and, conversely, whatever is not negative must be positive.

Addition and subtraction in algebra are independent of the signs, for negative quantities may be added to, or subtracted from, positive or negative quantities; also positive quantities may be subtracted from, or added to, negative as well as positive quantities.

The recognition of the fact that negative quantities actually exist and may be operated on as though they are positive is the first striking difference between algebra and common arithmetic.

The money a man possesses is positive. The money he owes is negative. These are elementary facts known to every person. When a bookkeeper balances his books he performs an operation in algebra, or, in mathematical language, "obtains the algebraic sum" of the money passing through his hands.

A surveyor calls the elevation of the first floor of a building zero, or ± 0 (plus or minus zero). All heights above are positive $(+)$ and all depths below are negative $(-)$. From the first floor to the cellar floor the distance is 9 ft. (-9) and from the first to the second floor the distance is 16 ft. $(+16)$.

Find the difference in elevation between the basement floor and the second floor.

Without any knowledge of algebra a person would say $9 + 16 = 25$ ft., which is correct.

The elevation of the second floor is $+16$ and the elevation of the basement floor is -9. How far is it from the second floor to the basement floor?

Ans. $\begin{array}{r} -\ 9 \\ \underline{+16} \\ -25 \end{array}$ that is $\begin{array}{r} -\ 9 \\ \underline{-16} \\ -25 \end{array}$

The negative sign prefixed to the result shows the measurement to be in a negative direction.

How far is it from the basement floor to the second floor?

Ans. $\begin{array}{r} +16 \\ \underline{-\ 9} \\ +25 \end{array}$ that is $\begin{array}{r} +16 \\ \underline{+\ 9} \\ +25 \end{array}$

The positive sign prefixed to the result shows the measurement to be in a positive direction.

In the first question the second floor elevation was subtracted from that of the basement floor. In the second question the basement floor elevation was subtracted from that of the second floor. Thus the answers show direction as well as amount. A study of the work shows that the distance between two points was found and one quantity had not been lessened by taking it from another as in arithmetic.

The algebraic sum is the difference between the sums of the positive and negative quantities or expressions, hence the rule for subtraction in algebra: *Change the sign of the subtrahend from + to −, or from − to +, and then obtain the algebraic sum.*

A man has \$7.00 and on pay day receives \$15.00. He owes a friend \$1.50, pays \$3.50 for a meal ticket, pays \$5.00 for a pair of shoes and \$6.00 for a hat. What has he left?

Place the amounts in two columns.

$$\begin{array}{r}
+\ 7.00 \\
\underline{+15.00} \\
+22.00 \\
\underline{-16.00} \\
+\ 6.00
\end{array}
\qquad\qquad
\begin{array}{r}
-\ 1.50 \\
-\ 3.50 \\
-\ 5.00 \\
\underline{-\ 6.00} \\
-16.00
\end{array}$$

In the above example the process of subtraction is purely arithmetical and the result obtained is the algebraic sum of the positive and negative sums of money.

Suppose the man purchased a suit of clothes for \$18.00, the result would be $+6.00 - 18.00 = -\$12.00$, showing he would be in debt to the amount of \$12.00.

A man swims up a stream at the rate of 4 miles per hour against a current of 2 miles per hour. How far does he go in 6 hours?

Let his rate be $+4$ and the current be -2, for the directions are opposite. Then in 6 hours he will go $6\,(+4\ -2) = +12$ miles. The positive sign indicates that he went up stream 12 miles. Had the current been -4 miles and the swimmer could only proceed at the rate of $+2$ miles per hour, then in 6 hours he would find himself 12 miles down stream, that is $6\,(-4 + 2) = -\ 12$ miles.

Letters as well as figures are used freely in algebraic work to represent either quantities or numbers. When a sign of operation is placed between figures or letters they are termed factors. Thus a, b and c are factors in the expression abc, or $a \times b \times c$; and 2, 6 and 7 are factors in the expression $2 \times 6 \times 7$. Factors are numbers from the multiplication of which a product results.

A figure and letter are often written together, as $5\,a$, $6\,h$, $7\,k$, meaning $5 \times a$, $6 \times h$, $7 \times k$. The figure and letter together form

one factor, each being a coefficient of the factor. The figure is the *numerical coefficient* and the letter is the *literal coefficient* of the factor. For the sake of clearness the word "coefficient" is used to indicate the numerical coefficient, the word "letter" indicating the literal coefficient. Figures in arithmetic are characters used to represent numbers. The word "figure" does not always mean a number, so it is common to use the word "number" in mathematical work instead of "figure," a custom to which the author will adhere.

ADDITION AND SUBTRACTION

The rule for obtaining the algebraic sum as a process of addition, the rule for subtraction having been already given, is as follows:

Case I. When the quantities are alike. If the signs are alike add the coefficients. If the signs are unlike take the difference between the coefficients. To the sum or difference prefix the sign of the greater and annex the common letter, or letters.

Case II. When the quantities are unlike. Write them one after another with their proper signs and coefficients.

EXAMPLES.

Addition.

$$\begin{array}{r} 3a - 5b + 4c - 3d - 2e \\ \underline{6a + 2b - 7c - 4d + 8e} \\ 9a - 3b - 3c - 7d + 6e \end{array}$$

Subtraction.

$$\begin{array}{l} 8ab - 2cd + 5ac - 7ad \\ \underline{3ab + 4cd + 5ac - 2ad} \\ 5ab - 6cd^{*} \qquad\quad - 5ad \end{array}$$

When no sign is written in front of a quantity it is understood that the quantity is positive.

If $a - b$ is to be added to $3a$ we may first subtract b from a and add the remainder to $3a$; or subtract b from $3a$ and add a to the remainder. The result in either case $= 4a - b$. Since a and b are unlike quantities the only way their sum can be represented is

* In the example in subtraction the signs in the subtrahend were mentally changed.

to write $a + b$; conversely their difference can only be represented by writing $a - b$, or $b - a$.

To subtract $a - c$ from $3a$ we may first subtract c from a and then subtract the remainder from $3a$; or add c to $3a$ and then subtract a. Thus, $3a - (a - c) = 2a + c$. If $a - c$ is to be subtracted from $3a + 2c$, subtract a from $3a$ and add c, the remainder being $2a + 3c$. Thus $3a + 2c - (a - c) = 2a + 3c$. In the foregoing illustrations of subtraction the work has all been done by changing the sign of the quantities to be subtracted.

A little thought will show that we may add or subtract any two terms, without regard to the other terms with which they may be connected.

DEFINITIONS

A term is a simple quantity; as a, ab, $4\,bc$, etc.

Like terms are those of which the literal parts are the same; as $4\,ab$, ab, $9\,ab$, etc.

Unlike terms are those which consist of different letters; as $2\,ab$, $3\,bc$, $5\,cd$, etc.

Compound quantities consist of several terms connected by the signs $+$ or $-$. A two term quantity is called a binomial; a three term quantity a trinomial; a four term quantity a quadrinomial; a quantity containing more than four terms is a polynomial, or multinomial. It is customary to speak of all quantities higher than binomials as polynomials.

Compound quantities are sometimes enclosed within parentheses, which indicates, as already explained, that the group thus enclosed must be first operated upon before using the other factors. When two or more negative signs are enclosed within a parenthesis it indicates continued subtraction.

When the parenthesis is preceded by a positive sign remove the parenthesis and proceed to work as indicated by the signs.

When the parenthesis is preceded by a negative sign it means the group as a factor is to be subtracted. Therefore change all the signs within the parenthesis, the $+$ to $-$ and the $-$ to $+$, after which remove the parenthesis.

When several parentheses are encountered first remove the inner one and work to the ends.

EXAMPLE. Remove the parentheses in the following expression:

$$\begin{aligned}5\,a &- [2\,a + (-3\,a - 4\,b) - (a - 8\,b) + 4\,a]\\ &= 5\,a - (2\,a - 3\,a - 4\,b - a + 8\,b + 4\,a)\\ &= 5\,a - (2\,a + 4\,b)\\ &= 5\,a - 2\,a - 4\,b\\ &= 3\,a - 4\,b.\end{aligned}$$

MULTIPLICATION

In algebraic work the signs of the quantities must always be considered. When multiplying algebraic quantities like signs produce $+$ and unlike signs $-$. That is, $+a \times -a = -a^2$; $aa = a^2$ and $-a \times -a = a^2$.

Multiply the coefficients and to the product annex the letters of both factors. If the multiplicand is compound multiply each term separately by the multiplier.

EXAMPLE.

$$\begin{array}{l} 5\,a - 4\,b + 3\,c - 2\,d + e - 1 \\ \underline{5\,a \qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\ 25\,a^2 - 20\,ab + 15\,ac - 10\,ad + 5\,ae - 5\,a \end{array}$$

If the multiplier is compound multiply first by one of its terms, then by another, etc., and add the products.

EXAMPLE.

$$\begin{array}{rrrrrr} 2\,x^2 & -\,3\,xy & +\,6 & & & \\ 3\,x^2 & +\,3\,xy & -\,5 & & & \\ \hline 6\,x^4 & -\,9\,x^3y & +\,18\,x^2 & & & \\ & +\,6\,x^3y & & -\,9\,x^2y^2 & +\,18\,xy & \\ & & -\,10\,x^2 & & +\,15\,xy & -\,30 \\ \hline 6\,x^4 & -\,3\,x^3y & +\,8\,x^2 & -\,9\,x^2y^2 & +\,33\,xy & -\,30 \end{array}$$

It helps if the quantities are arranged in some order. In the example given above the product comes out according to the descending values of x, both multiplicand and multiplier having been so

arranged. This carries out the principle that quantity is independent of order and that the positive and negative signs indicate a condition or state of being rather than an operation.

Powers of the same quantity are multiplied by adding their exponents, this fact being dealt with in the chapter where the use of logarithms is explained.

Multiplication is a shortened method of addition as division is a shortened method of subtraction. In multiplying $a - b$ by c, we may either first subtract and then multiply, or first multiply and then subtract. The latter is the order in algebra; we first multiply a by $c = ac$, and then b by $c = bc$. The latter is subtracted from the former, the result being $ac - bc$, with the signs the same as those of the multiplicand.

In multiplying $a - b$ by $c - d$, first multiply $a - b$ by c as before $= ac - bc$; then multiply $a - b$ by $d = ad - bd$, which subtract from the first product. This is accomplished by changing the signs, when it becomes $-ad + bd$, the signs being the opposite of those in the multiplicand.

The first and last terms show that quantities with like signs multiplied together produce $+$, and the other two terms show that those which have unlike signs produce $-$.

DIVISION

When the signs are alike the sign of the quotient is $+$, but if the signs be unlike the sign of the quotient is $-$.

The above statement is self-evident; for the divisor multiplied by the quotient must produce the dividend with its proper sign. The whole operation depends upon the principle that the value of a quantity is not altered by both multiplying and dividing it by the same quantity.

Powers of the same quantity are divided by subtracting the exponent of the divisor from that of the dividend; the remainder is the exponent of the quotient. See the chapter dealing with logarithms.

When the divisor is a simple quantity, write it under the dividend in the form of a fraction; cancel like quantities, and divide the coefficients by their greatest common divisor.

EXAMPLES.

$$1. \quad \frac{56\,a^2b^3c}{8\,ab^3} = 7\,ac.$$

$$2. \quad \frac{54\,xy^2}{36\,x^2y} = \frac{3\,y}{2\,x}.$$

If the dividend is compound divide each term separately by the divisor.

EXAMPLE.

$$\frac{63\,a^3b^2c^3 - 42\,a^2b^3c^3}{14\,a^2b^2c^2} = \frac{9\,ac}{2} - 3\,bc.$$

When the divisor is compound arrange the terms of the dividend and divisor according to the powers of some letter. Divide the first term of the dividend by the first term of the divisor to obtain the first term of the quotient, then multiply the whole divisor by this term, and subtract the product from the dividend; bring down as many terms to the remainder as may be requisite for a new partial dividend, with which proceed as before.

EXAMPLE.

$$\begin{array}{rrrr|l}
a^3 & -\,3\,a^2b & +\,3\,ab^2 & -\,b^3 & a - b \\
\underline{a^3} & \underline{-\quad a^2b} & & & \overline{a^2 - 2\,ab + b^2} \\
 & -\,2\,a^2b & +\,3\,ab^2 & & \\
 & \underline{-\,2\,a^2b} & \underline{+\,2\,ab^2} & & \\
 & & +\quad ab^2 & -\,b^3 & \\
 & & \underline{+\quad ab^2} & \underline{-\,b^3} &
\end{array}$$

Note that while the signs are written in the partial subtrahends as produced by the multiplication, they are changed, mentally, for each subtraction.

When in dividing the last remainder is a simple quantity, place the divisor below it in the form of a fraction, and annex it with its proper sign to the quotient.

FRACTIONS

Fractions are the same in algebra as in common arithmetic save for the fact that letters are used in algebra. They may be added,

subtracted, multiplied and divided by the rules already given for dealing with whole quantities. They are reduced to a common denominator exactly as in arithmetic.

NEGATIVE QUANTITIES

A negative quantity standing by itself has, strictly speaking, no meaning.

For example if c be the difference between a and b the algebraic expression is $a - b = c$, when a is greater than b; or $b - a = - c$, when a is smaller than b. The expression $-c$, however, is impossible arithmetically, for a greater quantity cannot be taken from a less.

Join the negative quantity to another, as $m - c$ and the expression may be subjected to all the operations of algebra, therefore it is proper. If there is any absurdity it does not appear until the result is obtained, when it shows that some condition inconsistent with the other conditions has been admitted into the question. A negative result when it agrees with the steps of the processes in solving a problem is a proper algebraic result, for it points out the impossibility of the conditions and thus has a use, in that it places a limit on the terms of the question.

In all algebraic work it is necessary to pay close attention to the positive and negative expressions and the forms which result from them. There must be no hesitation in the operations for it has been shown how quantities, and single terms may be added, subtracted, multiplied and divided by one another and how the signs of the results are obtained. These signs do not belong to the terms taken as isolated quantities, but express the relation existing between the terms of the result.

The product of $a - b$ by $a - b = a^2 - 2\,ab + b^2$ and the product of $b - a$ by $b - a = a^2 - 2\,ab + b^2$. There is nothing in the product to indicate whether a is greater or less than b, or, looking at it in another way, if $a - b = c$, whether the product has arisen from $+c$ or from $-c$, for each of these squared $= +c^2$. The square root of $+c^2$ may be either $+c$ or $-c$ and the square root of $-c^2$ is impossible.

The formula $a^2 - b^2 = (a + b)(a - b)$ is of great service in algebra and should be memorized. Let

$$a^2 + b^2 = a^2 - b^2 \times -1 = (a + b\sqrt{-1})(a - b\sqrt{-1}).$$

The use of negative exponents often simplifies work. We can divide a^5 by a^2 by writing them in the form of a fraction $\frac{a^5}{a^2}$ and canceling like quantities, $\frac{a^5}{a^2} - \frac{a^2}{a^2} = a^3$, or merely subtract the exponent of the divisor from that of the dividend $a^{5-2} = a^3$.

Now divide a^2 by a^5. By writing the work in fractional form $\frac{a^2}{a^5} = \frac{1}{a^3}$. By subtraction, $a^2 - a^5 = a^{-3}$; so that $a^{-3} = \frac{1}{a^3}$.

In such examples the negative exponent does not represent a negative quantity, but only shows that the quantity placed in the numerator should be in the denominator. In either place it can be subjected to all the rules of algebra, but in some problems the use of the negative exponent is a convenient method for getting rid of fractions. The examples given show that any quantity may be removed from the numerator to the denominator, or *vice versa*, by changing the sign of the exponent. Thus $\frac{a^2b}{c^2} = a^2bc^{-2}$ and $ab^{-2}c^2 = \frac{ac^2}{b^2}$.

Roots may be expressed by fractional exponents, as $\sqrt{a} = a^{\frac{1}{2}}$, and fractions by negative exponents as shown, so the following expressions should be carefully studied:

$$\sqrt[3]{a^2} = a^{\frac{2}{3}},$$

$$\frac{1}{a} = a^{-1},$$

$$\frac{x}{\sqrt[4]{a^3}} = xa^{-\frac{3}{4}}.$$

In arithmetic it is often found to be convenient to reduce a common fraction to a decimal, and the negative exponent of a fraction is the result of an equivalent operation.

Notice the ease with which the following expressions are handled by the use of fractional and negative exponents.

$$\sqrt{x} \times \sqrt[3]{x} = x^{\frac{1}{2}}x^{\frac{1}{3}} = x^{\frac{3}{6}}x^{\frac{2}{6}} = x^{\frac{3}{6}+\frac{2}{6}} = x^{\frac{5}{6}} = \sqrt[6]{x^5},$$

$$\frac{\sqrt{x}}{\sqrt[3]{x}} = x^{\frac{1}{2}-\frac{1}{3}} = x^{\frac{1}{6}} = \sqrt[6]{x},$$

$$\frac{\sqrt[3]{x^2}}{\sqrt[3]{x}} = x^{\frac{2}{3}-\frac{1}{3}} = x^{\frac{1}{3}} = \sqrt[3]{x}.$$

Exponents, the student must remember, are added and subtracted, being logarithms.

SECONDARY OPERATIONS

To facilitate work a number of rules have been evolved from a study of the development of algebraic expressions when raised regularly to higher powers. An expert algebraist is thus enabled to save a great deal of time because he remembers how one factor follows another. He not only can write quickly without actual multiplication, the expansion of an expression to any degree, but conversely can "factor" expressions of any degree. By factoring is meant to reduce an expression to its elemental factors.

EQUATIONS

A statement in algebraic language that one quantity equals another is an equation.

A simple equation is one of the first degree. That is, the radical sign and exponents are not used in either the statement or the solution.

A quadratic equation is of the second degree and the square root of a quantity appearing in it must be obtained.

A cubic equation is of the third degree; a bi-quadratic equation is of the fourth degree, etc.

The degree of an equation is the degree of its highest term.

To solve an equation the following rules must be observed *in the order given.*

1st. Clear of fractions.

If any term be divided by any quantity, multiply every term by this divisor (denominator).

2nd. Write all the known quantities on one side of the equation and the unknown quantities on the other side. Then collect by addition.

Any term may be transposed from one side of an equation to the other by changing its sign from + to −, or from − to +.

If a term be found on both sides with the same sign it should be canceled in both.

3rd. If the unknown quantity be multiplied by any other, divide both sides by the multiplier.

In this way the value of an unknown quantity is found when there are no surds or powers.

4th. If the equation contains a surd (an expression of a root of an algebraic quantity which is not a complete power) bring the surd to one side by itself, take away the radical sign and raise the other side to the corresponding power.

5th. If one side of the equation be a complete power take the corresponding root of both sides.

At no time during the process of solving an equation can the sides be unequal and the foregoing rules maintain equality of the sides as follows:

Rule 1. Both sides are multiplied by the same quantity.

Rule 2. In transposition the same quantity is subtracted from both sides.

Rule 3. Both sides are divided by the same quantity.

Rule 4. Both sides are raised to the same power.

Rule 5. The same root is taken of both sides.

Solve $$2x - \frac{19}{4} = \frac{3x}{4} + 4.$$

Clear of fractions, $$8x - 19 = 3x + 16.$$

Transpose, $$8x - 3x = 16 + 19.$$

Collect, $$5x = 35.$$

Divide by 5, $$x = \frac{35}{5} = 7.$$

$$
\begin{array}{ll}
\text{Solve,} & (3x+1)^{\frac{1}{2}} + 5 = 10. \\
\text{Transpose,} & (3x+1)^{\frac{1}{2}} = 10 - 5 = 5. \\
\text{Square by Rule 4,} & 3x + 1 = 5^2 = 25. \\
\text{Transpose,} & 3x = 25 - 1 = 24. \\
\text{Divide by 3,} & x = \dfrac{24}{3} = 8.
\end{array}
$$

$$
\begin{array}{ll}
\text{Solve,} & 9x^2 + 9 = 3x^2 + 63. \\
\text{Transpose,} & 9x^2 - 3x^2 = 63 - 9 = 54. \\
\text{Collect,} & 6x^2 = 54. \\
\text{Divide by 6,} & x^2 = \dfrac{54}{6} = 9. \\
\text{Extract the root,} & x = \sqrt{9} = 3.
\end{array}
$$

It has been stated that any proportion or analogy may be turned into an equation by making the product of the first and fourth terms equal to the product of the second and third terms. This will now be illustrated.

$$
\begin{array}{ll}
 & 2 + x : 6 - x :: 15 : 9. \\
\text{Then} & 9(2+x) = 15(6-x). \\
\text{or} & 18 + 9x = 90 - 15x. \\
\text{Transpose,} & 9x + 15x = 90 - 18 = 72. \\
\text{Collect,} & 24x = 72. \\
\text{Divide by 24,} & x = \dfrac{72}{24} = 3.
\end{array}
$$

$$
\begin{array}{ll}
 & x - 5 : 2x :: 5 : 20. \\
\text{Then} & 20 \times x - 5 = 5 \times 2x \\
\text{or} & 20x - 100 = 10x. \\
\text{Transpose,} & 20x - 10x = 100. \\
\text{Collect,} & 10x = 100. \\
\text{Divide by 10,} & x = \dfrac{100}{10} = 10.
\end{array}
$$

The age of a man is a and the age of his son is b. In how many years will the father be n times as old as the son?

Solution. As time passes there will be added to each age an increment, x. The ratio n between the ages of the father and son depends upon this factor, or increment. Then

$$n = \frac{a + x}{b + x}.$$

Transpose, $$a + x = nb + nx.$$
Collect, $$nx - x = a - nb.$$
$$x(n - 1) = a - nb.$$
Divide, $$x = \frac{a - nb}{n - 1}.$$

Assume the father's age to be 26 and that of the son to be 4. When will the father be twice as old as the son?

$$x = \frac{26 - (2 \times 4)}{2 - 1} = 18 \text{ years.}$$

Proof. $$n = \frac{a + x}{b + x} = \frac{26 + 18}{4 + 18} = \frac{44}{22} = 2.$$

Problems.	*Answers.*
1. $5x + 3 = 2x + 15.$	$x = 4.$
2. $6 - x = 4 - \frac{2x}{3}.$	$x = 6.$
3. $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = 10.$	$x = 9\frac{3}{13}.$
4. $x - a = \frac{x^2}{x - a}.$	$x = \frac{a}{2}.$
5. $\frac{a}{1 + x} + \frac{a}{1 - x} = b.$	$x = \left(\frac{b - 2a}{b}\right)^{\frac{1}{2}}.$
6. $x + (a^2 + x^2)^{\frac{1}{2}} = \frac{2a^2}{(a^2 + x^2)^{\frac{1}{2}}}.$	$x = \frac{a}{\sqrt{3}}.$

Equations may also be solved by factoring.

SIMULTANEOUS SIMPLE EQUATIONS

When there are several unknown quantities there must be an independent equation for each quantity; from these an equation must be deduced which contains only one of the unknowns and this equation may be solved by the preceding rules. Three rules are in general use for solving simultaneous simple equations. It is immaterial which is used. The object is to eliminate one unknown after another until but one is left.

Rule I. This appears to be the most regular.

(*a*) *Find a value for one of the unknowns, assuming all the rest to be known.*

(*b*) *Make these values equal to one another and from them find a value for another unknown.*

(*c*) *Repeat* (*b*) *successively with the remaining unknowns until the final equation results.*

Rule II. This is a little shorter than the first, but the reductions are more intricate.

(*a*) *Find a value for one of the unknown quantities in that equation in which it is the least involved.*

(*b*) *Substitute this value and its powers for that unknown quantity and its powers in all the other equations.*

(*c*) *Proceed as in* (*b*) *with these equations to get rid of other unknown quantities.*

Rule III. This is the most simple and expeditious.

(*a*) *Multiply the equations by such quantities as will make the coefficients of one of the unknown quantities, or of its highest power, the same in all the equations.*

(*b*) *If these equal terms have like signs subtract them, but if signs are unlike add them, thus giving rise to new equations.*

(*c*) *Continue as in* (*a*) *and* (*b*) *with the new equations.*

Examples under Rule III.

1. Let the equations be $\left.\begin{array}{r} x + y = 12 \\ 5x + 3y = 50 \end{array}\right\}$. Find x and y.

First equation multiplied by 5, $5x + 5y = 60$

$$\begin{array}{r} 5x + 3y = 50 \\ \hline 2y = 10 \end{array}$$

Divide, $y = \frac{10}{2} = 5.$

$x = 12 - 5 = 7.$

2. Let the equations be $\left.\begin{aligned} x + y + z &= 53 \\ x + 2y + 3z &= 105 \\ x + 3y + 4z &= 134 \end{aligned}\right\}$. Find x, y and z.

$$\begin{array}{lrcl} \text{Second equation,} & x + 2y + 3z & = & 105 \\ \text{First equation,} & x \times y + z & = & 53 \\ \hline \quad\text{Subtract,} & y + 2z & = & 52 = \text{Fourth equation.} \end{array}$$

$$\begin{array}{lrcl} \text{Third equation,} & x + 3y + 4z & = & 134 \\ \text{Second equation,} & x + 2y + 3z & = & 105 \\ \hline \quad\text{Subtract,} & y + z & = & 29 = \text{Fifth equation.} \end{array}$$

$$\begin{array}{lrcl} \text{Fourth equation,} & y + 2z & = & 52 \\ \text{Fifth equation,} & y + z & = & 29 \\ \hline & z & = & 23 \end{array}$$

$$y = 29 - z = 29 - 23 = 6.$$
$$x = 53 - (6 + 23) = 24.$$

By substituting these values in the original equations they may be solved and a proof obtained of the accuracy of the work.

Problems.	*Answers.*
1. $5x - 3y = 90.$	$x = 30.$
$2x + 5y = 160.$	$y = 20.$
2. $\frac{x}{2} + \frac{y}{3} = 16.$	$x = 20.$
$\frac{x}{5} - \frac{y}{9} = 2.$	$y = 18.$
3. $x + y = a.$	$x = \frac{1}{2}a + \frac{b}{2a}.$
$x^2 - y^2 = b.$	$y = \frac{1}{2}a - \frac{b}{2a}.$

4. $\dfrac{3x - 7y}{3} = \dfrac{2x + y + 1}{5}.$ $\qquad x = 13.$ $\quad y = 3.$

The student must remember that the solution of equations involving fractions is as easy as the solution of equations in which no fractions appear. They merely take a little more time, and care must be exercised that nothing is missed. It has been stated before that fractions in algebra are treated exactly as fractions are treated in common arithmetic.

5. $x + 100 = y + z.$ $\qquad x = 9\dfrac{1}{11}.$

$y + 100 = 2x + 2z.$ $\qquad y = 45\dfrac{5}{11}.$

$z + 100 = 3x + 3y.$ $\qquad z = 63\dfrac{7}{11}.$

6. $x + y = a.$ $\qquad x = \dfrac{b + a - c}{2}.$

$x + z = b.$ $\qquad y = \dfrac{a + c - b}{2}.$

$y + z = c.$ $\qquad z = \dfrac{c + b - a}{2}.$

7. $\dfrac{1}{2}x + \dfrac{1}{3}v + \dfrac{1}{4}z = 62.$ $\qquad x = 24.$

$\dfrac{1}{2}x + \dfrac{1}{4}v + \dfrac{1}{5}z = 47.$ $\qquad v = 60.$

$\dfrac{1}{4}x + \dfrac{1}{5}v + \dfrac{1}{6}z = 38.$ $\qquad z = 120.$

In problems containing fractions the work is simplified by recasting some of the expressions. For example $\dfrac{1}{2}x$ may be written $\dfrac{x}{2}$ and $\dfrac{2}{3}x$ may be written $\dfrac{2x}{3}$, etc.

QUADRATIC EQUATIONS

A quadratic equation often comes to the engineer in the following form

$$M = Rbh^2,$$

then

$$h = \sqrt{\frac{M}{Rb}},$$

in which M = resisting moment (should be equal to the bending moment),
R = moment factor (unit moment of resistance),
b = breadth of beam,
h = height (depth) of beam.

For a rectangular beam the resisting moment $= \dfrac{fbh^2}{6}$ in which f = maximum fiber stress (skin stress) and

$$\frac{f}{6} = R.$$

A pure quadratic equation contains only the second power of the unknown quantity.

An affected quadratic equation contains the first and second powers of the unknown quantity.

Affected quadratic equations assume one of the following three forms:

1. $$x^2 + ax = +b$$

in which $$x = \frac{-a \pm \sqrt{a^2 + 4b}}{2}.$$

2. $$x^2 - ax = +b$$

in which $$x = \frac{+a \pm \sqrt{a^2 + 4b}}{2}.$$

3. $$x^2 - ax = -b$$

in which $$x = \frac{+a \pm \sqrt{a^2 - 4b}}{2}.$$

The student should commit the above formulas to memory.

Since the square root of $x^2 - 2\,ax + a^2$ is either $a - x$ or $x - a$, the root of the known side of the equation must have both the signs $+$ and $-$ before it. Sometimes both give the proper solutions, and at other times only one of them is proper.

If a positive answer is required the sign of the radical in (1) and (2) must be $+$, but in (3) it may be either $+$ or $-$. There is however a limitation in this case for $4\,b$ must not be greater than a^2, otherwise the quantity below the radical sign would be negative and its root impossible. This happens when the absolute term b is greater than $\frac{1}{4}a^2$, the square of $\frac{1}{2}$ the coefficient of x.

In solving an affected quadratic several methods are at the service of the mathematician. Some problems are more readily solved by one method than by another. The method best retained by the memory is that of completing the square. The methods in the order of general custom are as follows:

1. By inspection.

The sum of the roots is equal to the coefficient of the first power of the unknown quantity with the sign changed, and the product of the roots is equal to the right-hand member of the equation with the sign changed.

$$x^2 - 6\,x = 72.$$

Sum of the roots, $= +6.$

Product of the roots, $= -72.$

$$x = \frac{-72}{+6} = -12.$$

2. By factoring.

$$x^2 - 6\,x = 72.$$
$$x^2 - 6\,x - 72 = 0.$$
$$(x + 6)(x - 12) = 0.$$
$$x = 12.$$

3. By the rule.

$$x^2 - 6\,x = 72.$$

$$x = \frac{\pm 6 \pm \sqrt{6^2 + 4 \times 72}}{2} = 12.$$

4. By completing the square.

The student should work the following by actual multiplication, observe the formation carefully and memorize the results.

$$(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2.$$
$$(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2.$$
$$(a - b)(a + b) = a^2 - b^2.$$

To solve a quadratic by completing the square add to both sides the square of half the coefficient of the unknown quantity.

$$x^2 - 6x + 9 = 72 + 9 = 81.$$

In the above line the square of one-half of 6 was added to both sides of the equation.

Extract the square root of both sides.

$$x - 3 = 9.$$

Solve for x.

$$x = 9 + 3 = 12.$$

Sometimes the square of the unknown quantity has a coefficient:

$$3x^2 - 6x = 72,$$

in which case the equation must first be divided by this coefficient;

$$x^2 - 2x = 14.$$

To avoid fractions multiply the equation by 4 times the coefficient of x^2 and then add the square of the original coefficient of x.

Let the equation be $7x^2 - 20x = 32.$

Multiply by $4 \times 7 = 28,\ 196x^2 - 560x = 896.$

Add $20^2 = 400,$

$$196x^2 - 560x + 400 = 896 + 400 = 1296.$$

Extract the root, $14x - 20 = \pm 36.$

$$x = +4, \quad \text{or} \quad -1\frac{1}{7}.$$

If an equation contains two powers of the unknown quantity and the exponent of one is double that of the other it may be solved like a quadratic.

Let the equation be $x^6 - 6x^3 = 16.$
Completing the square, $x^6 - 6x^3 + 9 = 16 + 9 = 25.$
Extracting the root, $x^3 - 3 = \pm 5.$
Transposing, $x^3 = 3 \pm 5 = 8.$
Extracting the cube root, $x = 2.$

Problems.	*Answers.*
1. $8 + x^2 - 6 = 80.$	$x = 12.$
2. $\frac{x}{3} + 42\frac{2}{3} = \frac{x^2}{2} + 20\frac{1}{2}.$	$x = 7.$
3. $\frac{7}{x+1} + \frac{2}{x} = 5.$	$x = \frac{2 + \sqrt{14}}{5}.$
4. $2x^4 - x^2 = 496.$	$x = 4.$
5. $x^4 + 2ax^2 = b.$	$x = (\sqrt{a^2 + b} - a)^{\frac{1}{2}}.$
6. $x^n - 6x^{\frac{n}{2}} = e.$	$x = (18 + e \pm 6\sqrt{e + 9})^{\frac{1}{n}}.$

7. A brick pier 30 ins. wide and 42 ins. long carries a load of 450,000 pounds. Design a foundation of the same shape resting on earth having a safe bearing value of 3000 pounds per square foot. What should be the length and breadth of the footing?

$$\text{Required area} = \frac{450{,}000}{3000} = 150 \text{ sq. ft.}$$

$$\text{Ratio of sides} = \frac{42}{30} = 1.4.$$

Let $x = \text{width},$
then $1.4\,x = \text{length}.$

$$x\,(1.4\,x) = 150.$$
$$1.4\,x^2 = 150.$$
$$x^2 = \frac{150}{1.4}.$$
$$x = \sqrt{\frac{150}{1.4}} = 10.35 \text{ feet.}$$
$$1.4\,x = 1.4 \times 10.35 = 14.45 \text{ feet.}$$

Make the footing $14'\,6'' \times 10'\,4''.$

8. Find a number of which the square shall be 4 times the number together with 5 times the same number. *Ans.* 9.

9. Find two numbers of which the sum is 133 and their difference is 47. *Ans.* 90 and 43.

10. Find two numbers in the ratio of 4 to 3, so that if 1 be added to each of them, the sums shall be in the ratio of 9 to 7.
Ans. 8 and 6.

11. Divide 72 into two parts so that three times the greater shall exceed twice the less by 121. *Ans.* 53 and 19.

12. A general sends 1500 of his troops to the West and $\frac{1}{3}$ of his entire army to the East. He retains $\frac{1}{2}$ of the army after detaching 1200 to the rear guard. How many men are in the army?
Ans. 16,200.

13. A and B started out in life with equal amounts of money. A spent annually \$600 more than his income, while B saved \$800 annually. In 12 years B was twice as rich as A. What was their original capital?

$$(x - 60 \times 12) = x + 80 \times 12.$$

Ans. \$24,000 each.

14. An aeroplane going at the rate of 90 miles an hour overtakes another having a start of 180 miles, but going at the rate of 70 miles an hour. How many hours did it require?
Ans. 9 hours.

15. A cistern can be filled with water by one faucet in 12 hours and by another in 8 hours. In what time will it be filled if both run together? *Ans.* $4\frac{4}{5}$ hours.

16. Two men can together do a piece of work in 8 days. One can do it alone in 12 days. How long would it take the other?
Ans. 24 days.

17. A can do a piece of work in 50 hours; B in 60 hours and C in 75 hours. In what time can the three do it when working together?
Ans. 20 hours.

18. A and B together can do a piece of work in 12 hours; A and C together in 20 hours; B and C together in 15 hours. In what time

will they do it, all working together, and in what time will each do it separately?

$$\frac{x}{12} + \frac{x}{20} + \frac{x}{15} = 2.$$

Ans. Together in 10 hours; A alone in 30 hours; B alone in 20 hours; C alone in 60 hours.

19. The sum of two numbers = 100 and their product = 2100. What are the numbers? *Ans.* 70 and 30.

20. Find two numbers of which the difference is 8 and the product is 240. *Ans.* 20 and 12.

21. Find two numbers of which the product is 108 and the sum of their squares is 225. *Ans.* 12 and 9.

22. An oblong pond was surrounded by a road 7 yards wide, the area of the pond being 15,000 square yards and the area of the road being 3696 square yards. Give the length and breadth of the pond.

$$xy = 15{,}000, \quad \text{and} \quad 14x + 14y + 196 = 3696.$$

Ans. 100×150 yards.

23. Find two numbers of which the sum is 13 and the sum of their cubes is 637. *Ans.* 8 and 5.

24. Sold a cow for $24 and gained as much in per cent as the first cost in dollars. What was paid for the cow?

$$x + \frac{x^2}{100} = 24.$$

Ans. $20.

25. A and B started at the same time for a place at a distance of 150 miles. A travels 3 miles an hour faster than B, and beats him by $8\frac{1}{3}$ hours. At what rate did each travel?

Ans. A 9 miles per hour, B 6 miles per hour.

26. A and B distribute each $1200 among some poor families. A gave to 40 more families than B, but B gave $5 more to each family. How many families were helped by each?

Ans. 120 by A, 80 by B.

27. A traveling to Boston, overtook at the 50th milestone a band of sheep traveling at the rate of 3 miles in 2 hours; and 2 hours later met a wagon moving at the rate of 9 miles in 4 hours. B, traveling at the same rate as A, overtook the sheep at the 45th

milestone and met the wagon 40 minutes before he came to the 31st milestone. Where would B be when A reached Boston?

Let x = distance between them,
y = rate of their traveling per hour.

$$\frac{10\,y}{3} - 5 = x.$$

$$50 - 2\,y - \frac{32\,y^2}{27} + \frac{76\,y}{9} = 31 + \frac{2\,y}{3} - x.$$

Ans. $x = 25.$
$y = 9.$

Problems involving ratio, proportion and variation are handled by the rules given for solving equations.

TABLE OF FORMULAS

The accompanying table is of great value in many problems. Let x and y be any two quantities.

$s = x + y$, their sum,
$d = x - y$, their difference,
$p = xy$, their product,
$q = x/y$, their quotient,
$Z = x^2 + y^2$, the sum of their squares,
$D = x^2 - y^2$, the difference of their squares.

The use of this table will teach how to state a problem and only a few hints will be here given as to its use. The student is advised to consult it when called upon to solve problems.

EXAMPLES.

1. The sum of two numbers is 277 and their difference is 115. Find the greater.

From the table $x = \left(\frac{s + d}{2}\right) = \left(\frac{277 + 115}{2}\right) = 196.$

2. The difference of two numbers is 10 and the product is 110. Find the greater.

TABLE OF ALGEBRAIC FORMULAS

Given \ Find	x	y	s	d	p	q
s, d	$\frac{s+d}{2}$	$\frac{s-d}{2}$	....	....	$\frac{s^2-d^2}{4}$	$\frac{s+d}{s-d}$
s, p	$\frac{s+(s^2-4p)^{\frac{1}{2}}}{2}$	$\frac{s-(s^2-4p)^{\frac{1}{2}}}{2}$	....	$(s^2-4p)^{\frac{1}{2}}$	....	$\frac{s+(s^2-4p)^{\frac{1}{2}}}{s-(s^2-4p)^{\frac{1}{2}}}$
s, q	$\frac{sq}{q+1}$	$\frac{s}{q+1}$	....	$\frac{q-1}{q+1}s$	$\frac{s^2q}{(q+1)^2}$	....
d, p	$\frac{d+(d^2+4p)^{\frac{1}{2}}}{2}$	$\frac{d-(d^2+4p)^{\frac{1}{2}}}{2}$	$(d^2+4p)^{\frac{1}{2}}$	....	....	$\frac{d+(d^2+4p)^{\frac{1}{2}}}{d-(d^2+4p)^{\frac{1}{2}}}$
d, q	$\frac{dq}{q-1}$	$\frac{d}{q-1}$	$\frac{q+1}{q-1}\times d$	....	$\frac{qd^2}{(q-1)^2}$	....
p, q	$(pq)^{\frac{1}{2}}$	$\sqrt{\frac{p}{q}}$	$(q+1)\sqrt{\frac{p}{q}}$	$(q-1)\sqrt{\frac{p}{q}}$	....	....
d, D	$\frac{d^2+D}{2d}$	$\frac{D-d^2}{2d}$	$\frac{D}{d}$	....	$\frac{D^2-d^4}{4d^2}$	$\frac{D+d^2}{D-d^2}$
Z, D	$\left(\frac{Z+D}{2}\right)^{\frac{1}{2}}$	$\left(\frac{Z-D}{2}\right)^{\frac{1}{2}}$	$(Z+\sqrt{Z^2-D^2})^{\frac{1}{2}}$	$(Z-\sqrt{Z^2-D^2})^{\frac{1}{2}}$	$\frac{\sqrt{Z^2-D^2}}{2}$	$\frac{\sqrt{Z^2-D^2}}{Z-D}$

From the table

$$x = \frac{d + (d^2 + 4p)^{\frac{1}{2}}}{2} = \frac{10 + (100 + 476)^{\frac{1}{2}}}{2}$$

$$= \frac{10 + \sqrt{576}}{2} = \frac{10 + 24}{2} = 17.$$

3. The sum of the squares of two numbers is 250 and the difference of the squares is 88. Find the numbers.

$$x = \left(\frac{Z + D}{2}\right)^{\frac{1}{2}} = \left(\frac{250 + 88}{2}\right)^{\frac{1}{2}} = \sqrt{169} = 13.$$

$$y = \left(\frac{Z - D}{2}\right)^{\frac{1}{2}} = \left(\frac{250 - 88}{2}\right)^{\frac{1}{2}} = \sqrt{81} = 9.$$

4. Given the sum s of the products of two quantities, by known multipliers m and n, and also the sum of their products c by other known multipliers p and q; to find the quantities.

Here $mx + ny = s$, and $px + qy = c$; multiplying the first equation by p, and the second by m, they become $pmx + pny = ps$, and $mpx \times mqy = mc$; subtracting we get $npy - mqy = ps - mc$; and dividing by $np - mq$, we obtain

$$y = \frac{ps - mc}{np - mq}.$$

In like manner we find

$$x = \frac{qs - nc}{mq - np}.$$

5. Given the sum s of the quotients of two quantities by known divisors m and n, and also the sum c of their quotients by other known divisors p and q; to find the quantities.

Here $\frac{x}{m} + \frac{y}{n} = s$, and $\frac{x}{p} + \frac{y}{q} = c$,

then $nx + ny = mns,$ and $qx + py = pqc;$
which, resolved as in (4), gives

$$x = \frac{pm\,(ns - qc)}{pn - qm},$$

$$y = \frac{nq\,(ms - pc)}{qm - pn}.$$

Equations may be solved graphically and an interesting book on the subject is entitled, "A Graphical Method for Solving Certain Algebraic Equations," by Prof. G. L. Vose, in Van Nostrand's Science Series, 50 cts.

The foregoing rapid presentation of the essentials of algebra gives a student all that is necessary in order to have a good working tool. If his interest is aroused and he desires to proceed farther in the study of mathematics he should purchase the text on algebra used in the nearest high school and thus when hard places are encountered assistance may readily be obtained. Correspondence courses in mathematics are given by the University of Chicago, Chicago, Ill.

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